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Engineering Thermodynamics

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Preface

The purpose of this book is to present the principles of engineering thermodynamics together with some of the more important applications of these principles. It is intended as a textbook for undergraduate students and is designed for a course covering a full year of instruc- on.

The book is divided into three parts: Part I is devoted to the First Law of Thermodynamics, Part II to the Second Law, and Part III to applications of the principles and methods developed in the first two parts. It is suggested that, to preserve the method of exposition, the material in the first two parts be taken up in the order presented, although Art. 63, the last part of Art. 70, and Art. 80 may be omitted if time does not permit their inclusion. The seven chapters in Part III may be taken up in any order, and any of them may be omitted entirely since each is complete in itself.

Attention is directed to the fact that the material in Parts I and II is divided, not according to the kinds of systems considered, but according to the thermodynamic principles involved. In Chapter I, for example, the relation among the pressure, specific volume, and temperature of a perfect gas is developed; and in the same chapter the relation among these properties of actual fluids, such as steam, is considered. As another example, in Chapter 5 methods are developed for determining the heat and work transferred when any fluid, actual or ideal, undergoes various simple processes. This arrangement of the subject matter makes it possible to confront students in a single assignment with some problems involving perfect gases and with others which are similar but which involve actual fluids. Students can thereby be encouraged to think of thermodynamics, not as a subject consisting of perfect gas problems on the one hand and of *Steam Table* problems on the other, but as a subject involving only a few basic principles.

On the basis of some fifteen years' experience in teaching thermodynamics, I have concluded that one of the most effective devices for helping students understand new principles or methods is to require them to solve problems in which they must apply these principles or methods. For this reason I have inserted problems after more than half the articles in the book. These problems are graded in difficulty. The first ones in each list are simple, but others are likely to tax, and thereby help to develop, the students' powers of analysis. I have also inserted a list of general problems at the end of each chapter in Parts I and II.

An abridgment of the Bureau of Standards' *Tables of Thermodynamic Properties of Ammonia* is presented in the Appendix. This abridgment is sufficiently complete to permit students to solve any of the problems which involve ammonia. To be able to solve the problems which involve steam, each student should have a copy of *Thermodynamic Properties of Steam*, by Joseph H. Keenan and Frederick G. Keyes.

In writing this book I have consulted many of the standard books on thermodynamics and should like to acknowledge my indebtedness to them. I should like in particular to acknowledge the help I obtained from the works of Keenan, Zemansky, and Kiefer and Stuart. I wish also to acknowledge my special indebtedness to Professor C. R. G. Dougherty, of Rutgers University, who studied the entire manuscript and made many valuable suggestions and criticisms. I should like to thank the reviewers, known only to the publishers, who read all or part of the manuscript and who assisted me materially by their comments. It is a pleasure also to acknowledge the help of Mr. Clarence R. Thompson and Mr. Jerrold Peace, who prepared most of the illustrations. Finally, I should like to acknowledge the very great help given me by my wife. She helped me by her interest and encouragement and spent many hours assisting me on matters of English and expression.

Although I have tried earnestly to eliminate errors, no doubt some remain. If users of the book will notify me of those they discover, I shall appreciate their kindness.

H. J. Stoever

Ames, Iowa
April, 1951

Contents

Introduction 1

PART I. THE FIRST LAW OF THERMODYNAMICS

1. Pressure, Specific Volume, and Temperature 7
2. Mechanical Work and Heat 35
3. The First Law of Thermodynamics 61
4. The Internal Energy and Enthalpy of Fluids 87
5. The Analysis of Simple Processes 114

PART II. THE SECOND LAW OF THERMODYNAMICS

6. Power Cycles and Refrigeration Cycles 151
7. The Second Law of Thermodynamics 176
8. The Thermodynamic Scale of Temperature 187
9. Entropy 197

PART III. APPLICATIONS

10. Steam Power Plants 243
11. Internal-Combustion Engines 268
12. Refrigeration 290
13. Compressors 321
14. Nozzles and Turbines 345
15. Venturi Meters, Flow Nozzles, Orifices, and Pitot Tubes . . 379
16. Air Conditioning 397

Appendix. Thermodynamic Properties of Ammonia 441

Index 449

INTRODUCTION

In beginning the study of any subject, it is helpful to know what the subject deals with and to learn what its principal objectives are. Accordingly, the purpose of this Introduction is to explain briefly what the science of **thermodynamics** deals with and to state some of the principal objectives of that branch of the general science known as **engineering thermodynamics.**

Let us consider a given mass of any substance. The condition, or *state*, of the substance at any instant may be adequately described for some purposes by stating its length, width, and height. For other purposes an adequate description may require that its pressure, specific volume, temperature, velocity, and elevation be stated. For still other purposes it may be necessary to state the position and the velocity of each of the molecules making up the substance. These various characteristics describing the state of a substance are called *properties*. Thermodynamics is concerned, first, with properties of substances. It is concerned, however, only with those properties which can in general be directly measured. For example, thermodynamics has to do with the pressure, specific volume, temperature, velocity, and elevation of substances, all of which can be directly measured. It is not concerned with the positions and velocities of individual molecules, which cannot be directly measured.

Consider next methods by which a substance can be caused to change from one state to another. If the substance undergoes a change of state, that is, if at least one of its properties changes, it is said to undergo a *process*. It can be caused to undergo a process in various ways. One way is to let some external agent exert on it a force which acts through a distance. As a result of this action, the substance might be compressed, stirred, accelerated, or elevated. *Mechanical work* is said to have been done on it. Instead of mechanical work, the agent might do electrical or magnetic work on it. Or, the substance can also be caused to undergo a process by bringing it into contact with some other substance whose temperature is higher or lower. If the change in state is caused by this difference in temperature, *heat* is said to have flowed from the one substance to the other. Thermodynamics is concerned, second, with the quantities work and heat.

INTRODUCTION

In brief, thermodynamics is the science that deals with the properties of substances and with the quantities work and heat, which can cause a substance to change from one state to another. It is based on two fundamental postulates known as the First Law and Second Law of Thermodynamics. The first of these is the Principle of the Conservation of Energy. The second may be stated in various ways. One way has to do with the fact that heat will not flow spontaneously from a substance at one temperature to another at a higher temperature. It is of interest to know that as a consequence of the First Law an additional property called *internal energy* is discovered, and that as a consequence of the Second Law still another property called *entropy* is discovered.

Engineering thermodynamics is the name frequently given to those aspects of the general science that are of particular interest to the engineer. It is to this branch of thermodynamics that this book is devoted. Engineering thermodynamics is characterized by the facts that (*a*) only work of the mechanical type is considered, and (*b*) only substances that undergo no changes in chemical composition and that are subject to no surface, electric, or magnetic effects are considered.

One objective of engineering thermodynamics is to develop methods for determining the changes that will take place in the state of a substance when work or heat is added to or removed from it. Another objective is to develop methods for determining how much work or heat must be added to or removed from a substance in order to produce a given change in its state. The development of these methods is based on the First Law of Thermodynamics and is considered in Part I. Some practical applications of these methods include the determination of the heat required to generate steam in a boiler, the work done by steam in flowing through a turbine, the work required to compress air in a compressor, and the heat that must be removed from atmospheric air to cool and dehumidify it.

Another objective of engineering thermodynamics has to do with the conversion of heat into work. Suppose that a substance undergoes a series of processes such that its final state is identical with its initial state. The substance is said to undergo a *cycle*. Since its final state is the same as its initial state, the amount of energy stored within the substance in its final state is the same as the amount stored within it initially. In accordance with the First Law, therefore, the amount of energy added to the substance during the cycle must be exactly equal to the amount removed. Now, if all the energy added is in the form of heat, some of the energy which leaves, *but not all,* might be in the form of work; that is, the net result might be that heat is partially converted into work. Methods, based on the Second Law of Thermodynamics, for determining how complete the conversion of heat into work can be made are developed in Part II of the book. This information is of value to the designer of power-generating equipment, such as steam power plants and internal-combustion engines.

Still another objective of engineering thermodynamics has to do with refrigeration. If a substance undergoes a cycle, it may absorb heat at a low temperature and reject heat at a higher temperature. If this takes place, it is always found that some work (or its equivalent) has been supplied to the substance during the cycle. Methods, again based on the Second Law, are developed in Part II for determining how much work must be supplied to effect the transfer of a given amount of heat from a low temperature to a higher temperature region. This information is of value to the designer of refrigeration equipment.

A detailed analysis of various types of engineering equipment to which the science of thermodynamics is applicable is presented in Part III. This part of the book includes a discussion of the processes that take place in steam power plants, internal-combustion engines, refrigeration plants, compressors, nozzles and turbines, certain flow-measuring devices, and air-conditioning equipment.

PART I

THE FIRST LAW OF THERMODYNAMICS

Chapter 1

PRESSURE, SPECIFIC VOLUME, AND TEMPERATURE

1. A System and Its Surroundings. In thermodynamics the term **system** is used to designate any portion of matter that is separated from its surroundings by either real or imaginary boundaries. The term **surroundings** is used to designate everything that is outside these boundaries but that affects the behavior of the system. To illustrate, let us consider the apparatus shown in Fig. 1. The apparatus consists of a cylinder equipped with a piston upon which weights are placed. If the system is considered to be the contents of the cylinder, a pound of any gas for example, the boundaries of the system are then the cylinder walls and the piston face. The behavior of the system is affected by the weight of the piston itself, the weights placed on the piston, and the atmospheric pressure acting on the upper face of the piston. If the cylinder rests upon a body that is hotter or colder than the system, and if the bottom of the cylinder is made, say, of copper, the behavior of the system is also affected by this body. Under these conditions, therefore, the surroundings are the piston, the weights, the atmosphere, and the body upon which the cylinder rests.

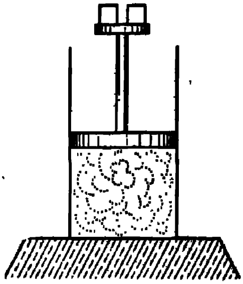

FIG. 1.

The cylinder in the foregoing example might contain some other fluid, such as a pure liquid, a pure liquid and its vapor,* the vapor alone, or a mixture of various gases or liquids. These are the usual systems considered in engineering thermodynamics. Other systems, such as solids, surface films, electric cells, stretched wires, electric capacitors, and magnetic substances, are considered in chemical or in physical thermodynamics.

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* A sharp distinction between the terms *gas* and *vapor* will not be made. However, if both liquid and gaseous states of a substance are likely to be considered, the substance will usually be referred to as a *vapor* when it is in the gaseous state. If only gaseous states of the substance are considered, the substance will be called a *gas*.

2. State, Property, and Phase. Certain quantities are used in thermodynamics to describe the **state**, or condition, of a system. These quantities are called **properties.** They are any observable characteristic of the system. *Pressure, specific volume* (or its reciprocal, *density*), and *temperature*, for example, are all properties of the system.

Consider again the apparatus shown in Fig. 1, and suppose that the system confined within it consists of a quantity of air. The state of the air may be described by specifying its pressure, specific volume, and temperature. If one or more of these properties are caused to change by any means, the air is said to undergo **a change of state.** For example, if another weight is added to the piston, the air will be compressed; its pressure, specific volume, and temperature will all change; and the air will reach a new state.

The air can be caused to undergo a change of state also by other means. For example, the weights may be left unchanged, but the cylinder may be brought into contact with another body that is hotter or colder than the air. The air will then reach a state in which its pressure is the same as in the original state but its specific volume and temperature are different. There are many other ways by which at least one property of a system can be caused to change and the system thereby be caused to undergo a change of state.

The properties pressure, specific volume, and temperature are of particular interest since they are frequently the only ones needed to describe the state of a system. Under some conditions other properties, such as elevation and velocity, may be needed to give an adequate description. There are still other properties, such as shape and color, that are never of any interest in thermodynamics since they have no bearing on the heat added to or the work performed by the system. The heat required to raise the temperature of 1 pound of water from one temperature to another, for example, does not depend upon the shape of the container in which the water is heated.

A substance may exist as a solid, liquid, or gas. These various forms of the substance are called **phases.** If a substance undergoes a change of state during which it passes from the liquid to the vapor phase, it is said to **evaporate.** The change from the vapor to the liquid phase is known as **condensation**; the change from the liquid to the solid phase, as **freezing**; the change from the solid to the liquid phase, as **melting**; and the change from the solid directly to the vapor phase, as **sublimation.** A substance may, of course, undergo a change of state without undergoing a change of phase.

3. Pressure. The pressure of a system is defined as the force that the system exerts on a unit area of its boundaries. It may not be constant over all parts of the boundaries. For example, suppose that the system consists of a quantity of water, and suppose that the water is confined inside the apparatus shown in Fig. 1. Because of the effect of gravity, the pressure of the water at the bottom of the cylinder is greater than at the top. If the piston face is, say, 1 ft above the bottom of the cylinder, the pressure of the water at the bottom of the cylinder is greater than at the top by an amount equal to the weight of a column of water 1 ft high. If the water is at room temperature, this amounts to 62.4 psf or $62.4/144 = 0.433$ psi. Variations in pressure due to gravity depend upon the density of the fluid and are small if the density of the fluid is low. For the sake of simplicity, variations in pressure due to this effect will usually be neglected.

It may be noted that, in order for pressure to serve as one of the properties for describing the state of a system, the system must be large enough to consist of many molecules. This is true since, according to the kinetic theory of matter, the force exerted by any fluid on the walls of its container is due to the bombardment of these walls by the molecules of the fluid. If the system were so small that it consisted of only a few molecules, the force exerted by the system on its boundaries would be finite at each point where a molecule strikes and zero at all other points. For such a system the pressure would be continuous with neither time nor distance and would be of no value in describing the state. On the other hand, if the system is large enough to consist of many molecules, the *average* rate of bombardment on each small part of the container is constant, and therefore the pressure on this small part is also constant. By a small part of the container is meant a part that is small compared to the size of the system but large compared to the size of and distance between individual molecules. Since pressure is one of the properties used in thermodynamics to describe the state of systems, the systems considered are necessarily only those consisting of many molecules.

In engineering, pressures are designated as gage, vacuum, or absolute, depending upon the basis on which they are measured. A Bourdon tube gage, for example, measures the *difference* between the pressure inside the vessel to which it is attached and atmospheric pressure. If the pressure inside is higher than atmospheric pressure, the gage reading is called the **gage pressure.** Such pressures can be converted to **absolute pressures** by adding atmospheric pressure to them. A pressure of 100.0 psi gage, for instance, is the same as 114.7 psi absolute (psia) if the atmospheric pressure is 14.7 psia. If the pressure inside the vessel is less than atmospheric pressure, the difference is called the **vacuum.** The absolute pressure inside the vessel is determined by subtracting the vacuum from atmospheric pressure. For example, if a mercury manometer shows that the pressure inside a vessel is 10.00 in. Hg less than atmospheric pressure, it may be referred to as a 10.00 in. Hg vacuum; or, if atmospheric pressure is 29.92 in. Hg abs, it may be referred to as a pressure of (29.92 − 10.00 =) 19.92 in. Hg abs.

4. Specific Volume and Density. The **specific volume** of a system is defined as the volume of a unit mass of the system. The **density** of a system is defined as the mass of a unit volume; thus, density is the reciprocal of specific volume. To illustrate, 1 cu ft of water at room temperature has a mass of 62.4 lb. Therefore, its density is 62.4 lb per cu ft, and its specific volume is $1/62.4 = 0.01606$ cu ft per lb.

Since specific volume and density are used in thermodynamics to describe the state of systems, it is again necessary that the systems considered be large enough to consist of many rather than few molecules. Otherwise, these properties would vary discontinuously, depending upon the number of molecules present within each small element of the system. By considering only systems that consist of many molecules, the number within each small element (small compared to the size of the system) is large; therefore, each will contain the same number of molecules, or the number will vary continuously from point to point. Hence, the specific volume and density will be constant or will vary continuously, and they may be used to describe the state of the system. Although they may not actually be constant throughout the system because of the effect of gravity, for the sake of simplicity they will usually be assumed to be so.

5. Temperature. Two systems are said to be at the same **temperature** if on being brought into contact with each other no change takes place in either. For example, suppose that one of the systems is a quantity of air confined within the apparatus shown in Fig. 1, and suppose that the other is a block of iron. If the cylinder is brought into contact with the iron, and if the weights on the piston are left unchanged, the pressure of the air will remain constant, but its volume may change. If its volume remains constant, the air and the iron are at the same temperature. But, if the volume of the air changes, the air and the iron are initially not at the same temperature. Note that the foregoing is a definition, not of temperature itself, but of the conditions that must be satisfied in order to have **equality of temperature.**

Next, suppose that the cylinder is brought into contact first with a block of iron and then with a block of copper and that the air undergoes no change in volume either time. The temperatures of the air and of the iron are therefore equal, and likewise the temperatures of the air and of the copper are equal. Experience shows that, if the iron and the copper are each at the same temperature as the air, they are at the same temperature as each other; that is, if the iron and the copper were brought into contact with each other, no change would take place in either. This fact, that two systems at the same temperature as a third system are also at the same temperature as each other, allows a **scale of temperature** to be defined.

Suppose, for example, that the cylinder containing the air is brought into contact with a cake of melting ice at atmospheric pressure. If the air is not at the same temperature as the ice initially, some of the ice will melt or some of the water will freeze. After a sufficient interval of time has elapsed, however, no further changes will take place either in the ice or in the air, and they will be at the same temperature. Let the position of the piston face be marked on the cylinder, and let this position be designated by an *arbitrarily* chosen number, say 32°. Suppose that the cylinder, with the weights on the piston left unchanged, is next brought into contact with steam condensing at atmospheric pressure. Again, if the air and the steam are not at the same temperature, some of the steam will condense or some of the condensate will evaporate, but after an interval of time no further changes will take place. The air and the steam will then be at the same temperature. Let the position of the piston face again be marked on the cylinder, and let this position also be designated by some arbitrarily chosen number, say 212°. If the distance between the two marks is divided into (212 − 32 =) 180 *arbitrarily* chosen divisions, and if each dividing mark is numbered consecutively from 32° to 212°, an instrument for measuring temperature will have been devised, and a scale of temperature will have been defined. Such an instrument is called a **thermometer,** and by means of it the temperatures of other systems can be compared without the necessity of bringing them into contact with each other.

Perhaps the simplest way of dividing the distance between the 32° and the 212° marks would be to divide it arbitrarily into 180 *equally* spaced divisions. Regardless of the method of division, the marks could be extended both above the 212° mark and below the 32° mark, so that the instrument could be used to measure a wide range of temperatures.

It is interesting to note that, since the choice of 32° for the temperature of the ice and of 212° for the temperature of the steam is entirely arbitrary, these values could have been reversed. Had this been done, the temperature of molten steel, for example, would be "lower" than the temperature of liquid air. It is only because the temperature of condensing steam has arbitrarily been chosen as higher then the temperature of ice that "higher" temperatures are associated with hotter bodies and "lower" temperatures with colder ones.

6. Thermometers. The apparatus described in the preceding section could be called a **constant pressure gas thermometer.** The gas used in it is called the **thermometric fluid,** and the volume of the gas is called the **thermometric property.** Other thermometers can be devised by using other fluids for the thermometric fluid and by using other properties for the thermometric property. Note that, in order for any thermometer to yield reproducible values, its thermometric fluid must be constrained to act always in the same way. The gas in a constant pressure gas thermometer, for example, is constrained in such a way that its pressure always remains constant.

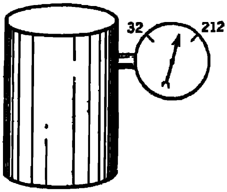


FIG. 2.

The apparatus shown in Fig. 2 could also be used as a thermometer. It consists of a rigid vessel provided with a pressure gage, and the thermometric fluid confined within it could be any gas. The pressures that the gas will attain when the vessel is brought into contact first with melting ice and then with steam condensing at atmospheric pressure may be marked on the gage, and these two marks may be designated as 32° and 212° respectively. The distance between the two marks may be divided in any way into 180 divisions, and the dividing marks may be numbered consecutively from 32° to 212°. This thermometer could be called a **constant volume gas thermometer.** Note that the gas used in it is constrained in such a way that its volume remains constant, and that its thermometric property is pressure.

If the two thermometers described so far are brought into contact with a system which is at the temperature of melting ice, both will indicate a temperature of 32°. If they are brought into contact with a system at the temperature of condensing steam, both will read 212°. However, if they are brought into contact with a system at any other temperature, it would be only a coincidence if both indicated exactly the same value for the temperature. This is true because the method used to subdivide the volume scale on the one thermometer and that used to subdivide the pressure scale on the other are arbitrarily selected. Even if both scales were arbitrarily divided into 180 *equal* divisions, the two thermometers would not necessarily agree at any temperature other than at the 32° and 212° values. For example, suppose that the constant pressure gas thermometer is brought into contact with a system that causes it to indicate a temperature of 122°; that is, the piston face rises to a position exactly halfway between the 32° and 212° marks.

In order that the constant volume gas thermometer also read 122° when brought into contact with the system, the gas in it would have to assume a pressure exactly halfway between the pressures corresponding to the 32° and 212° marks. There is no reason to expect that this will happen.

Consider what the effect would be on either thermometer if the thermometric fluid were changed. Suppose, for example, that the air in the constant pressure gas thermometer (Fig. 1) is replaced by carbon dioxide gas, and that just enough of the carbon dioxide is used to bring the piston face to the 32° mark when the cylinder is in contact with melting ice; that is, the carbon dioxide thermometer is made to agree with the air thermometer at one temperature by controlling the amount of carbon dioxide introduced into the cylinder. Now, if the marks on the cylinder are left unchanged, and if the thermometer is brought into contact with some substance other than melting ice, there is no reason to expect that the thermometer will indicate the same temperature as it would if it still contained air. For example, if the thermometer containing carbon dioxide is brought into contact with condensing steam, there is no reason to expect that the piston face will rise to exactly the same level that it did when it contained air. In fact, it will not.

Similarly, if the gas in the constant volume gas thermometer (Fig. 2) is replaced by some other gas, the thermometer could be made to read 32° when in contact with melting ice by using just the right amount of gas. But, if the marks on the gage are left unchanged, and if the thermometer is brought into contact with some substance other than melting ice, there is no reason to suppose that the thermometer will indicate the same temperature as it would if it still contained the first gas.

Summarizing these comments, we are led to the conclusion that the numerical value of the temperature of any system depends upon (*a*) the kind of thermometer used to measure it, (*b*) the kind of thermometric fluid used, and (*c*) the method arbitrarily selected for specifying the relation between the thermometric property of the fluid and the temperature.

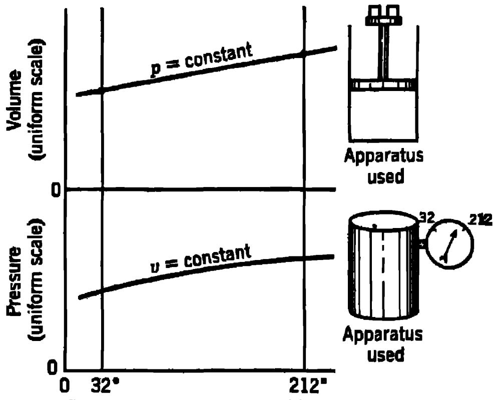
One way of making all thermometers agree at all temperatures would be to select any one kind of thermometer and one thermometric fluid as a standard, to agree on some arbitrarily chosen method of specifying the relation between the thermometric property of this fluid and the temperature, and then to calibrate all other thermometers by comparing them with this standard thermometer. This procedure is actually used. It will be discussed further in Art. 68.

There are many other kinds of thermometers. Perhaps the most familiar is the **liquid-in-glass thermometer.** The thermometric fluid usually used in this kind of thermometer is either mercury or alcohol, and the thermometric property is volume. The changes in volume are made easily visible by letting the liquid expand into a long capillary tube.

Another common type of thermometer is the **thermocouple.** These thermometers consist of two wires made of different metals connected at the ends. When the two junctions are at different temperatures, a difference in electrical potential is set up, and this emf is used as the thermometric property.

A type of thermometer frequently used in physical laboratories where precise measurements of temperature are necessary is the **platinum resistance thermometer.** These thermometers use the electrical resistance of platinum wire as the thermometric property.

7. The Absolute Perfect Gas Scale of Temperature. Consider again the constant pressure gas thermometer (Fig. 1). Let the volumes attained by the gas when the cylinder is brought into contact first with melting ice and then with steam condensing at atmospheric pressure be plotted as ordinates against temperature as abscissa, as shown in the upper part of Fig. 3. Uniform scales (that is, scales with equally spaced graduations) are used for both axes, and the temperatures of the ice and steam are again arbitrarily designated as 32° and 212° respectively. If a straight line is drawn through the two points obtained, the relation between temperature and volume, represented by this line, is the same as that established by dividing the distance between the 32° and 212° marks on the cylinder itself into 180 *equal* divisions.



Temperature, as determined by a *constant pressure* gas thermometer (uniform scale)

FIG. 3.

Now consider the constant volume gas thermometer (Fig. 2), and suppose that the same gas is used in it as in the constant pressure gas thermometer. If the constant volume gas thermometer is brought into contact with various systems, and if the pressures attained are plotted against the temperatures of these systems *as determined by the constant pressure gas thermometer*, a curve such as the one shown in the lower part of Fig. 3 might be obtained. Uniform scales are used for both the pressure and temperature axes, the scale of the temperature axis being the same as that of the temperature axis in the upper part of the figure. If the relation between pressure and temperature represented by this curve is marked on the pressure gage of the constant volume gas thermometer, the marks on the gage will not be equally spaced. Thus, if the two thermometers are graduated in such a way that they read the same, and if the graduations on one of them are equally spaced, the graduations on the other will in general not be equally spaced.

It has been found by experiment that, if the gas in these thermometers is at a pressure approaching zero, the curve shown in the lower part of Fig. 3 becomes a straight line. Therefore, *if the gas in these thermometers is any gas at a pressure approaching zero, and if the graduations on each thermometer are equally spaced, both thermometers will indicate the same value for the temperature of any system.*

The question might be asked, how is it possible to measure temperatures with either of these thermometers if the pressure of the thermometric gas approaches zero? This could be done as follows: Consider first the constant pressure gas thermometer. Let the volume of the gas be measured when the cylinder is brought into contact (*a*) with melting ice, (*b*) with steam condensing at atmospheric pressure, and finally (*c*) with the system whose temperature is to be determined; and let these measurements be made with the gas at pressures of, say, 10, 20, and 30 psia. These various pressures could be obtained by changing the number of weights on the piston. The results of these measurements, when plotted on a volume-temperature diagram, might appear as shown in Fig. 4. The constant pressure lines in this figure are drawn by first plotting the volumes of the gas when the cylinder is in contact with the ice and with the steam and then connecting these points with straight lines. The volumes attained when the cylinder is in contact with the system whose temperature is to be measured are then marked on these lines. As shown on the figure, the temperature indicated by the thermometer will be found to depend upon the pressure of the thermometric gas. Now, if these values of the temperature are plotted against pressure, as shown in Fig. 5, and if the curve through the resulting points is extrapolated to zero pressure, the desired value of the temperature corresponding to zero pressure of the thermometric gas is obtained. The temperature need not, of course, be between 32° and 212°. A similar procedure could be used to determine the temperature that

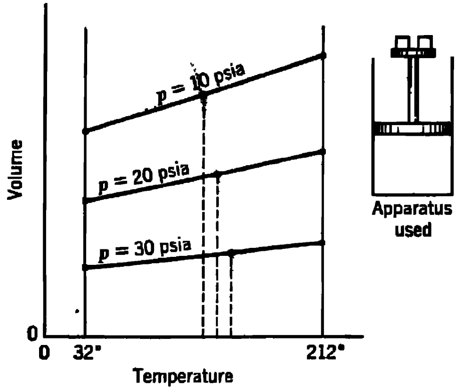


Fig. 4.

a constant volume gas thermometer will indicate as the pressure of its thermometric gas approaches zero.

Now suppose that the two types of thermometers contain the same gas at some finite pressure and that they are again brought into contact first with melting ice and then with steam condensing at atmospheric

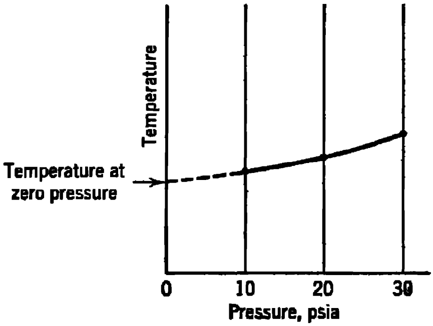


Fig. 5.

pressure. If, as shown in Fig. 6, the volumes and pressures attained are plotted as ordinates against temperature as abscissa, and if straight lines are drawn through the two pairs of points, it will be found in general that these lines intersect the temperatures axes at different points.

Repeating the experiment at other pressures and specific volumes, or with other gases, will produce still other points of intersection. It has been found by experiment, however, that, as the pressure of the thermometric gas is made to approach zero, the straight lines for both types of thermometers, and for all gases, intersect the temperature axis at the *same* point. This point of intersection is found to be at −459.69°, or, for simplicity in writing, at approximately −460°.

In summary, it has been found by experiment that, if any actual gas at a pressure approaching zero is used as the thermometric fluid in a constant pressure and in a constant volume gas thermometer, and if uniform scales are used on each, (*a*) both thermometers will indicate the same value for the temperature of any system, (*b*) the curved line shown in the lower part of Fig. 3 will become a straight line, and (*c*) the extensions of the straight lines for the two types of thermometers will both intersect the temperature axis at approximately −460°. These results are shown graphically in Fig. 7.

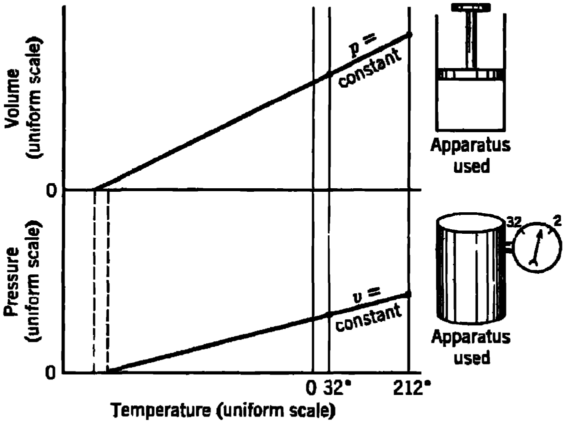


Fig. 6.

It is evident from Fig. 7 that, for any gas at a pressure approaching zero,

$$\left(\frac{v_2}{v_1}\right)_{p=\text{const.}} = \left(\frac{p_2}{p_1}\right)_{v=\text{const.}}, \tag{1}$$

where p_1 and v_1 are the pressure and volume of the gas at any one temperature, and p_2 and v_2 are the pressure and volume at any other temperature. No actual gases obey this relation at all pressures, although all gases obey it approximately at moderate pressures. The hypothetical gas which obeys it at all pressures is called a **perfect gas.***

The temperatures indicated by either a constant pressure or a constant volume gas thermometer using a perfect gas as the thermometric fluid (or using any actual gas at a pressure approaching zero) are called **perfect gas scale temperatures.** Temperatures on this scale will be represented by a small letter t. Temperatures indicated by the thermometers ordinarily used in engineering work, such as the mercury-in-glass thermometer and the thermocouple, usually agree with perfect gas scale temperatures to within a fraction of a degree. If the same uniform scale is used for the temperature axis of Fig. 7, but if the zero value of temperature is moved down to the point where the straight lines on this figure intersect the temperature axis, the resulting scale is called the **absolute perfect gas scale of temperature.** Temperatures on this scale will be represented by a capital letter T.

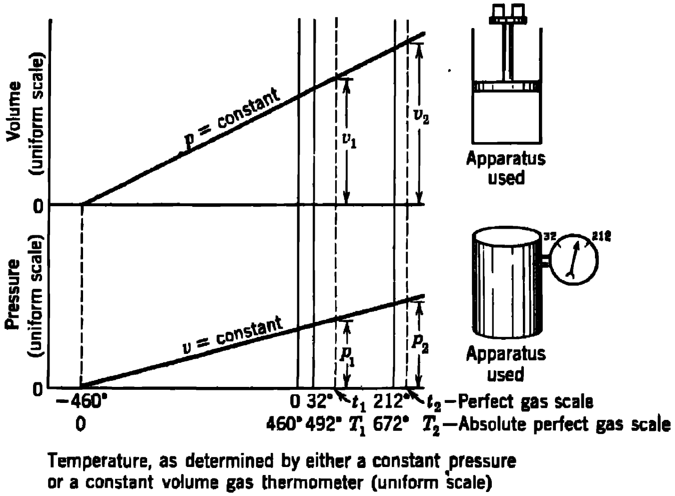

Volume (uniform scale)
0
Pressure (uniform scale)
0
p = constant
v = constant
v_1
v_2
p_1
p_2
Apparatus used
32
212
Apparatus used
−460°
0
0 32° t_1 212° t_2 —Perfect gas scale
460° 492° T_1 672° T_2 —Absolute perfect gas scale
Temperature, as determined by either a constant pressure or a constant volume gas thermometer (uniform scale)

FIG. 7.

The student might ask why temperatures measured on the absolute perfect gas scale are called "absolute" temperatures. This name would be logical if it were known that the zero value on this scale is the lowest temperature possible. This happens to be true, although it cannot be proved until after the Second Law of Thermodynamics has been introduced. The fact that temperatures lower than zero on the absolute perfect gas scale are impossible will be proved in Chapter 8.

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* A second relation which must be obeyed in order for a gas to be a "perfect gas" is introduced in Art. 35.

8. Fahrenheit and Centigrade Temperatures. In the three preceding articles the temperatures of melting ice and of steam condensing at atmospheric pressure were arbitrarily designated, by way of example, as 32° and 212° respectively. If this is done, the resulting gas scale of temperature is known as the **Fahrenheit** scale, and values measured on this scale are indicated by the letter F. The corresponding absolute gas scale (for which the *difference* between the temperature of the steam and the ice is arbitrarily chosen as 180°) is called the **Fahrenheit absolute** or the **Rankine** scale of temperature. It is evident from Fig. 7 that these two scales are related by the equation

$$T_F = t_F + 460.$$

If, instead, the temperatures of melting ice and of steam condensing at atmospheric pressure are arbitrarily designated as 0° and 100° respectively, the resulting gas scale of temperature is known as the **Centigrade** scale. Values measured on this scale are indicated by the letter C. The corresponding absolute gas scale (for which the difference between the temperature of the steam and the ice is arbitrarily chosen as 100°) is called the **Centigrade absolute** or the **Kelvin** scale of temperature. If the temperatures of melting ice and of condensing steam are marked 0° and 100° on Fig. 7, the straight lines are found to intersect the temperature axis at approximately $-273°$. Hence, the relation between the Centigrade and the Centigrade absolute scales is

$$T_C = t_C + 273.$$

Values on the Fahrenheit scale can be readily converted to the Centigrade scale, and vice versa, by noting that 180 degrees on the Fahrenheit scale are equal to 100 degrees on the Centigrade scale. Hence,

$$t_F = 1.8t_C + 32,$$

and

$$t_C = (t_F - 32) \div 1.8.$$

The relation between the Fahrenheit absolute scale and the Centigrade absolute scale is

$$T_F = 1.8T_C.$$

9. Equations of State. The usual systems considered in engineering thermodynamics are of two types: (a) single chemical substances in the form of a liquid, a vapor, or a mixture of a liquid and its vapor, and (b) mixtures of various gases. Systems of these types are considered, for example, in the analysis of such engineering equipment as the following: steam power plants, in which the system is water in the form of a liquid, a vapor, or a mixture of liquid and vapor; refrigeration plants, in which the system is some refrigerant, such as ammonia, again in the form of a liquid, a vapor, or a mixture of liquid and vapor; compressors, in which the system is either a single gas, such as oxygen, or a mixture of gases, such as air; nozzles and orifices, in which the system might be either a single gas or a mixture of gases; and air-conditioning equipment, in which the system is a mixture of water vapor and air. None of the foregoing systems is subject to any surface effects, such as capillarity, or to any electric or magnetic effects. Under these circumstances, and the additional condition that the system be at rest, it has been found by experience that *only two properties are required to specify the state of the system.* In other words, if two properties of the system are specified, all the other properties of the system are fixed and may not be arbitrarily selected. Suppose, for example, that the system is 1 lb of air. We might specify that the pressure and temperature of the air are to be, say, 100 psia and 80°F; but then we may not specify arbitrarily what the specific volume is to be. Experiments show that at this pressure and temperature the specific volume of air is always 2.00 cu ft per lb. Similarly, we might specify that the temperature and specific volume are to be 80°F and 2.00 cu ft per lb; but then the pressure will be 100 psia and may not be selected arbitrarily.

The fact that only two properties are required to specify the state of a system under the aforenamed conditions permits a simple graphical representation of any state of a system to be made; namely, if any two properties, such as pressure and specific volume, are used as coordinates, each point on the resulting diagram represents one state of the system.

In general, the two properties used to specify the state of any of the systems ordinarily considered in engineering thermodynamics may be *any* two, although there are some exceptions in which certain pairs of properties are not independent of each other. For example, water at atmospheric pressure will boil only at 212°F. Since at this pressure and temperature it can exist as a liquid, a vapor, or a mixture of liquid and vapor, this particular pair of properties does not serve to identify the state of the system. The point on a pressure-temperature diagram, determined by this pair of properties, represents all these states.

An equation stating the relationship that exists among the three properties of any system, pressure, specific volume, and temperature, is called an **equation of state** or a **characteristic equation** for that system. Such equations cannot be deduced from theoretical considerations alone but must be determined by experiment. Equations of state are available for most fluids of importance in engineering.

10. The p-v-T Relation for Perfect Gases. A perfect gas is defined in Art. 7 as one that obeys the relation

$$\left(\frac{v_2}{v_1}\right)_{p=\text{const.}} = \left(\frac{p_2}{p_1}\right)_{v=\text{const.}} \tag{1}$$

where v_2/v_1 is the ratio of the specific volumes assumed by the gas at any two temperatures when its pressure is kept constant, and p_2/p_1 is the ratio of the absolute pressures exerted by the gas at the same two temperatures when its volume is kept constant. It is evident from Fig. 7 that we may also write, for a perfect gas,

$$\left(\frac{v_2}{v_1}\right)_{p=\text{const.}} = \frac{T_2}{T_1} \tag{2a}$$

and

$$\left(\frac{p_2}{p_1}\right)_{v=\text{const.}} = \frac{T_2}{T_1}, \tag{2b}$$

where T_1 and T_2 are the two temperatures of the gas, measured on the absolute perfect gas scale It has already been stated that no actual gases obey these relations exactly but that all gases obey them approximately at moderate pressures The approach to perfect gas behavior improves as the pressure of the gas is decreased and as its temperature is increased Under the conditions ordinarily encountered in engineering, the deviation for most gases is small enough that for many purposes they may be assumed to be perfect gases. Consequently, the equation of state for perfect gases is of considerable value in solving many problems involving actual gases. This equation can be derived from Eqs. 2a and 2b as follows:

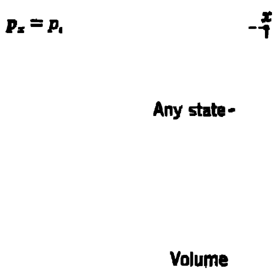

FIG. 8.

Consider first the state of the gas represented by point a on the pressure-volume diagram shown in Fig. 8. Although the temperature does not appear on this diagram, there is only one absolute temperature T_a that the gas can have when its pressure is p_a and its specific volume is v_a. Consequently, any combination of these properties, such as the product of p_a and v_a divided by T_a (that is, p_av_a/T_a), can also have only one value. Let this particular constant be represented by the letter R.

Consider next *any* state of the gas, and let the absolute pressure, specific volume, and absolute temperature of the gas at this state be denoted by p, v, and T without subscripts. Let a third state x be such that $p_x = p_a$ and $v_x = v$, as shown in Fig. 8. Since the pressure is the same at states a and x, by Eq. 2*a*,

$$\frac{v_a}{v_x} = \frac{T_a}{T_x},$$

or

$$\frac{v_a}{T_a} = \frac{v_x}{T_x}.$$

Multiplying the left side by p_a and the right side by p_x, we obtain

$$\frac{p_a v_a}{T_a} = \frac{p_x v_x}{T_x}.$$

Similarly, since $v_x = v$, by Eq. 2*b*,

$$\frac{p}{p_x} = \frac{T}{T_x},$$

or

$$\frac{p}{T} = \frac{p_x}{T_x}.$$

Multiplying the left side by v and the right side by v_x, we obtain

$$\frac{pv}{T} = \frac{p_x v_x}{T_x}.$$

Therefore, since $p_a v_a/T_a$ and pv/T both equal $p_x v_x/T_x$,

$$\frac{pv}{T} = \frac{p_a v_a}{T_a}.$$

Thus, it is shown that for *any* state of a perfect gas the product of the absolute pressure and specific volume divided by the absolute temperature always equals the same quantity, $p_a v_a/T_a$ (= the constant R). The desired equation of state for a perfect gas is, therefore,

$$\frac{pv}{T} = R,$$

or

$$pv = RT. \tag{3}$$

The constant R is called the **gas constant.**

Solution. By Eq. 3,

$$R = \frac{pv}{T} = \frac{35.75 \times 144 \times 1.121}{(20 + 460)} = 12.03 \text{ ft per deg F.}$$

The molecular weight of CCl_2F_2 is approximately 121. Hence, by Eq. 4,

$$R = \frac{R_0}{M} = \frac{1545}{121} = 12.75 \text{ ft per deg F.}$$

These two values differ by about 6%. This rather poor agreement is not surprising because the data given are for the condition where condensation of the vapor begins, and therefore the deviation from perfect gas behavior is the greatest. Moreover, the molecular weight is relatively high, and gases having high molecular weights deviate from perfect gas behavior more than do lighter ones. The least deviation is observed with monatomic and diatomic gases.

2. A steel tank contains 2 cu ft of oxygen gas at a pressure of 2000 psi gage and a temperature of 70°F. (*a*) Determine the weight of oxygen in the tank. (*b*) If the temperature is kept constant at 70°F, what will the pressure be after 10.0 lb of the gas have been released from the tank?

Solution. (*a*) Assuming that atmospheric pressure is 14.7 psia, and taking R for oxygen from Table I, we find by Eq. 3 that

$$2014.7 \times 144 \times \frac{2}{m} = 48.3 \times (70 + 460),$$

whence

$$m = 22.7 \text{ lb.}$$

(*b*) The weight of oxygen remaining in the tank will be (22.7 − 10.0 =) 12.7 lb. Hence, by Eq. 3,

$$p \times \frac{2}{12.7} = 48.3 \times (70 + 460),$$

whence

$$p = 162{,}500 \text{ psfa (or 1130 psia).}$$

PROBLEMS

3. The Bureau of Standards has found that the specific volume of ammonia vapor ($M = 17$) at 50 psia and 200°F is 8.185 cu ft per lb. How does the gas constant for ammonia based on these data compare (*a*) with the value listed in Table I, and (*b*) with the value calculated by Eq. 4?

4. An automobile tire contains 750 cu in. of air at 60°F and 30 psia. If the temperature of the air rises to 130°F, (*a*) what will its pressure be, and (*b*) how much air must be let out of the tire to reduce the pressure back to 30 psia? Assume that the tire does not stretch.

5. Fifty cubic feet of a gas at atmospheric pressure and 70°F are to be compressed to a pressure of 120 psi gage. What will the volume of the compressed gas be if its temperature is 160°F and atmospheric pressure is 14.2 psia? *Ans.* 6.19 cu ft.

6. A 2-cu-ft tank contains air at 100 psia and 90°F, and a 3-cu-ft tank contains air at 50 psia and 60°F. If a valve between the tanks is opened, and if the temperature of the air in both tanks is 70°F after equilibrium is established, what will the final pressure be?

7. If atmospheric pressure at sea level is 14.7 psia, and if the atmosphere is at a constant temperature of 70°F at all altitudes, what is the atmospheric pressure at an altitude of 5000 ft? *Ans.* 12.3 psia.

12. The *p-v-t* Relation for Liquids and Vapors.

It has been found that the relation between pressure, specific volume, and temperature for all actual gases (or vapors) approaches the relation expressed by Eq. 3 as the pressure of the gas approaches zero or as its temperature becomes very high. At higher pressures and lower temperatures, however, the *p-v-t* relation for actual gases may deviate considerably from this equation. At any given pressure the deviation becomes greatest when the temperature of the gas is lowered to the point where condensation begins.

For many vapors the *p-v-t* equation of state, based on experimentally measured values, is so complex that its use for ordinary engineering calculations is impractical. For example, the equation of state for water vapor proposed by Smith, Keyes, and Gerry * contains terms involving the pressure raised to powers as high as the twelfth and the reciprocal of the absolute temperature raised to powers as high as the twenty-fourth. Because of the complexity of the equations of state for many of the liquids and vapors of importance in engineering, tables of properties based on them have been prepared. By means of these tables values of the third property can be determined easily and quickly when two of the three properties, pressure, specific volume, and temperature, are known. These tables also list certain other properties of importance in thermodynamics which will be discussed in subsequent chapters.

As an example of the method of using tables of thermodynamic properties, the tables listing the properties of water and steam entitled *Thermodynamic Properties of Steam*, by Keenan and Keyes,† will be discussed in detail. These particular tables will be referred to hereafter as the *Steam Tables*. Before discussing the method of using them, however, it is necessary to introduce certain definitions. In order to do this, let us first consider what happens when any pure substance initially in the liquid phase is heated at constant pressure and changed into the vapor phase.

13. The Critical Point and the Triple Point.

Suppose that the apparatus shown in Fig. 1 contains 1 lb of water initially at a temperature of 70°F and at a pressure of 1 standard atmosphere (= 14.696 psia). If the pressure is kept constant, and if the cylinder is brought into contact with some large system at a temperature of, say, 500°F, the temperature of the water within the cylinder will rise until it reaches 212°F. At this temperature it will begin to vaporize. The temperatures of the water and of the steam formed will each remain constant at 212°F as long as any of the water remains unevaporated. After the water is completely vaporized, the temperature of the steam will begin to rise above 212°F and will continue to rise until it reaches the temperature of the system with which the apparatus has been brought into contact.

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* L. B. Smith, F. G. Keyes, and H. T. Gerry, *Proc. Am. Acad. Arts and Sci.*, **69**, 137 (1934).

† J. H. Keenan and F. G. Keyes, *Thermodynamic Properties of Steam*, John Wiley and Sons, Inc., 1936.

If the experiment is repeated with the pressure kept constant at 1 in. Hg abs instead of at atmospheric pressure, the water will remain in the liquid phase only until its temperature rises to 79.03°F. At this temperature the water will evaporate, and a further rise in temperature will not take place until all the water has changed into steam. If the experiment is repeated a third time with the pressure kept constant at 100 psia, the change from the liquid to the vapor phase will take place at 327.81°F.

The foregoing experiments show that the temperature at which the change from the liquid to the vapor phase takes place increases as the pressure is increased. This is true for all pure fluids. There is an upper limiting pressure, however, above which evaporation at constant temperature no longer takes place. For water this pressure is 3206.2 psia. At this pressure the density of the vapor formed is the same as that of the liquid; and consequently, at this or at any higher pressure, a clearly defined change from the liquid to the vapor phase no longer takes place. For any fluid the pressure at which the density of the vapor formed is the same as that of the liquid is known as the **critical pressure**, and the corresponding temperature (705.4°F for water) is known as the **critical temperature**. The state of the fluid at the critical pressure and critical temperature is known as the **critical state** or the **critical point**.

There is also a lower limiting pressure for each fluid below which it cannot exist as a liquid *in a stable state*, and, consequently, below which a change from the liquid to the vapor phase cannot take place. For water this pressure is 0.08854 psia, and the corresponding temperature is 32.02°F. At this pressure and temperature all three phases, solid, liquid, and vapor, can coexist in equilibrium. At higher pressures the liquid and vapor phases, but not the solid phase, can exist together in equilibrium; and at lower pressures the solid and vapor phases, but not the liquid phase, can coexist in equilibrium. For example, if an insulated tank contains water and steam both at atmospheric pressure and 212°F, none of the water will evaporate and none of the steam will condense.

If any ice is introduced into the tank, however, it will be at a lower temperature and will melt. Likewise, if the insulated tank contains ice and water vapor both at a pressure of 0.0185 psia and at a temperature of 0°F, none of the ice will sublime and none of the vapor will change to ice. If any water is introduced into the tank, however, it will be at a higher temperature and consequently will freeze. For any pure fluid the condition at which all three phases can coexist in equilibrium is known as the **triple point.** It may be remarked that most engineering equipment operates at pressures between the critical and the triple point pressures.

The definition of certain terms, associated with the change of any pure fluid from the liquid or solid to the vapor phase, can now be introduced.

14. Definitions Associated with Phase Changes. If any pure fluid is heated at constant pressure, the temperature at which it changes from the liquid to the vapor phase (that is, the temperature at which it boils) is called its **saturation temperature.** It depends upon the pressure, as explained in the preceding article. Liquid at the saturation temperature is called **saturated liquid**; at the same pressure but at any lower temperature, it is called **subcooled liquid.** Vapor at the saturation temperature is called **saturated vapor**; at the same pressure but at any higher temperature, it is called **superheated vapor.** In a mixture of saturated liquid and saturated vapor, the weight of the saturated vapor divided by the weight of the mixture (that is, the fraction of the mixture, by weight, which is vapor) is called the **quality** of the mixture and is represented by the letter x.

To illustrate these various terms, consider again the experiment in which 1 lb of water initially at 70°F is heated at atmospheric pressure. The water at 70°F is subcooled water and continues to be subcooled until it reaches a temperature of 212°F. At this temperature it becomes saturated water. The steam at a temperature of 212°F is saturated steam. When the water is only partially evaporated, and the cylinder contains, say, 0.9 lb of saturated steam and 0.1 lb of saturated water, the mixture has a quality of 0.9 (or a quality of 90%). After the water is completely evaporated, and the temperature of the steam rises above 212°F, the steam is superheated steam.

Note that temperature or pressure alone is not enough to determine whether a liquid is subcooled or saturated, or to determine whether a vapor is saturated or superheated. For example, water at a temperature of 212°F is saturated water if the pressure is atmospheric, but it is subcooled water if the pressure is higher. Similarly, steam at 212°F is saturated steam if the pressure is atmospheric, but it is superheated at this temperature if the pressure is lower.

Finally, note that it is impossible for any fluid to exist as a liquid in a stable state if its temperature is higher than the saturation temperature corresponding to its pressure; likewise, it cannot exist as a vapor in a stable state if its temperature is below the saturation temperature. For example, at atmospheric pressure water cannot exist as a stable liquid at 220°F, and at this pressure steam cannot exist as a stable vapor at 200°F. Under special conditions, however, such unstable states can be realized. One example of such a condition is discussed in Art. 123.

15. Determination of p-v-t Values by Means of Tables of Properties. If at any state of a fluid two of the three properties, pressure, specific volume, and temperature, are known, and if tables of thermodynamic properties of the fluid are available, the third one of these properties can be determined from the tables. The method of doing this for water and steam by means of the Keenan and Keyes *Steam Tables* is explained in this article. Since tables of thermodynamic properties of other fluids are similar to the *Steam Tables*, except for minor differences in arrangement, the procedures explained may be used for other fluids as well.

Tables 1 and 2 of the *Steam Tables* both list the specific volumes of saturated water and saturated steam. The two tables differ only in that one lists these values for integral values of the saturation temperature, and the other, for integral values of the saturation pressure. Either table alone would be sufficient, but both are included for convenience. Some sample lines from Table 2 are given below.

Table 2. Saturation: Pressures

Abs. Press. Lb./Sq. In.	Temp. Fahr.	Specific Volume		Enthalpy			Entropy			Internal Energy			Abs. Press. Lb./Sq. In.
		Sat. Liquid	Sat. Vapor	Sat. Liquid	Evap.	Sat. Vapor	Sat. Liquid	Evap.	Sat. Vapor	Sat. Liquid	Evap.	Sat. Vapor	
p	t	v_f	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	u_f	u_{fg}	u_g	p
50	281.01	0.01727	8.515	250.09	924.0	1174.1	0.4110	1.2474	1.6585	249 93	845.4	1095.3	50
51	282.26	0.01728	8.359	251.37	923 0	1174.4	0.4127	1.2442	1.6569	251.21	844.3	1095.5	51
52	283.49	0.01729	8.208	252.63	922.2	1174.8	0.4144	1.2409	1.6553	252.46	843.3	1095.8	52
53	284.70	0.01730	8.062	253.87	921.3	1175.2	0.4161	1.2377	1.6538	253 70	842.4	1096.1	53
54	285.90	0.01731	7.922	255.09	920.5	1175.6	0.4177	1.2346	1.6523	254.92	841.5	1096.4	54

Values of the specific volume of saturated liquid (denoted by the symbol v_f) and of saturated vapor (denoted by the symbol v_g) can be read directly from these tables if either the saturation temperature or saturation pressure is known. In parts of these tables the difference between v_g and v_f (denoted by the symbol v_{fg}) is also listed for convenience.

If the quality and the temperature, or the quality and the pressure, of a mixture of saturated liquid and saturated vapor are known, the specific volume of the mixture can be easily determined. It is equal to the volume of the liquid plus the volume of the vapor in each pound of the mixture. Since 1 lb of the mixture consists of x pounds of saturated vapor and $(1 - x)$ pounds of saturated liquid, the volume of the liquid is $(1 - x)v_f$, the volume of the vapor is xv_g, and the specific volume v of the mixture is

$$v = (1 - x)v_f + xv_g. \tag{5}$$

Unless the quality is extremely low, the volume of the liquid is likely to be negligible compared to the volume of the vapor; and, consequently, for most purposes a sufficiently accurate value of the volume of the mixture may be calculated by the equation

$$v = xv_g. \tag{5a}$$

As an example, let us find the specific volume of a mixture of saturated water and saturated steam having a quality of 90% at a pressure of 50 psia. Taking the values of v_f and v_g from the foregoing excerpt from Table 2, we find by Eq. 5 that

$$\begin{aligned} v &= 0.1 \times 0.01727 + 0.9 \times 8.515 \\ &= 0.002 \qquad\quad + 7.664 \\ &= 7.666 \text{ cu ft per lb.} \end{aligned}$$

Had Eq. 5*a* been used, the value 7.664 cu ft per lb would have been obtained.

Table 3 lists values of the specific volume of superheated steam. Some sample lines from this table follow:

Table 3. Superheated Vapor

Lb./Sq. In. (Sat Temp)		Sat. Liquid	Sat. Vapor	Temperature—Degrees Fahrenheit 290°	300°	320°	340°	360°	380°	400°	420°	440°	460°	480°	500°	520°	540°
	v	0.017	8.208	8.293	8.423	8.680	8.932	9.181	9.427	9.671	9.914	10.154	10.394	10.632	10.869	11.106	11.342
52	h	252.6	1174.8														1303.1
(283.49)	s	0.4144	1.6553	1.6601	1.6672	1.6809	1.6940	1.7066	1.7186	1.7303	1.7417	1.7527	1.7635	1.7739	1.7842	1.7942	1.8040
	v	0.017	8.062	8.131	8.259	8.511	8.759	9.004	9.246	9.485	9.723	9.960	10.195	10.429	10.662	10.894	11.126
53	h	253.9	1175.2	1178.1	1183.5	1194.1	1204.5	1214.6	1224.7	1234.6	1244.5	1254.3	1264.1	1273.9	1283.6	1293.3	1303.1
(284.70)	s	0.4161	1.6538	1.6577	1.6648	1.6786	1.6917	1.7043	1.7164	1.7281	1.7395	1.7505	1.7613	1.7718	1.7820	1.7920	1.8019
	v	0.017	7.922	7.974	8.100	8.348	8.592	8.833	9.071	9.306	9.540	9.772	10.103	10.233	10.462	10.690	10.917
54	h	255.1	1175.6	1177.8	1183.2	1193.9	1204.3	1214.4	1224.5	1234.5	1244.4	1254.2	1264.0	1273.8	1283.5	1293.3	1303.0
(285.90)	s	0.4177	1.6523	1.6553	1.6625	1.6763	1.6894	1.7021	1.7142	1.7259	1.7373	1.7483	1.7591	1.7696	1.7799	1.7899	1.7998

Values of the third property can be read directly from this table if two of the three properties, pressure, specific volume, and temperature, are known. For example, at a pressure of 52 psia and a temperature of 400°F, the specific volume of superheated steam is 9.671 cu ft per lb.

Approximate values of the specific volume of subcooled water can be determined either from Table 1 or from Table 2 by assuming that the specific volume of subcooled water is the same as that of saturated water *at the same temperature.* For example, the specific volume of subcooled water at a pressure of 400 psia and a temperature of 200°F is approximately 0.01663 cu ft per lb (the value of v_f at 200°F). The assumption made is that water is incompressible. At the pressures ordinarily encountered in engineering, this assumption introduces little error and is usually permissible. Note that the value 0.01663 cu ft per lb is the *exact* specific volume of water at 11.526 psia and 200°F.

Exact values of the specific volume of subcooled water can be determined from Table 4. The following sample lines are reproduced from this table:

Table 4. Compressed Liquid

		Temperature—Degrees Fahrenheit										
		32°	100°	200°	300°	400°	500°	600°	620°	640°	660°	680°
	p	0.08854	0.9492	11.526	67.013	247.31	680.8	1542.9	1786.6	2059.7	2365.4	2708.1
Saturated	v_f	0.016022	0.016132	0.016634	0.017449	0.018639	0.020432	0.023629	0.02466	0.02598	0.02777	0.03054
Liquid	h_f	0	67.97	167.99	269.59	374.97	487.82	617.0	646.7	678.6	714.2	757.3
Abs. Press. Lb./Sq. In. (Sat. Temp.)	s_f	0	0.12948	0.29382	0.43694	0.56638	0.68871	0.8131	0.8398	0.8679	0.8987	0.9351
200	$(v-v_f)\cdot 10^5$	−1.1	−1.1	−1.1	−1.1							
	$(h-h_f)$	+0.61	+0.54	+0.41	+0.23							
(381.79)	$(s-s_f)\cdot 10^3$	+0.03	−0.05	−0.21	−0.21							
400	$(v-v_f)\cdot 10^5$	−2.3	−2.1	−2.2	−2.8	−2.1						
	$(h-h_f)$	+1.21	+1.09	+0.88	+0.61	+0.16						
(444.59)	$(s-s_f)\cdot 10^3$	+0.04	−0.16	−0.47	−0.56	−0.40						
.	.	.	.	.	.	.	.	.	.	.	.	.
4000	$(v-v_f)\cdot 10^5$	−21.5	−19.2	−21.0	−27.5	−40.0	−64.5	−132.2	−160	−202	−270	−400
	$(h-h_f)$	+11.88	+10.49	+9.03	+7.41	+4.71	−0.16	−10.0	−13.3	−17.4	−22.5	−32.4
	$(s-s_f)\cdot 10^3$	+0.29	−2.42	−4.74	−6.77	−9.40	−13.03	−19.3	−21.4	−24.0	−27.2	−34.3
4500	$(v-v_f)\cdot 10^5$	−24.1	−21.4	−23.5	−30.7	−44.9	−72.5	−151.5	−184	−234	−313	−464
	$(h-h_f)$	+13.32	+11.80	+10.15	+8.40	+5.40	+0.02	−11.1	−15.1	−19.9	−26.4	−38.4
	$(s-s_f)\cdot 10^3$	+0.26	−2.74	−5.33	−7.60	−10.58	−14.80	−22.4	−25.1	−28.3	−32.7	−41.8

Instead of listing the specific volume itself, this table lists the difference between the specific volume of the subcooled (or "compressed") water and the specific volume of saturated water at the same temperature. As an example of the method of using this table, let us find the exact specific volume of subcooled water at 400 psia and 200°F. From Table 4,

$$(v - v_f) \times 10^5 = -2.2,$$

or

$$v = v_f - 0.000022.$$

The specific volume v_f of saturated water at 200°F can be found either from Table 1 (= 0.01663 cu ft per lb) or from the top line of Table 4 (= 0.016634 cu ft per lb). Hence, the exact specific volume of the subcooled water is (0.016634 − 0.000022 =) 0.016612 cu ft per lb. Notice that the decrease in the specific volume of the water caused by increasing its pressure from 11.526 to 400 psia is only a little more than 0.1%.

Before its specific volume can be found, it is necessary to know whether the fluid is subcooled water, a mixture of saturated water and saturated steam, or superheated steam. If the pressure and temperature are known, this can be determined by comparing the temperature with the saturation temperature corresponding to the pressure. For example, if the pressure of the fluid is 50 psia, its saturation temperature is 281.01°F (from Table 2). If the temperature of the fluid is higher than this, the fluid is superheated steam; if it is lower, the fluid is subcooled water; if it is the same, the fluid could be saturated water, saturated steam, or a mixture of the two. In the last case additional datum (the quality) is necessary to identify the condition.

If the specific volume and either the temperature or the pressure are known, the condition of the fluid can be determined by comparing the specific volume with the specific volumes of saturated water and saturated steam at the given temperature or pressure. For example, if the pressure is again 50 psia, v_f and v_g are 0.01727 and 8.515 cu ft per lb respectively (from Table 2). If the specific volume of the fluid is less than 0.01727 cu ft per lb, the fluid is subcooled water; if the specific volume is between 0.01727 and 8.515 cu ft per lb, it is a mixture; and if the specific volume is larger than 8.515 cu ft per lb, it is superheated steam.

ILLUSTRATIVE PROBLEMS

✓8. A 5-cu-ft tank contains 2 lb of H_2O at a temperature of 300°F. Determine the condition of the H_2O.

Solution. The specific volume of the H_2O is (5/2 =) 2.5 cu ft per lb. From Table 1 of the *Steam Tables*, at 300°F, $v_f = 0.01745$ and $v_g = 6.466$ cu ft per lb. Since the specific volume of the H_2O is between these two values, the tank contains a mixture of saturated water and saturated steam. The quality can be determined by Eq. 5*a*:

$$v = xv_g, \tag{5a}$$

or

$$2.5 = x6.466,$$

whence

$$x = 0.387.$$

Therefore, the tank contains (2 × 0.387 =) 0.774 lb of saturated steam and (2 × 0.613 =) 1.226 lb of saturated water at a pressure of 67.013 psia.

9. A rigid tank contains 2 lb of saturated water and 3 lb of saturated steam at 40 psia. If the temperature is raised to 500°F, what will the pressure be?

Solution. In the initial state, the quality is (3/5 =) 0.60. From Table 2 of the *Steam Tables*, at 40 psia, $v_g = 10.498$ cu ft per lb. Hence, the initial specific volume is, by Eq. 5*a*,

$$v = 0.60 \times 10.498$$

$$= 6.299 \text{ cu ft per lb.}$$

Since the tank is rigid, the final specific volume is the same as the initial. Hence, in the final state the two properties, temperature and specific volume, are known. To determine the final condition of the steam, let us compare the known specific volume with the values of v_f and v_g at 500°F. From Table 1, $v_f = 0.0204$ and $v_g = 0.6794$ cu ft per lb. Since the specific volume is larger than v_g, the steam is superheated in its final state. Turning to Table 3 and reading under the 500°F column, we find that at 89 psia the specific volume is 6.297 cu ft per lb. Therefore, in the final state, the pressure in the tank is about 89 psia.

PROBLEMS

10. Determine the temperature of steam at 150 psia if its specific volume is (*a*) 2.000 cu ft per lb, and (*b*) 4.000 cu ft per lb.

11. Determine the pressure of steam at 500°F if its specific volume is (*a*) 0.500 cu ft per lb, and (*b*) 2.000 cu ft per lb.

12. A rigid tank contains steam at 200 psia and 800°F. If the steam is cooled to 400°F, what will its pressure be? *Ans.* 132 psia.

13. A rigid tank contains 3 lb of saturated water and 1 lb of saturated steam at 60 psia. If just enough heat is added to evaporate all the water, what will the final pressure be?

14. Determine the specific volume of (*a*) saturated water at 100°F, and (*b*) subcooled water at 100°F and 1000 psia. (*c*) Increasing the pressure on the water from 0.0492 psia to 1000 psia causes its volume to decrease by what per cent?

Ans. (*c*) 0.316%.

15. A quantity of water is compressed at a constant temperature of 300°F until its specific volume is 0.017300 cu ft per lb. What is its final pressure?

GENERAL PROBLEMS

16. Superheated steam behaves approximately as a perfect gas. (*a*) Calculate the gas constant R for steam by making use of the specific volume at 15 psia and 500°F listed in the *Steam Tables*. (*b*) Use this value of R to calculate the specific volume of steam at 100 psia and 750°F, and compare the result with the value given in the tables. (*c*) Compare your value of R with that calculated by Eq. 4.

17. Calculate the density of air at 14.7 psia and 70°F. *Ans.* 0.0750 lb per cu ft.

18. A quantity of superheated steam initially at 130 psia and 580°F is cooled at constant volume until it becomes saturated. What is its final temperature?

19. Fifteen pounds of argon ($M = 40$) are to be stored at a temperature of 65°F in a 10-cu-ft tank. To what pressure must the gas be compressed? *Ans.* 211 psia.

20. A 10-cu-ft tank contains a mixture of saturated water and saturated steam at a pressure of 150 psia. The mixture has a quality of 60%. If the temperature is raised to 600°F, what weight of steam must be released from the tank to keep the pressure at 150 psia? *Ans.* 3.08 lb.

21. Compressed air is stored at a pressure of 100 psi gage in a 30-cu-ft tank. What is the weight of air in the tank if the temperature of the air is 60°F and the barometric pressure is 29.2 in. Hg abs?

22. If a quantity of water initially saturated and at a pressure of 40 psia is heated at constant volume to a temperature of 300°F, what will its final pressure be?

23. A quantity of air initially at 70°F is compressed from 15 to 100 psia in such a way that the product $pv^{1.2}$ remains constant. What is the final temperature of the air? *Ans.* 268°F.

24. A 5-cu-ft tank contains a mixture of saturated water and saturated steam. If the mixture weighs 100 lb and is at 600°F, what fraction of the total volume of the tank is filled by the water? What is the quality of the mixture?

Ans. 0.420; 0.109.

25. Suppose that atmospheric pressure and temperature at sea level are 14.7 psia and 70°F respectively, that the temperature at an altitude of 5000 ft is 45°F, and that the temperature varies linearly with altitude. Calculate the atmospheric pressure at an altitude of 5000 ft.

26. A 20-cu-ft tank contains 9 lb of steam at 500°F. If enough heat is absorbed from the steam to condense six-tenths of it (by weight), by how much will the pressure decrease?

SYMBOLS

M	molecular weight
p	pressure
R	gas constant
R_0	universal gas constant
t	temperature
T	absolute temperature
v	specific volume; v_f = specific volume of saturated liquid; v_g = specific volume of saturated vapor; $v_{fg} = v_g - v_f$
x	quality, weight of saturated vapor per unit weight of mixture

Chapter 2

MECHANICAL WORK AND HEAT

Chapter 1 dealt only with various states of systems and with some of the properties associated with those states. It did not deal with the methods by which systems might be caused to change from one state to another. In the present chapter we shall consider some of these methods.

16. Processes. If a system changes from one state to another, it is said to undergo a **process.** It can be caused to undergo a process in many ways. Two ways are (*a*) by letting *mechanical work* be done on or by the system, and (*b*) by letting *heat* flow into or out of it. Mechanical work and heat are discussed further in Arts. 17 and 18.

Two general types of processes are of particular interest in engineering thermodynamics. In the first type, the system consists of a quantity of some substance which (*a*) is neither raised nor lowered during the process, (*b*) is not in motion either initially or at the end of the process, and (*c*) is not subject to any surface effects, such as capillarity, or to any magnetic or electric effects. A process taking place under these conditions is known as a **non-flow process.** As an example of this type of process, consider a system consisting of a quantity of fluid confined inside the cylinder-and-piston apparatus shown in Fig. 1. The fluid undergoes a non-flow process if no fluid is added to or removed from the cylinder during the process, and if the cylinder remains at rest while the fluid is compressed or expanded or while it is heated or cooled.

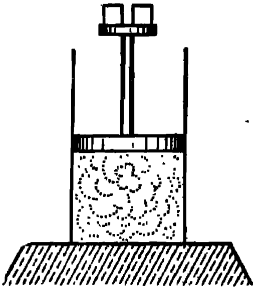

FIG. 1.

It was explained in Art. 9 that, if the system is one of the types usually considered in engineering thermodynamcis, only two properties are required to describe its state if it is at rest and is subject to no surface effects nor to any magnetic or electric effects. If such a system undergoes a non-flow process, its initial and final states can be represented by two points on a diagram constructed by using any two properties, such as pressure and specific volume, as coordinates. Moreover, if during the process the system is ever in a state of equilibrium, this intermediate equilibrium state can also be represented by a point on the diagram. If the system should pass through an infinite number of equilibrium states, as it does if the process is of a type called a "reversible" process (discussed in Art. 20), the points representing all these states will form a continuous curve. Such curves will be used frequently throughout the text to represent graphically processes that are non-flow and reversible.

The second type of process of particular interest in engineering thermodynamics is one in which the system consists of one or more fluids that flow into and out of some kind of apparatus during the process. Examples of this type are the processes that take place in steam turbines, air compressors, and internal-combustion engines. These processes are known as **flow processes.** A further discussion of them will be deferred until the next chapter.

17. Mechanical Work. **Mechanical work** is defined as "something" that is transferred between a system and its surroundings when either of them exerts a force that acts through a distance on the other. The amount of the "something" transferred will be represented by the letter W. It is measured by the product of the force and the distance through which the force acts, the distance being measured in the direction of the force. For example, if the force is measured in pounds, and the distance through which it acts is measured in feet, the amount of mechanical work transferred is measured in foot-pounds. These are the units usually used in engineering, although the product of any unit of force and any unit of distance may be used. Mechanical work done *by a system* on its surroundings will be designated as positive; and mechanical work done by the surroundings *on the system* will be designated as negative.

To illustrate the foregoing sign convention, suppose that the system consists of a quantity of air confined in the cylinder-and-piston apparatus shown in Fig. 1. Suppose further that the system undergoes a non-flow process during which the piston *rises* 3 in., while the downward force (due to the weight of the piston itself, the weights placed on the piston, and the atmospheric pressure acting on the upper face of the piston) is 100 lb. Under these conditions, $W = +100 \times 3/12 = +25$ ft-lb. The positive sign is used because the system (the air) does mechanical work on its surroundings (the piston, the weights, and the atmosphere). Had the piston descended 3 in. while the downward force was 100 lb, the surroundings would have done work on the system; and we would write $W = -25$ ft-lb.

When mechanical work is done on or by any system confined inside the apparatus shown in Fig. 1, the volume of the system may change. This is not the only effect that may result, however. For example, if a force acts through a distance on the cylinder itself, the entire apparatus might be elevated, or it might be given a velocity. Note that neither of these processes would be classified as a non-flow process because the system is elevated in the one and is in motion at the end of the other.

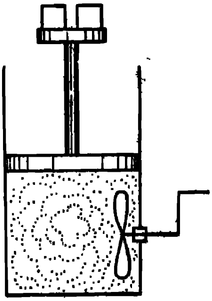
FIG. 9.

As another example of the possible effect of doing mechanical work on a system, suppose that the system consists of a quantity of water placed in an apparatus similar to that shown in Fig. 1, but with a stirrer added, as shown in Fig. 9. If mechanical work is done on the water by turning the crank of the stirrer, the temperature of the water might be raised; some of the water might be changed to steam; and the system (the water and the steam) might expand and do mechanical work on the piston, on the weights, and on the atmosphere. Or, if the piston is held stationary while mechanical work is done on the water with the stirrer, both the pressure and the temperature of the water will be raised. These two processes are both non-flow processes if the cylinder remains stationary and if, in the final state, the system is not in motion.

There are forms of work other than mechanical work. **Work,** in general, may be defined as "something" that is transferred from a system to its surroundings (or vice versa) if the sole effect external to the system *could be* that mechanical work is done on or by the surroundings. For example, electricity flowing from any system is a form of work because the sole effect on the surroundings could be that mechanical work is supplied to the surroundings. This effect could be realized by interposing a frictionless and resistanceless electric motor between the system and its surroundings. Electricity may be called **electrical work.**

Although there are various forms of work, only mechanical work is considered in engineering thermodynamics. For simplicity in writing, it will be referred to hereafter simply as "work," but it should be remembered that by this term only mechanical work is meant.

18. Heat. **Heat** is defined as "something" that is transferred between two systems because of a difference in their temperatures. There are two methods by which the transfer may take place: (*a*) the systems may be brought into contact with each other, in which case the transfer is said to take place by *conduction;* or (*b*) the systems may be exposed to each other but not be in actual contact, in which case the transfer is said to take place by *radiation.* The method by which the transfer takes place is not important in thermodynamics, however. The fact that the "something" which passes from one system to the other is transferred *because of a difference in temperature* is important.

The amount of the "something" transferred will be represented by the letter Q. It may be measured in various units. The unit most commonly used in engineering is the **British thermal unit** (Btu), defined as the amount of heat required to raise the temperature of 1 lb of water at constant atmospheric pressure from 63° to 64°F. This definition, sometimes called the "calorimetric" definition of the British thermal unit, is subject to certain objections. These are explained in the next article, and a preferable definition is stated.

Another unit of heat is the **calorie.** This unit may be defined as the amount of heat required to raise the temperature of 1 gram of water at constant atmospheric pressure from 14.5° to 15.5°C. The calorie is the unit of heat regularly used in scientific work.

To indicate whether heat is supplied to or removed from a system, we shall designate heat supplied *to the system* as positive and heat removed *from the system* as negative. For example, if 10 Btu of heat are added to a system during a certain process, we shall write $Q = +10$ Btu, the positive sign indicating that the heat is supplied to the system. If 10 Btu of heat are removed from the system, we shall indicate this by writing $Q = -10$ Btu for the process. If heat is neither supplied to nor removed from the system during a process (that is, if $Q = 0$), the system is said to undergo an **adiabatic process.**

In order for heat to be transferred from one system to another, the system receiving the heat must be at a temperature lower than that of the system supplying the heat. The temperature of the system receiving the heat may rise but does not necessarily do so. For example, if the cylinder-and-piston apparatus shown in Fig. 1 contains saturated water, and if the pressure is kept constant by leaving the weights on the piston unchanged, the addition of heat could cause some of the water to change into saturated steam. Since the steam would be at the same temperature as the water, the temperature of the system (the water and the steam) would remain constant during the process. The transfer of heat may also be accompanied by a decrease in the temperature of the system. For example, suppose that the system consists of a quantity of air confined inside the apparatus shown in Fig. 1. If one of the weights is removed from the piston, the piston will rise, and the air will do work on the piston, on the remaining weights, and on the atmosphere. This will be accompanied by a decrease in the temperature of the air. Now, if during the process heat is supplied to the air, but if not enough is supplied either to raise the temperature of the air or to keep its temperature constant, the transfer of heat to the system will be accompanied by a decrease in its temperature.

19. Energy. It is sometimes possible to produce the same change in the state of a system either by adding heat to it or by doing work on it. For example, consider a quantity of water at constant atmospheric pressure, and suppose that its temperature initially is 70°F. The temperature of the water can be raised to, say, 80°F either by letting heat flow into it (from another system at a higher temperature), or by stirring it with a paddle and thereby doing work on it. Since the "something" added to the water as heat and the "something" added as work produce exactly the same change in the state of the water, it is evident that heat and work must be different forms of the same thing. We call the "something" added to the water "energy." Thus, **energy is defined** as the "something" that is transferred to or from a system (*a*) when work is done on or by the system, and (*b*) when heat is added to or removed from the system.

Since work and heat are both energy, it follows that they may be measured in the same units. For example, they may be measured in foot-pounds or in British thermal units. If the calorimetric definition of the British thermal unit is used, the relation between this unit and the foot-pound must be determined by experiment. Thus, by definition, 1 Btu of energy in the form of heat is required to raise the temperature of 1 lb of water at constant atmospheric pressure from 63° to 64°F. Experiments show that the amount of energy in the form of work required to produce the same change is approximately 778 ft-lb. Hence, 1 Btu = 778 ft-lb.

Since the relation between the British thermal unit (as defined by the calorimetric definition) and the foot-pound must be determined by experiment, it is evident that the accuracy of the relation depends upon how accurately the experiment can be carried out. This is undesirable because it leads to a constantly changing relation as the accuracy of the experimental work is increased. In order to avoid this, it was proposed, in effect, by the First International Steam Table Conference at London in 1929 * that the British thermal unit be *defined* by the relation

$$1 \text{ Btu} = 778.26 \text{ ft-lb}.$$

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* *Mechanical Engineering*, **52**, 120 (1930).

This particular value was chosen in order to make the new unit approximately the same as the old one, defined by the calorimetric definition. This new definition of the British thermal unit is now the generally accepted one.

The number 778.26 is frequently called the **mechanical equivalent of heat.** It will be represented by the letter J. For most engineering calculations, the value of J may be taken simply as 778 ft-lb per Btu.

20. Reversible Processes. If any system undergoes a process, the work and heat transferred to or from the system cannot in general be calculated from the properties of the system if the properties are known only for the initial and final states. Even if the properties at all intermediate states are known, it is still not always possible to calculate the amount of work or heat involved in the process. Suppose, for example, that a pound of air has an initial temperature of 100°F and a final temperature of 200°F, and that the volume of the air remains constant while the air is passing from one state to the other. The impossibility of calculating from these data alone the amount of work and heat involved is evident from the fact that the change could be effected by adding only heat to the system (by bringing it into contact with another system at a higher temperature) or by doing only work on it (by stirring it with a paddle). For certain kinds of processes, however, the work and heat involved can be calculated from a knowledge of the manner in which certain properties of the system behave. One of these is the so-called "reversible process." This type of process is of great importtance in thermodynamics and will be discussed in the next several articles in detail.

A **reversible process** may be defined as one during which

(*a*) *no part of the system or its surroundings ever departs from a state of equilibrium by more than an infinitesimal amount, and*

(*b*) *no friction of any kind occurs.*

✓To satisfy the first of these two requirements, it is necessary (1) that the forces within the system be very nearly balanced throughout, (2) that the forces within each part of the surroundings be very nearly balanced throughout, and (3) that the forces exerted by the system on its surroundings very nearly equal the forces exerted by the surroundings on the system. It is also necessary (4) that the temperature of the system be very nearly uniform, (5) that the temperature of each part of the surroundings be very nearly uniform, and (6) that the temperature of the system be very nearly the same as that of any part of the surroundings to which it may be exposed.

To satisfy the second requirement, (1) there must be no mechanical friction between the moving parts of any apparatus involved in the process, and (2) one portion of any fluid involved in the process must not have a finite velocity relative to an adjacent portion, since this would result in shearing forces (or friction) within the fluid. Also, there must be no inelastic deformation of any solid involved in the process, and no electrical resistance or magnetic hysteresis. The latter conditions need not concern us, however, since the usual systems considered in engineering thermodynamics are fluids and are assumed to be subject to no electric or magnetic effects (see Art. 9).

Another type of process of interest is one that will be called an "internally reversible process." **An internally reversible process may** be defined as one in which (*a*) the system itself behaves exactly as it would if the entire process were reversible, but (*b*) irreversibilities (friction and finite departures from equilibrium) occur *within the surroundings.* The distinction between reversible and internally reversible processes is introduced because many of the equations that will be developed, although correct for reversible processes, require only that the process be internally reversible.

To see how a system might be made to undergo either a reversible or an internally reversible process, and to understand why such processes are of interest, we shall consider the performance of work and the transfer of heat during these types of processes in the next two articles. The reason for calling reversible processes by this name will be explained in Art. 63 after the Second Law of Thermodynamics has been introduced.

21. Reversible Processes and Work. A **heat reservoir** is defined as any body which is at a uniform temperature and is so large that the transfer of a finite amount of heat to or from it produces no perceptible change in its temperature. For example, the atmosphere or a large lake may be considered a heat reservoir since the addition or removal of moderate amounts of heat will not appreciably affect the temperature of either.

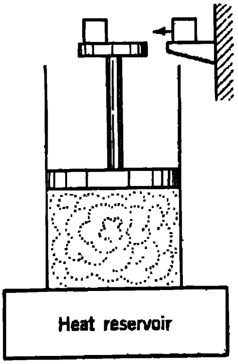

FIG. 10.

Now consider again a system consisting of a gas confined inside a cylinder equipped with a piston, and suppose that the cylinder is in contact with a heat reservoir, as shown in Fig. 10. Heat will flow from the gas to the reservoir if the temperature of the gas rises above the temperature of the reservoir, or heat will flow from the reservoir to the gas if the temperature of the gas falls below that of the reservoir.

The gas can be caused to undergo such changes in temperature either by compressing it or by letting it expand and do work.

One way of compressing the gas is to place an additional, finite weight on the piston. This will cause the piston to move downward, perhaps to oscillate for a while, but eventually to come to rest at a lower level. Let us consider whether this process is reversible. We note the following things: First, there will inevitably be some friction between the piston and the cylinder. Second, the downward force (due to the weight of the piston, the weights placed on the piston, and the atmospheric pressure acting on the upper face of the piston) will equal the upward force (exerted by the gas on the lower face of the piston) only when the gas is at rest before the additional weight is placed on the piston, and again when the gas is at rest after the weight is added. At all intermediate times, these two forces will differ by a finite amount; and the system (the gas) will therefore depart from equilibrium with one part of its surroundings (the piston, the weights, and the atmosphere) by more than an infinitesimal amount. Third, the temperature of the gas will change rapidly during the process and will not be the same as that of the heat reservoir, except when the gas is in its initial and final states. At all intermediate states, the temperatures of the gas and the heat reservoir will differ by a finite amount; and, therefore, the gas will also depart from equilibrium with another part of its surroundings (the heat reservoir) by more than an infinitesimal amount. Fourth, some parts of the gas will have finite velocities relative to other parts, and all parts will not be at the same temperature. Therefore, the system will depart by more than an infinitesimal amount from a state of internal equilibrium. For these various reasons, we must conclude that the process is not reversible.

The process could be made reversible (and therefore also internally reversible) (*a*) if the friction between the piston and the cylinder could be eliminated, and (*b*) if infinitesimal weights could be added to the piston one at a time by sliding them horizontally and without friction off a series of platforms, arranged one above the other an infinitesimal distance apart. The system and its surroundings must be allowed to return to equilibrium before each successive weight is added. If the compression could be carried out in this way, the downward force (due to the weight of the piston, the weights placed on the piston, and the atmospheric pressure acting on the upper face of the piston) would never differ from the upward force (exerted by the gas on the lower face of the piston) by more than an infinitesimal amount. Moreover, the temperature of the gas would never differ from that of the heat reservoir by more than an infinitesimal amount. Finally, the system would not depart by more than an infinitesimal amount from a state of internal equilibrium because no portion of the gas would ever have a finite velocity relative to any other portion, and no finite temperature differences would ever exist within the gas. It is evident that the compression under these conditions could be carried out only in theory, since, in actual practice, it would be impossible to eliminate completely all friction; and, even if this were possible, the time required to complete any finite compression of the gas would be infinite. The question arises then, why should this type of process be of interest? One of the reasons can be brought out by an example.

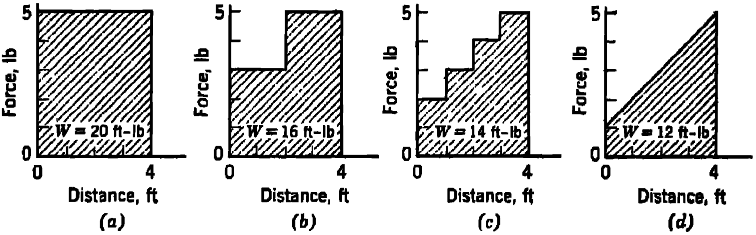

Fig. 11. Force-distance diagrams.

Consider again the apparatus shown in Fig. 10, and suppose that the mechanical friction involved in its operation can be made vanishingly small. Suppose also that initially the downward force is 1 lb. Finally, suppose that the cylinder contains a gas which, for the sake of simplicity, is of such nature that, every time another 1-lb weight is added to the piston, the piston reaches a new equilibrium position exactly 1 ft lower than before. Consider, now, three ways in which the piston might be caused to descend, say, 4 ft.

First, let the compression be accomplished by sliding one 4-lb weight horizontally onto the piston. The total downward force after this weight is added is 5 lb, and it acts through a distance of 4 ft. Hence, the work done on the gas is (5 × 4 =) 20 ft-lb. This amount of work can be represented graphically by the shaded area on the force-distance diagram shown in Fig. 11*a*. This figure is obtained by plotting the downward force acting on the gas against the distance through which this force acts.

Second, instead of placing one 4-lb weight on the piston, let the compression be accomplished by (*a*) sliding a 2-lb weight horizontally onto the piston, and letting the piston descend 2 ft (under the action of a total downward force of 3 lb); and (*b*) then sliding a second 2-lb weight onto the piston from a level 2 ft lower than the first, and letting the piston descend 2 ft more (under the action of a total downward force of 5 lb). The total work done on the gas this time is ($3 \times 2 + 5 \times 2 =$) 16 ft-lb. This amount of work, represented graphically by the shaded area on the force-distance diagram shown in Fig. 11*b*, is less than the work required when the compression to the same final state is accomplished by adding a single 4-lb weight.

Third, let the compression be accomplished by sliding four 1-lb weights onto the piston one at a time, allowing the piston to descend 1 ft before each successive weight is added. The total work done on the gas is now ($2 \times 1 + 3 \times 1 + 4 \times 1 + 5 \times 1 =$) 14 ft-lb. This amount of work, represented by the shaded area in Fig. 11*c*, is less than the work required when the compression is accomplished by adding two 2-lb weights.

It is apparent that, under the prescribed conditions, the work required to make the piston descend 4 ft becomes less as the weights added to the piston are made smaller. The *minimum* amount of work would evidently be required if the weights added were infinitesimal in size and if the system and its surroundings were allowed to return to equilibrium before each successive weight was added. But, if this were done, the process would be a *reversible process*. Hence, we conclude that, if a system confined inside a cylinder equipped with a piston is compressed and if the system can attain only one equilibrium state corresponding to each total external force exerted on the piston, *the amount of work required to compress the system is a minimum if the process is reversible.*

Using for the moment the force exerted on the piston and the position of the piston to describe the state of the system, we may represent the intermediate equilibrium states that the system can attain in this example by the infinite series of points forming the straight line shown in Fig. 11*d*. The minimum work required to compress the system is represented by the shaded area under this line and is equal to 12 ft-lb. It is not necessary, of course, that the intermediate equilibrium states fall on a straight line on a force-distance diagram; this particular behavior was assumed only for the sake of simplicity.

It should be noted that the mere fact that a compression is carried out infinitely slowly is no assurance that the process is reversible. For example, suppose that an experiment is carried out with the same gas and the same apparatus (Fig. 10) but with a controllable amount of friction introduced between the piston and the cylinder (by means of a set screw, for example). With such an arrangement, the compression can be made to take place as slowly as desired even though a finite weight, such as a 4-lb weight, is placed on the piston. The process is not reversible since it involves friction. It is *internally reversible*, however, because the friction is confined to the surroundings and because the system itself (the gas) passes through exactly the same series of equilibrium states (and has exactly the same amount of work done on it) as it does if the process is reversible.

Let us consider next the work done by a system that undergoes an expansion. Suppose that the same gas and the same apparatus (Fig. 10) are used and that the mechanical friction involved in the operation of the apparatus can again be made vanishingly small. Suppose further that the total downward force is initially 5 lb and that, each time a 1-lb weight is removed from the piston, the piston reaches a new equilibrium position exactly 1 ft higher. Consider now three ways in which the piston might be caused to rise 4 ft:

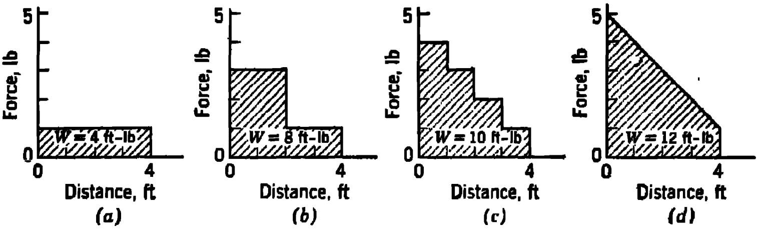

FIG. 12. Force-distance diagrams.

First, let one 4-lb weight be removed from the piston. The total downward force is then reduced to 1 lb, and the system acts against this force through a distance of 4 ft. The work done by the system is (1 × 4 =) 4 ft-lb. This amount of work is represented by the shaded area shown in Fig. 12*a*. Second, let the expansion be carried out by removing two 2-lb weights from the piston, one at a time, allowing the piston to rise 2 ft before the second weight is removed. The work done by the system this time is (3 × 2 + 1 × 2 =) 8 ft-lb, represented by the shaded area in Fig. 12*b*. Third, let the expansion be carried out by removing four 1-lb weights from the piston, one at a time, allowing the piston to rise 1 ft after each weight is removed. The work done by the system is (4 × 1 + 3 × 1 + 2 × 1 + 1 × 1 =) 10 ft-lb, represented by the shaded area in Fig. 12*c*.

Since the work done by the system in expanding becomes larger as the size of the weights removed from the piston is made smaller, it is evident that the *maximum* possible amount of work would be done by the system if the weights removed were infinitesimal in size. But, as in the preceding example, if this is done, the process is a *reversible process*. Hence, we conclude that if a system confined inside a cylinder equipped with a piston expands and if the system can attain only one equilibrium state corresponding to each total external force exerted on the piston, *the amount of work done by the system in expanding is a maximum if the process is reversible.*

If the force exerted on the piston and the position of the piston are again used to describe the state of the system, the intermediate equilibrium states that the system can attain are represented by the infinite series of points forming the straight line shown in Fig. 12*d*. The maximum possible work the system can do is represented by the shaded area under this line and is equal to 12 ft-lb.

If a controllable amount of friction is introduced between the piston and the cylinder, the expansion can be made to proceed as slowly as desired, but the process is not reversible because of the friction. The system itself again undergoes an *internally reversible process,* however, and passes through exactly the same states and does exactly the same amount of work as if the process were reversible.

Note that the work required to compress the system and the work done by it in reexpanding to its original state are exactly equal if both processes are either reversible or internally reversible. If they are neither of these kinds of processes, more work must be supplied to the system to compress it than is done by the system in reexpanding.

Note also that the external force acting on the system need not be exerted by weights placed on the piston. This arrangement was assumed so that the effect of the work on the surroundings could be easily visualized. Actually, the external resisting force may be exerted by any agent.

One reason why reversible and internally reversible processes are of interest, even though they cannot actually be realized in practice, is now apparent. Such processes represent the ideal or optimum in that the minimum work is needed to compress the system and the maximum work is done by the system in expanding if the processes are either reversible or internally reversible. By assuming that each of the processes involved in the operation of various kinds of equipment is such a process, we can calculate the ideal or best possible performance of the equipment. This information is useful in several ways. For example, it may serve as a criterion for determining whether an existing piece of equipment is operating satisfactorily. Or, if the ideal performance of some proposed new type of equipment is found to be better than the ideal performance of some existing type, in all probability the actual performance of the new type will be better than that of the old. Thus, improvements can sometimes be made without recourse to expensive experimental work. The ability to calculate the ideal performance of certain types of equipment is also of great value to the designer. The actual work done by a given type of turbine, for example, can be determined by experiment, and the ratio of the actual to the ideal can be calculated. This ratio can then be used in designing other turbines of the same type.

22. Reversible Processes and Heat. Although both work and heat were involved in the examples discussed in the preceding article, we directed our attention in particular to the work done. Let us now examine more carefully the requirements that must be satisfied in order that heat be transferred reversibly, and let us consider an example in which the temperature of the system changes.

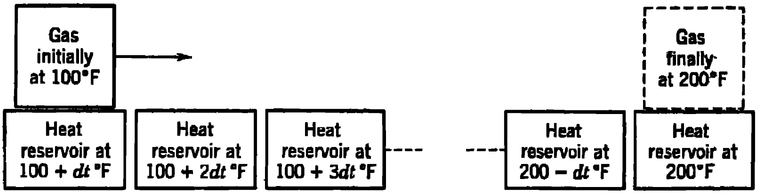
Gas initially at 100°F
Gas finally at 200°F
Heat reservoir at 100 + dt °F
Heat reservoir at 100 + 2dt °F
Heat reservoir at 100 + 3dt °F
Heat reservoir at 200 − dt °F
Heat reservoir at 200°F

FIG. 13.

Suppose that a system consisting of a quantity of gas initially at 100°F is caused to undergo a non-flow process because of the addition of heat alone; that is, the gas is not stirred during the process and is contained in a rigid vessel so that it can be neither compressed nor expanded. The addition of heat will cause both the pressure and the temperature of the gas to rise. Let us suppose that at the end of the process the temperature of the gas is 200°F.

The heat necessary to accomplish the foregoing process can be transferred to the gas by bringing the vessel containing it into contact with a heat reservoir whose temperature is, say, 500°F. Since the temperature of the system (the gas) and the temperature of its surroundings (the heat reservoir) differ by a finite amount, the system departs from a state of equilibrium with its surroundings by more than an infinitesimal amount. Moreover, the system departs by more than an infinitesimal amount from a state of internal equilibrium because the temperature of the gas at the side of the container in contact with the heat reservoir will be a finite amount higher than the temperature of the gas at the opposite side. For these reasons the process is neither reversible nor internally reversible.

The process can be carried out reversibly if the heat is transferred to the gas from a series of heat reservoirs, as shown in Fig. 13. The gas is first brought into contact with a heat reservoir whose temperature is just an infinitesimal amount higher than 100°F, and heat is allowed to flow into the gas until its temperature reaches that of the reservoir. The gas is then brought into contact with a second heat reservoir at a temperature just an infinitesimal amount higher than the first, and again heat is allowed to flow into the gas until its temperature reaches that of the second reservoir. This procedure is repeated with an infinite number of heat reservoirs, each at an infinitesimally higher temperature than the one before, until the temperature of the gas finally reaches 200°F. The process is reversible (and therefore also internally reversible) because at no time does the temperature of the system differ by more than an infinitesimal amount from the temperature of that part of its surroundings to which it is exposed; and therefore the system never departs from a state of equilibrium with its surroundings by more than an infinitesimal amount. Also, a finite temperature difference never exists between any two portions of the gas, and therefore the system never departs by more than an infinitesimal amount from a state of internal equilibrium.

The process just described is evidently one that can be carried out only in theory because, with an infinitesimal temperature difference between the gas and the heat reservoirs, an infinite amount of time is required for the temperature of the gas to reach that of each reservoir with which it is brought into contact. Moreover, an infinite number of such reservoirs is required. Since such a process cannot actually be carried out in practice, the question again arises, why should this type of process be of interest?

From the conclusions drawn in the preceding article regarding reversible processes and work, the student is likely to infer that less heat is required to raise the temperature of the gas and more heat is rejected by the gas in undergoing a drop in temperature if these processes are reversible. This is not true, however. Instead, exactly the same amount of heat is required whether the process is reversible or not, provided that the same amount of work is involved each time (none in the present example).

To understand why reversible processes involving heat are of interest, it is necessary first to know that heat can be converted partially into work and partially into heat at a lower temperature. For example, as shown in Fig. 14*a*, if 100 Btu of heat flow from a heat reservoir at a temperature of 500°F into an apparatus known as a **heat engine,** it is theoretically possible for the heat engine to use this energy to do 41.7 Btu of work on some other system and to reject 58.3 Btu of heat to another heat reservoir at a temperature of 100°F. Second, it is necessary to know that, as the temperature of the source from which the heat is obtained decreases, a smaller fraction of the heat taken from it can theoretically be converted into work. For example, as shown in Fig. 14*b*, if 100 Btu of heat are supplied to an ideal heat engine from a heat reservoir at a temperature of 200°F, instead of 500°F, only 15.2 Btu of this energy can theoretically be converted into work (if the temperature of the reservoir to which the remainder of the energy is rejected as heat is again at 100°F). These facts will be discussed in greater detail in Chapter 7 after the First and Second Laws of Thermodynamics have been introduced. For the present, the student is asked to accept the statement that *the higher the temperature of the system from which heat is taken, the greater is the fraction of this heat that can theoretically be converted into work.*

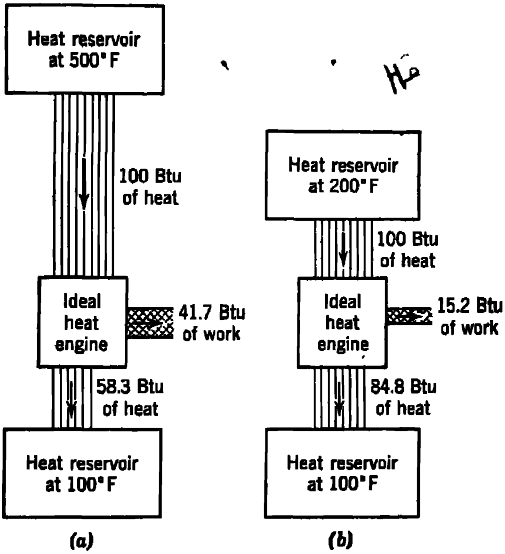

FIG. 14. Diagram showing how the conversion of heat into work depends upon the temperature of the source.

It follows from the foregoing statement that, since heat always flows from one system to another at a *lower* temperature, the transfer of energy in the form of heat is accompanied by a decrease in the fraction of this energy which can theoretically be converted into work; that is, the transfer of energy in the form of heat is accompanied by a deterioration in what might be called its "grade." Furthermore, the lower the temperature of the receiving system, the greater is the deterioration in the grade of the energy transferred. One reason why reversible processes involving heat are of interest is now apparent. In such processes, the temperature of the receiving system is only an infinitesimal amount lower than that of the emitting system; therefore, *if the process is reversible, the transfer of heat from one system to another is accompanied by no finite deterioration in the grade of the energy transferred.*

Just as a system may undergo an internally reversible process while being irreversibly compressed or expanded, a system may also undergo an internally reversible process while being irreversibly heated or cooled. Suppose, for example, that the gas in the foregoing illustration is initially at 100°F, that it receives heat from a reservoir at 500°F, but that a poor conductor of heat is interposed between the gas and the reservoir. If the conductor has a low enough thermal conductivity, the face of the conductor in contact with the vessel containing the gas will assume a temperature that is always just an infinitesimal amount higher than that of the gas. Therefore, even though the process is not reversible, the gas itself will behave exactly as though it were. Since the source of the irreversibility is confined to the surroundings, the system will undergo an internally reversible process.

Some of the reasons why reversible and internally reversible processes are of interest have been discussed in this and in the preceding article. In addition to these reasons, reversible processes are of importance in thermodynamics because they permit a scale of temperature known as the "thermodynamic scale" to be defined; and this temperature scale in turn leads to the concept of a very useful property called "entropy." Part II of this text is devoted almost entirely to a discussion of these concepts.

23. Work Done during a Non-Flow Process. If a system is confined inside a stationary cylinder equipped with a movable piston, the work done on or by it can be calculated from its properties if (*a*) the piston moves slowly enough so that the system exerts the same pressure on the moving piston as it does on the stationary walls of the cylinder, and if (*b*) no work is done on or by the system except by or on the piston (the system is not stirred, for example). These conditions are satisfied exactly if the process is reversible or if the system itself undergoes an internally reversible process. They are satisfied approximately if the system consists of a fluid undergoing an expansion or compression in a reciprocating engine, such as a steam engine, internal-combustion engine, or an air compressor. An expression for the work done under these conditions can be derived as follows:

Suppose that the system consists of 1 lb of any fluid. The pressure of the fluid may change during the process, but, while the piston is moving only an infinitesimal distance dx, the pressure will remain very nearly constant. The force exerted by the fluid on the piston (or vice versa) while this infinitesimal change in the position of the piston is taking place is pA, where p denotes the absolute pressure of the fluid and A the cross-sectional area of the piston. The work dW done by the system while the piston moves the infinitesimal distance dx is equal to the product of the force pA and the distance dx through which it acts; that is,

$$dW = pA\,dx.$$

But $A\,dx$ is the change dv in the specific volume of the fluid. Therefore,

$$dW = p\,dv. \tag{6}$$

The total work W done by the system while the piston moves a finite distance is determined by integrating Eq. 6:

$$W = \int p\,dv. \tag{7}$$

Note that, if the system expands, the work dW or W is done *by the system*. In accordance with the convention adopted in Art. 17, this work should be positive in sign. It will be positive since its sign is the same as that of dv, and dv is positive when the system expands. Similarly, if the system is compressed, the work dW or W is done *on the system* and should be negative in sign. It will be negative because dv is negative when the system is compressed. Regarding units, if the pressure p is expressed in pounds per square foot absolute and the specific volume v is expressed in cubic feet per pound, the units of dW or W are foot-pounds per pound of the system.

To be able to integrate Eq. 7, it is necessary to know how the pressure p varies with the specific volume v. If this is known, p can be expressed in terms of v, this expression can be substituted into Eq. 7, and the integration can then be performed. This procedure is illustrated in the Illustrative Problems which follow this article.

It is of interest to note that, whereas the integral of $p\,dv$ is equal to the work done by the system under the forenamed conditions, under other conditions this integral may not represent work at all. For example, suppose that the system consists of a gas confined inside a vertical cylinder equipped with a piston; and suppose that the piston is made so thin that it is virtually weightless, that there is no friction between the piston and the cylinder, and that the upper face of the piston is not acted upon by atmospheric pressure. Suppose further that the external resisting force is exerted, not by weights placed on the piston, but by a series of stops spaced an infinitesimal distance apart, as shown in Fig. 15. If the first stop is withdrawn, the piston will rise until it meets the second stop, and the gas will expand an infinitesimal amount. After a certain length of time, the turbulence or waves resulting from this sudden expansion of the gas will be dissipated, and the gas will return to a state of equilibrium. If this procedure is repeated with each successive stop, the gas will pass through an infinite series of equilibrium states, and the integral of $p\,dv$ for the process can be evaluated. This integral will not represent work, however, since no work will have been done. Note that, although the *average* velocity of the piston for the entire process will be very low, its velocity *while moving from one stop to the next* will be infinite.* Note also that, since the piston is both frictionless and weightless, the pressure the gas exerts on it while it is moving from one stop to the next is zero. The pressure on the piston is the same as that on the stationary walls of the cylinder only when the gas is in equilibrium just before each stop is withdrawn.

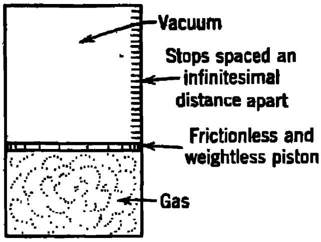
Vacuum
Stops spaced an infinitesimal distance apart
Frictionless and weightless piston
Gas

FIG. 15.

ILLUSTRATIVE PROBLEMS

27. A system consisting of 3 lb of water confined inside a cylinder equipped with a piston is heated from 80° to 500°F. If the pressure of the system remains constant at 100 psia during the process, and if the piston moves slowly, how much work is done by the system?

Solution. The work done per pound of the system can be determined by Eq. 7. Since the pressure remains constant, we may take p out from under the integral sign and write

$$W = p\int dv$$

$$= p(v_2 - v_1).$$

The saturation temperature corresponding to 100 psia is 327.81°F (from Table 2 of the *Steam Tables*). Hence, in the initial state the system is subcooled water,

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* After a stop has been withdrawn, but before the piston has started to move, the force F acting on the piston is still finite. But, since the piston is assumed to have no mass, it follows from the law of motion,

$$F = ma,$$

that its acceleration a must be infinite. Therefore, the piston will move instantaneously from one stop to the next.

and in the final state it is superheated steam. The initial specific volume is 0.01608 cu ft per lb (from Table 1, neglecting the effect of pressure) and the final specific volume is 5.589 cu ft per lb (from Table 3). Therefore,

$$W = 100 \times 144 \times (5.589 - 0.01608)$$

$$= 80{,}300 \text{ ft-lb per lb.}$$

The total work done by the system is ($3 \times 80{,}300 =$) 241,000 ft-lb.

28. Two pounds of air are compressed reversibly from a pressure of 14.7 psia to a pressure of 60 psia in a cylinder equipped with a piston. If the temperature of the air remains constant at 70°F during the process, how much work is required?

Solution. If we assume that air behaves as a perfect gas, the relation between p, v, and T for the system is (by Eq. 3, page 22)

$$pv = RT.$$

Hence, $p = RT/v$. Substituting this expression for p into Eq. 7, and noting that T remains constant, we get

$$W = RT \int \frac{dv}{v}$$

$$= RT \ln \left(\frac{v_2}{v_1}\right).$$

The values of v_1 and v_2 could be determined by Eq. 3. However, since T remains constant, by Eq. 3, $p_1 v_1 = p_2 v_2$. Hence, we may replace (v_2/v_1) with (p_1/p_2) and write

$$W = RT \ln \left(\frac{p_1}{p_2}\right).$$

Therefore, using $R = 53.3$ ft per deg F for air (Table I, page 23), we obtain

$$W = 53.3 \times (70 + 460) \ln \left(\frac{14.7}{60}\right)$$

$$= 53.3 \times 530 \times (-1.408)$$

$$= -39{,}700 \text{ ft-lb per lb.}$$

The negative sign indicates that the work is done *on the system.* The total work done is ($2 \times 39{,}700 =$) 79,400 ft-lb.

29. A certain fluid, initially at a pressure of 200 psia and having a volume of 3 cu ft, undergoes a non-flow reversible process during which the product $pV^{1.3}$ remains constant. If the final pressure is 40 psia, how much work does the system do?

Solution. Equation 7 may be written in the form

$$W = \int p \, dV, \qquad (7a)$$

where V represents the *total* volume of the system (cu ft) and W, the total work done by the system (ft-lb). Since the product $pV^{1.3}$ remains constant, we may write

$$pV^{1.3} = p_1V_1^{1.3} = p_2V_2^{1.3}.$$

Hence,

$$p = \frac{p_1V_1^{1.3}}{V^{1.3}}$$

$$= \frac{200 \times 144 \times 3^{1.3}}{V^{1.3}}$$

$$= \frac{120{,}000}{V^{1.3}}.$$

Substituting this expression for p in Eq. 7a and integrating, we get

$$W = 120{,}000\int \frac{dV}{V^{1.3}}$$

$$= 120{,}000\left[\frac{V_2^{-0.3}}{-0.3} - \frac{V_1^{-0.3}}{-0.3}\right]$$

$$= 400{,}000\left[\frac{1}{V_1^{0.3}} - \frac{1}{V_2^{0.3}}\right].$$

The final volume V_2 can be determined from the relation $p_1V_1^{1.3} = p_2V_2^{1.3}$. Rearranging this equation, we may write

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{1/1.3}$$

$$= 3 \times (\tfrac{200}{40})^{1/1.3}$$

$$= 10.35 \text{ cu ft.}$$

Therefore,

$$W = 400{,}000\left[\frac{1}{3^{0.3}} \quad 10.35^{0.3}\right]$$

$$= 400{,}000[0.719 - 0.496]$$

$$= +89{,}200 \text{ ft-lb.}$$

The positive sign indicates that the work is done *by the system.*

PROBLEMS

30. Three pounds of air having an initial volume of 18 cu ft undergo a non-flow reversible process during which the pressure remains constant at 50 psia. If the final temperature of the air is 300°F, how much work does the air do?

Ans. −7920 ft-lb.

31. Two pounds of steam initially at 75 psia and 350°F undergo a non-flow reversible process during which the pressure drops to 25 psia. If the process takes place in such a way that the product pv remains constant, how much work does the steam do?

32. Two pounds of air initially at 320°F undergo a non-flow reversible process during which the pressure remains constant at 150 psia. If 25,000 ft-lb of work are done on the air, what is its final volume? *Ans.* 2.69 cu ft.

33. Three cubic feet of a gas initially at 20 psia are compressed by a non-flow reversible process to a volume of 0.6 cu ft. If the process takes place in such a way that the product $pV^{1.35}$ remains constant, how much work is done on the gas?

34. A pound of steam initially at 100 psia and 500°F undergoes a non-flow reversible process during which the product of its pressure times its volume raised to a constant power n remains constant (that is, pv^n = a constant). If in its final state the steam is at 35 psia and 290°F, how much work does it do? *Ans.* 56,900 ft-lb.

24. Pressure-Volume Diagrams. Suppose that a system confined inside the apparatus shown in Fig. 1 undergoes a non-flow process; and suppose that the process proceeds so slowly that at each instant the system exerts very nearly the same pressure on the moving piston as on the stationary walls of the cylinder. The relation between the pressure and the volume of the system during the process can be shown graphically by plotting the pressure at each instant against the volume at the same instant. An infinite series of points, spaced infinitesimal distances apart, will be obtained. These points will appear as a continuous line and might form a curve such as the one shown connecting states 1 and 2 in Fig. 16. The area under the curve (the shaded area in Fig. 16) is equal to the integral of $p\,dv$. But in the preceding article it was shown that the work done by a system confined inside a cylinder equipped with a movable piston is also equal to the integral of $p\,dv$ (Eq. 7) if (*a*) the piston moves slowly enough so that the system exerts the same pressure on both the moving piston and the stationary walls of the cylinder, and (*b*) no work is done on or by the system except by or on the piston. It follows, therefore, that the work done under these conditions is represented by the area under the curve which shows on a pressure-volume diagram the relation between these properties during the process.

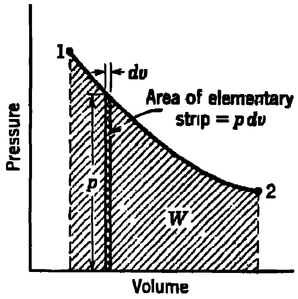


FIG. 16. Pressure-volume diagram.

If the system expands, the area under the pressure-volume curve for the process is considered positive because work will be done *by the system*, and such work has been designated as positive. Referring to Fig. 16, for example, if the system expands from 1 to 2, the area under the curve is considered a positive area. If the system is compressed, the area is considered negative because work will be done *on the system*, and such work has been designated as negative.

It is of interest to note that, if a system undergoes a non-flow process during which its volume changes, the work done depends upon the kind of process connecting its initial and final states. For example, if points 1 and 2 in Fig. 16 represent the initial and final states, any number of lines can be drawn connecting them. Each of these lines represents a relation which may exist between the pressure and the volume. The system can be made to follow any of these relations by properly controlling both the external force acting on the piston and the heat added to or withdrawn from the system. Since the areas under these lines will in general be different, and since these areas represent the work done by the system, the work evidently depends upon the process connecting the given end states.

25. Specific Heat. Suppose that a system having a mass m undergoes a non-flow process during which an amount of heat Q is added to it and its temperature changes from t_1 to t_2. The *average* amount of heat added per unit mass and per degree rise in temperature is called the **average specific heat** (c_{ave}) of the system for the given process: that is,

$$c_{\text{ave}} = \frac{Q}{m(t_2 - t_1)}. \tag{8a}$$

In general, if any system undergoes a non-flow process, the amount of heat added to it per unit mass and per degree rise in temperature is different for various parts of the process. The temperature of the system might rise continuously throughout the process, for example, but the heat added during the first one-degree rise might be more or less than the heat added during the last one-degree rise. On account of this, it is necessary to introduce what is known either as the **instantaneous specific heat** or simply as the **specific heat** of the system for each part of the process. This term, denoted by the symbol c, is defined as the amount of heat dQ added to the system during each infinitesimal part of the process divided by the product of the mass m times the temperature rise dt of the system; that is,

$$c = \frac{dQ}{m\,dt}. \tag{8b}$$

For many of the processes of interest in engineering, the instantaneous specific heats remain practically constant. For such processes, the instantaneous specific heat and the average specific heat are equal. If the mass m is measured in pounds, Q and dQ in British thermal units, and the temperature t in degrees Fahrenheit, the units of c_{ave} and of c are British thermal units per pound degree Fahrenheit. These are the units usually used in engineering.

Since the temperature of a system may increase, may remain constant, or may decrease while heat is being added to the system (see Art. 18), the average specific heat c_{ave} and the specific heat c may have values ranging from positive to negative infinity. For example, they are positive and infinite if the temperature of the system remains constant while heat is added; they are positive and finite if the temperature increases while heat is added; they are zero for adiabatic processes; and they are negative and finite if the temperature decreases while heat is added.

If either the average specific heat c_{ave} or the specific heat c of a system undergoing a non-flow process is known, the amount of heat added to or removed from the system during the process can be determined from the following equations (obtained by rearranging Eqs. 8*a* and 8*b*):

$$Q = mc_{\text{ave}}(t_2 - t_1), \tag{9a}$$

and

$$Q = m\int c\,dt. \tag{9b}$$

In order to integrate Eq. 9*b*, the specific heat c must be expressed as a function of the temperature t. This procedure is illustrated in Illustrative Problem 36 following this article. If c is a constant, the integration yields Eq. 9*a*.

The specific heats for the following two types of processes are of particular importance: In the first, the system undergoes a non-flow process during which (*a*) its volume remains constant, and (*b*) no work of any kind is done by or on it (the system is not stirred, for example). The specific heat of the system for this process is known as the **specific heat at constant volume** and is denoted by the symbol c_v. In the second, the system undergoes a non-flow process during which (*a*) its pressure remains constant, and (*b*) no work is done by or on it except on or by one of its boundaries, which moves slowly enough so that the system exerts the same pressure on both its moving and stationary boundaries (that is, the only work done is equal to the integral of $p\,dv$). The specific heat of the system for this process is known as the **specific heat at constant pressure** and is denoted by the symbol c_p.

In general, c_v and c_p are not constant for any system but depend upon the pressure and the temperature. The specific heats at constant volume and at constant pressure for dry air, for example, depend upon both the pressure and the temperature, as shown in Fig. 27 on page 94. For many of the systems of interest in engineering, however, both c_v and c_p may be considered approximately constant over moderate ranges of pressure and temperature.

ILLUSTRATIVE PROBLEMS

35. If c_p for air is assumed constant and equal to 0.241 Btu per lb deg F, how much heat is required to raise the temperature of a quantity of atmospheric air having an initial volume of 1000 cu ft from 70° to 150°F? Assume that the barometric pressure is 14.7 psia.

Solution. The mass of the air can be determined from the relation $pv = RT$:

$$14.7 \times 144 \times \frac{1000}{m} = 53.3 \times (70 + 460),$$

whence,

$$m = 75.0 \text{ lb}.$$

The amount of heat required can now be determined by Eq. 9*a*:

$$\begin{aligned} Q &= 75.0 \times 0.241 \times (150 - 70) \\ &= 1445 \text{ Btu}. \end{aligned}$$

Note that, although the air expands during the process and does work on the surrounding atmosphere, it always exerts the same pressure on all parts of the imaginary envelope enclosing it. Therefore, the specific heat for the process is c_p.

36. The specific heat of air for a certain process is given by the equation

$$c = 0.15 + 0.000003t^2,$$

where c has the units British thermal units per pound degree Fahrenheit and t has the units degrees Fahrenheit. Determine the values of c at 100°F and at 300°F, and determine the value of c_{ave} for the range of temperature from 100° to 300°F.

Solution. At 100°F,

$$\begin{aligned} c &= 0.15 + 0.000003 \times 100^2 \\ &= 0.18 \text{ Btu per lb deg F}, \end{aligned}$$

and, at 300°F,

$$\begin{aligned} c &= 0.15 + 0.000003 \times 300^2 \\ &= 0.42 \text{ Btu per lb deg F}. \end{aligned}$$

The desired value of c_{ave} can be determined by Eq. 8*a* after the heat added to the air in heating it from 100° to 300°F has been calculated by Eq. 9*b*. For 1 lb of air,

$$\begin{aligned} Q &= \int_{100}^{300} (0.15 + 0.000003t^2)\,dt \\ &= 0.15(300 - 100) + 0.000003 \times \frac{(300^3 - 100^3)}{3} \\ &= 56 \text{ Btu per lb}. \end{aligned}$$

Hence,

$$\begin{aligned} c_{\text{ave}} &= \frac{56}{(300 - 100)} \\ &= 0.28 \text{ Btu per lb deg F}. \end{aligned}$$

Note that c_{ave} is not equal to the arithmetic average of the values of c at 100°F and at 300°F.

PROBLEMS

37. The value of c_v for gaseous oxygen is approximately independent of the pressure but is dependent upon the temperature according to the equation

$$c_v = 0.1545 + 0.375 \times 10^{-8}T^2,$$

where the units of c_v and T are British thermal units per pound degree Fahrenheit and degrees Fahrenheit absolute respectively. Determine the average value of c_v for the temperature range 0° to 1000°F.

38. Five pounds of air initially at 70°F undergo a non-flow reversible process during which the pressure remains constant at 90 psia. If the final volume of the air is 14 cu ft, how much work does the air do during the process, and how much heat is added to it? Assume that c_p for air is constant and equal to 0.241 Btu per lb deg F. *Ans.* 40,000 ft-lb; 181 Btu.

39. A 12-cu-ft tank contains air initially at 20 psia and 80°F. How much heat must be added to the air to raise its pressure to 30 psia? Assume that c_v for air is constant and equal to 0.172 Btu per lb deg F.

40. A quantity of air undergoes a non-flow reversible process during which its pressure remains constant. If 60 Btu of heat are added to the air, how much work does the air do? Assume that the value of c_p for air is constant and equal to 0.241 Btu per lb deg F.

41. One pound of a certain substance undergoes a non-flow process during which its temperature rises from 50° to 200°F. If the substance absorbs 47 Btu of heat while its temperature changes from 50° to 150°F and absorbs 28 Btu while its temperature changes from 150° to 200°F, and if the relation between its specific heat for the process and its temperature is known to be linear, what is its specific heat at 200°F? *Ans.* 0.59 Btu per lb deg F.

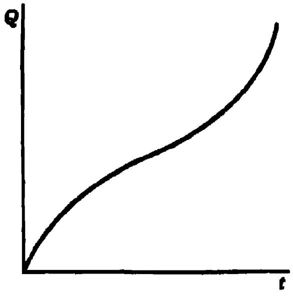
GENERAL PROBLEMS

42. Five pounds of air are heated from 70° to 250°F by a constant pressure, non-flow process. How much work does the air do?

43. Two pounds of air undergo a non-flow process during which the pressure increases from 15 to 50 psia while the volume decreases from 40 to 9 cu ft. If 150 Btu of heat are absorbed from the air, what is the value of c_{ave} for the process? *Ans.* 0.370 Btu per lb deg F.

44. A pound of air initially at 60 psia and having an initial volume of 4 cu ft undergoes a non-flow process during which its temperature remains constant. If the air does 30,000 ft-lb of work, what is its final pressure? *Ans.* 25.2 psia.

45. A certain fluid undergoes a non-flow process during which its pressure remains constant. If the quantity of heat required to raise the temperature of the substance to any value t is plotted against t, a curve having the form shown is obtained. Draw a rough graph showing how the value of c_p for the substance depends upon the temperature.



46. A pound of a gas initially at 15 psia and having a volume of 12 cu ft undergoes a non-flow reversible compression. In its final state the gas is at 45 psia and has a volume of 5 cu ft. (*a*) How much work is required to compress the gas if the process which it undergoes appears as a straight line on a pressure-volume diagram? (*b*) How much work is required if the gas is first compressed at constant pressure to a volume of 5 cu ft and then heated at constant volume until its pressure rises to 45 psia?

47. A gas having a molecular weight of 25 undergoes a constant pressure, non-flow process during which it absorbs 40 Btu of heat and does 12,000 ft-lb of work. If c_p for the gas is a constant, what is its value? *Ans.* 0.206 Btu per lb deg F.

48. A pound of air initially at 50 psia and 200°F undergoes a non-flow reversible process during which the product $pv^{1.4}$ remains constant. If the air does 16,000 ft-lb of work during the process, what is its final temperature?

49. A 10-cu-ft tank contains air initially at 15 psia and 60°F. A pipe line, in which the pressure remains constant at 20 psia, is connected to the tank, but a check valve in the line prevents air from flowing from the line into the tank. If heat is supplied *only to the air within the tank*, how much heat is required to raise the temperature within the tank to 500°F? The value of c_v for air may be taken as 0.172 Btu per lb deg F, and the value of c_p as 0.241 Btu per lb deg F. [Note that the weight of air within the tank does not remain constant after the pressure reaches 20 psia.] *Ans.* 65.6 Btu

SYMBOLS

a	acceleration
A	cross-sectional area of piston
c	specific heat; c_{ave} = average specific heat; c_p = specific heat at constant pressure; c_v = specific heat at constant volume
F	force
J	mechanical equivalent of heat
m	mass
p	pressure
Q	heat added to system
R	gas constant
t	temperature
T	absolute temperature
v	specific volume
V	total volume
W	work done by system
x	distance

Chapter 3

THE FIRST LAW OF THERMODYNAMICS

The science of engineering thermodynamics is built upon two basic principles known as the First and Second Laws of Thermodynamics. These laws cannot be proved, but they are believed to be true because no exceptions have ever been found either to the laws themselves or to any of the concepts developed from them. The purpose of the present chapter is to discuss the first of these laws and to develop certain equations that are based upon it.

26. The First Law of Thermodynamics. The **First Law of Thermodynamics** is the Principle of the Conservation of Energy applied to finite systems. It may be stated as follows: *If any system undergoes a process during which energy is added to or removed from it* (in the form of work or heat), *none of the energy added is destroyed within the system and none of the energy removed is created within the system.**

The validity of this statement of the First Law cannot be tested experimentally since, although it is possible to measure the amounts of heat and work added to or removed from a system during a process, it is not possible to determine directly whether energy has been created or destroyed within the system. However, two other statements of the First Law that can be tested experimentally are given in Art. 28. These statements are derived from and are equivalent to the foregoing statement. They involve only the relations that exist between the work and heat added to or removed from a system during any process. Experiments to test their validity have been carried out, and none of them has yielded results that contradict these statements.

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* One apparent exception is the conversion of matter into energy. Modern physics teaches that matter itself is a form of energy and that the conversion of matter into energy therefore does not violate the First Law of Thermodynamics. This apparent exception need not concern us, however, since the destruction of matter does not occur in the usual applications of thermodynamics to engineering.

27. The Existence of Internal Energy. When any system undergoes a non-flow process, energy may be both added to and removed from it during the process, perhaps added in the form of heat and removed in the form of work. Experience shows that in general the amounts of energy added to and removed from the system are not equal. For example, if the system consists of a fluid confined inside a rigid container, and if the system undergoes a non-flow process during which heat is supplied to it, the energy added in the form of heat does not equal the energy leaving in the form of work because the work done is zero. As another example, suppose that the system consists of a gas confined inside the cylinder-and-piston apparatus shown in Fig. 1. If the gas is allowed to expand by removing some of the weights from the piston, energy will leave the system in the form of work. But if the cylinder is insulated, no energy in the form of heat will be added (that is, the process will be *adiabatic*), and the amounts of energy added to and removed from the system during the process will again be unequal.

In the first of the foregoing examples the energy leaving the system in the form of work is zero, and in the second the energy added to the system in the form of heat is zero. Let us consider still another example in which neither of these quantities is zero.

Suppose that a system consisting of 1 lb of air is confined inside a cylinder equipped with a piston and that initially the air is at a pressure of 15 psia and a temperature of 70°F. Now suppose that sufficient heat is added to the air to raise its temperature to 200°F and that during the process the piston moves slowly outward while the pressure remains constant. The amount of energy added to the air in the form of heat can be calculated from the specific heat at constant pressure for air. Thus, since c_p for air is approximately equal to the constant value 0.241 Btu per lb deg F for this range of temperature, by Eq. 9*a*

$$Q = 0.241 \times (200 - 70)$$
$$= 31.3 \text{ Btu per lb.}$$

The amount of energy leaving the air in the form of work can be calculated by integrating Eq. 7. Thus, since the pressure p remains constant during the process,

$$W = \int p \, dv \tag{7}$$
$$= p(v_2 - v_1).$$

The initial and final specific volumes of the air can be determined from the relation $pv = RT$:

$$v_1 = \frac{53.3 \times (70 + 460)}{15 \times 144}$$

$$= 13.1 \text{ cu ft per lb},$$

and

$$v_2 = \frac{53.3 \times (200 + 460)}{15 \times 144}$$

$$= 16.3 \text{ cu ft per lb}.$$

Hence,

$$W = 15 \times 144 \times (16.3 - 13.1)$$

$$= 6910 \text{ ft-lb per lb},$$

or

$$\frac{W}{J} = \frac{6910}{778}$$

$$= 8.9 \text{ Btu per lb}.$$

As in the two preceding examples, the amount of energy added to the system (31.3 Btu in the form of heat) is not equal to the amount of energy leaving it (8.9 Btu in the form of work).

It is evident from these examples that, in general, the amounts of energy added to and removed from any system during a non-flow process are not equal. The First Law of Thermodynamics tells us, however, that energy cannot be created or destroyed within the system. Therefore, *if more energy is added to any system during a given process than is removed, the difference must remain within the system.* This energy residing within any system is called **internal energy**; and the **change in the internal energy** of any system undergoing a non-flow process is defined as the difference between the net heat added to it and the net work done by it during the process. The change may be positive or negative. For instance, in the first of the foregoing examples, in which heat is added to the system while its volume remains constant, the work done by the system is zero. Hence, the internal energy of the system increases by the amount of heat added. In the second example, in which the system expands adiabatically and does work, its internal energy decreases by the amount of work done. In the third example, the internal energy of the air increases by (31.3 − 8.9 =) 22.4 Btu per lb. Note that the existence of internal energy is known as a consequence of the First Law of Thermodynamics.

28. Internal Energy a Property. Suppose that a system changes from some initial state to some final state first by one and then by another non-flow process. Let us determine how the change in the internal energy of the system for the first process compares with that for the second. Before considering the general case, let us consider a specific example.

In the preceding article it was shown that when 1 lb of air initially at 15 psia and 70°F undergoes a non-flow process during which its pressure remains constant while its temperature rises to 200°F, the internal energy of the air increases by 22.4 Btu per lb. As explained in Art. 16, these initial and final states may be represented by points 1 and 2 on the pressure-volume diagram shown in Fig. 17; and the constant pressure process joining them may be represented by the horizontal line marked *A*.

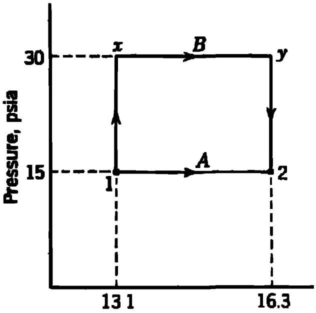

FIG. 17.

The air may also be caused to change from state 1 to state 2 by (*a*) first heating it at constant volume until its pressure rises to 30 psia, (*b*) then heating it at constant pressure until its volume is equal to the volume at state 2, and finally (*c*) cooling it at constant volume until it reaches state 2. This process is also shown on Fig. 17 (1 to *x* to *y* to 2) and is marked *B*.

To determine the change in the internal energy of the air for process *B*, it is necessary to calculate the net heat added to the air and the net work done by it during this process. The net heat added can be calculated from the specific heats of air at constant volume and constant pressure after the temperature of the air at states *x* and *y* have been determined from the relation $pv = RT$:

$$T_x = \frac{30 \times 144 \times 13.1}{53.3}$$

$$= 1060°\text{F abs (or } t_x = 600°\text{F}),$$

and

$$T_y = \frac{30 \times 144 \times 16.3}{53.3}$$

$$= 1320°\text{F abs (or } t_y = 860°\text{F}).$$

The specific heats at constant volume and constant pressure are approximately constant for air at the temperatures involved and are equal to 0.172 and 0.241 Btu per lb deg F respectively. Hence, for process *B*, the net heat added to the air is

$$\begin{aligned} Q &= 0.172 \times (600 - 70) + 0.241 \times (860 - 600) + 0.172 \times (200 - 860) \\ &= 91.1 + 62.6 - 113.5 \\ &= 40.2 \text{ Btu per lb.} \end{aligned}$$

Since the work done during the two constant volume portions of process B is zero, the net work done by the air during this process is equal to the work done in expanding from state x to state y at constant pressure; that is,

$$\begin{aligned} W &= p(v_y - v_x) \\ &= 30 \times 144 \times (16.3 - 13.1) \\ &= 13{,}800 \text{ ft-lb per lb,} \end{aligned}$$

or

$$\begin{aligned} \frac{W}{J} &= \frac{13{,}800}{778} \\ &= 17.8 \text{ Btu per lb.} \end{aligned}$$

Therefore, the change in the internal energy of the air during process B is $(40.2 - 17.8 =)$ 22.4 Btu per lb, which is also the change in the internal energy of the air during process A.

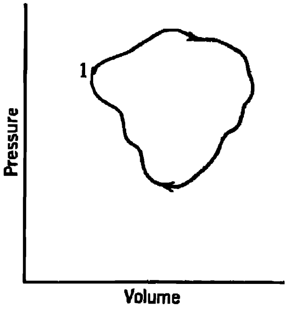

FIG. 18. Pressure-volume diagram for a system undergoing a cycle.

The foregoing result leads us to suspect that perhaps the change in the internal energy of any system depends only upon the initial and final states of the system and not upon the process connecting them. To determine whether this is true, let us consider the following:

Suppose that a system undergoes a series of processes such that its final state is exactly the same as its initial state. Such a series of processes is called a cycle and might appear on a pressure-volume diagram as the closed figure shown in Fig. 18. For any system undergoing a cycle, the net energy added in the form of heat must equal the net energy leaving in the form of work; that is,

$$Q_{\text{cycle}} = \frac{W_{\text{cycle}}}{J}, \tag{10}$$ ✓

where Q_{cycle} and W_{cycle} denote the net amount of heat added and the net amount of work done during one complete cycle. If this were not true, it would be possible either to create or to destroy energy within the system and thus to violate the First Law of Thermodynamics. For example, suppose that at the end of the cycle more energy has left the system in the form of work than was added to it in the form of heat. The net result, then, is that energy has left the system and must have come from within it. But, since the system has returned to its initial state, it could again undergo the same cycle, and again some energy would leave it. It is evident that, if this were possible, by having the system repeat the same cycle over and over, an unlimited amount of energy could be taken from it without producing any change in it; that is, energy could be created. Since this is impossible, it follows that W_{cycle}/J cannot be greater than Q_{cycle}. Similarly, W_{cycle}/J cannot be less than Q_{cycle} because then energy could be destroyed within the system. The truth of Eq. 10 is therefore established.

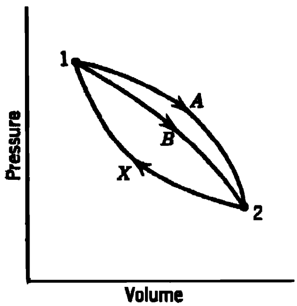

FIG. 19. Pressure-volume diagram for a system undergoing two cycles with one process common to both.

Now consider any system, and suppose that it can change from any state 1 to any other state 2 either by the non-flow process A or by any other non-flow process B, as shown in Fig. 19. Let process X be a non-flow process by which the system can return from state 2 to state 1. Let the net heat added during each of these processes be denoted by Q_A, Q_B, and Q_X, and the net work done by W_A, W_B, and W_X. If the system is now allowed to undergo two cycles, one consisting of processes A and X and the other of processes B and X, by Eq. 10 we may write

$$Q_A + Q_X = \frac{W_A + W_X}{J},$$

and

$$Q_B + Q_X = \frac{W_B + W_X}{J}.$$

Subtracting the second equation from the first and rearranging, we get

$$Q_A - \frac{W_A}{J} = Q_B - \frac{W_B}{J}. \qquad \checkmark \qquad (11)$$

But the left side of this equation is the change in the internal energy of the system when it passes from state 1 to state 2 by process A, and the right side is its change in internal energy when it passes from the same initial to the same final state by any other process B. Therefore it follows that *the change in the internal energy of any system depends only upon the initial and final states of the system and not upon the process connecting these states.*

The foregoing behavior is characteristic of a change in a *property* of a system. For example, the change in the pressure or in the specific volume of a system depends only upon its initial and final pressures or specific volumes and not upon the process by which the system passes from one state to the other. Conversely, any quantity that depends only upon the initial and final states of the system, and not upon the process connecting these states, is the change in a property. Therefore, *the internal energy of any system is a property of the system.*

If a system is at rest and is subject to no surface effects and to no magnetic or electric effects (that is, if the state of the system is determined when any two of its properties are specified), its internal energy will be denoted by the symbol u; and the change in the internal energy of such a system when it passes from any state 1 to any other state 2 will be denoted by $u_2 - u_1$.

Equations 10 and 11 are based only upon the statement of the First Law of Thermodynamics given in Art. 26, and either equation may be taken as an equivalent statement of this law. Equation 10 states that, *if any system undergoes a cycle, the net heat added to it is equal to the net work done by it* (both quantities being measured in the same units). Equation 11 states that, *if any system changes from one state to another, the difference between the net heat added and the net work done is a constant, regardless of the process* (both quantities again being measured in the same units). As explained in Art. 26, these two statements of the First Law involve only the relation existing between the heat added to and the work done by the system, and therefore their validity may be readily tested experimentally.

29. The Non-Flow Energy Equation. In the two preceding articles it was shown that when any system undergoes a non-flow process, (*a*) the heat added to the system is not in general equal to the work done by the system, but (*b*) the difference between the heat added and the work done is a constant for all processes by which the system can change from its initial to its final state. It was also explained that, since the difference between the heat added and the work done does not depend upon the process, this difference is the change in a property of the system. The property was given the name "internal energy." Expressed algebraically, the foregoing statements may be written

$$Q - \frac{W}{J} = u_2 - u_1,$$

or

$$Q = u_2 - u_1 + \frac{W}{J}. \tag{12}$$

This equation is a statement in mathematical form of the First Law of Thermodynamics applied to a non-flow process. It will be called the **non-flow energy equation.**

It should be noted that the heat Q and the work W which appear in Eq. 12 are associated with a process, whereas the internal energy u is associated with a state; that is, Q and W are quantities of energy added to or removed from the system *during a process*, whereas u is the amount of energy stored within the system *at any state* of the system.

The difference between the internal energies at any two states of a system can be determined experimentally by measuring the heat added to and the work done by the system when it changes by any conveneint process from one state to the other. The difference in the internal energies is then calculated by subtracting the work done from the heat added. Such experiments (together with certain other experiments *) have been carried out for most systems of importance in engineering, and the results are available either in the form of equations or in the form of tables or graphs. For some systems, such as gases that behave approximately as perfect gases, the difference between the internal energies at any two states can be expressed satisfactorily by simple equations. For other systems, such as liquids and their vapors, values of internal energy are more conveniently presented in the form of tables or graphs. Such equations and tables are discussed further in the next chapter.

It should be noted that only *differences* between the internal energies at any two states of a system can be determined by the method described and that absolute values of the internal energy at any one state cannot be found. The values of internal energy listed in tables of thermodynamic properties, for example, are never absolute values but are the differences between the internal energy at each state of the system and the internal energy at some arbitrarily selected reference state. This causes no difficulty in the engineering applications of thermodynamics, however, because only the difference between the internal energies at two states of a system are ever needed.

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* By means of the so-called **general thermodynamic equations,** it is possible to calculate the difference between the internal energies at any two states of a system if the p-v-T relation for the system is known. Hence, internal energy differences can be found by determining experimentally the p-v-T relation for the system.

ILLUSTRATIVE PROBLEMS

50. A system consisting of 1 lb of water initially at 70°F is heated at a constant pressure of 30 psia while confined inside a cylinder equipped with a piston. If the addition of 1200 Btu of heat raises the temperature to 400°F, what is the change in the internal energy of the system?

Solution. Since the pressure remains constant during the process, the work done by the system is, by Eq. 7,

$$W = \int p\,dv \tag{7}$$

$$= p(v_2 - v_1).$$

The initial volume of the system is 0.01606 cu ft per lb (v_f for water at 70°F, from Table 1 of the *Steam Tables*), and the final volume is 16.897 cu ft per lb (v for superheated steam at 30 psia and 400°F, from Table 3). Hence,

$$W = 30 \times 144 \times (16.897 - 0.016)$$

$$= 72{,}900 \text{ ft-lb per lb.}$$

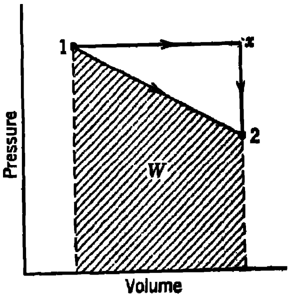
Therefore, the change in the internal energy of the system is, by Eq. 12,

$$u_2 - u_1 = Q - \frac{W}{J}$$

$$= 1200 - \frac{72{,}900}{778}$$

$$= 1106.4 \text{ Btu per lb.}$$

51. One pound of air initially at 50 psia and 300°F undergoes a non-flow process during which the pressure varies linearly with the volume, as shown in the accompanying figure. If in the final state the pressure is 30 psia and the temperature is 100°F, how much heat is added to the air during the process?



Solution. The amount of heat Q added to the air during the process can be calculated by Eq. 12 if (*a*) the work W done by the air during the process and (*b*) the difference $u_2 - u_1$ between the internal energies of the air at its initial and final states can be determined.

Since the work done by the air is represented graphically by the shaded area shown in the figure, it is evident that for the given process the work done is equal to the arithmetic average of the initial and final pressures times the change in the volume of the air. The initial and final volumes of the air can be calculated from the relation $pv = RT$:

$$v_1 = \frac{53.3 \times (300 + 460)}{50 \times 144}$$

$$= 5.62 \text{ cu ft per lb,}$$

and

$$v_2 = \frac{53.3 \times (100 + 460)}{30 \times 144}$$

$$= 6.90 \text{ cu ft per lb.}$$

Hence,

$$W = \left(\frac{50 + 30}{2}\right) \times 144 \times (6.90 - 5.62)$$

$$= 7380 \text{ ft-lb per lb,}$$

or

$$\frac{W}{J} = \frac{7380}{778}$$

$$= 9.5 \text{ Btu per lb.}$$

Since the change in the internal energy $u_2 - u_1$ is the same for all processes connecting states 1 and 2, it can be determined by substituting for the given process some other process for which both the heat added and the work done (and therefore $u_2 - u_1$) can be readily calculated. The process 1 to x to 2 shown on the figure is such a process. The temperature at state x can be determined from $pv = RT$:

$$T_x = \frac{50 \times 144 \times 6.90}{53.3}$$

$$= 933°\text{F abs } (t_x = 473°\text{F}).$$

Since c_p and c_v for air are approximately 0.241 and 0.172 Btu per lb°F respectively, for the process 1 to x to 2,

$$u_2 - u_1 = (u_x - u_1) + (u_2 - u_x)$$

$$= \left(Q_{1-x} - \frac{W_{1-x}}{J}\right) + \left(Q_{x-2} - \frac{W_{x-2}}{J}\right)$$

$$= \left[0.241 \times (473 - 300) - \frac{50 \times 144 \times (6.90 - 5.62)}{778}\right]$$

$$+ [0.172 \times (100 - 473) - 0]$$

$$= 41.6 - 11.8 - 64.2$$

$$= -34.4 \text{ Btu per lb.}$$

Thus, the internal energy of the air at state 2 is 34.4 Btu per lb less than at state 1.

Finally, the amount of heat added to the air during the direct process from 1 to 2 is, by Eq. 12,

$$Q = u_2 - u_1 + \frac{W}{J}$$

$$= -34.4 + 9.5$$

$$= -24.9 \text{ Btu per lb,}$$

the negative sign indicating that the heat is *removed* from the air during the process.

PROBLEMS

52. A pound of saturated water at 300°F is confined inside a cylinder equipped with a piston. If its pressure is kept constant while 910.1 Btu of heat are added, the water will change into saturated steam. By how much does the internal energy of saturated steam at 300°F differ from that of saturated water at the same temperature?

53. A certain fluid initially at 40 psia and 70°F and having an initial specific volume of 3 cu ft per lb undergoes a non-flow process which takes place in such a way that the product pv remains constant. If in its final state the fluid is at 120°F and has a specific volume of 7 cu ft per lb, by how much does its internal energy change? The average specific heat of the fluid for the process is 0.32 Btu per lb deg F. *Ans.* −2.8 Btu per lb.

54. Three pounds of air initially at 250°F undergo a non-flow process during which the pressure remains constant at 60 psia. If the internal energy of the air decreases by 100 Btu, what is its final volume? Assume that c_p for air is constant and equal to 0.241 Btu per lb deg F.

55. By how much does the internal energy of a pound of gaseous oxygen at 25 psia and 60°F differ from the internal energy at 40 psia and 300°F? Assume that the values of c_v and c_p for oxygen are constant and equal to 0.156 and 0.218 Btu per lb deg F respectively, and check your answer by making calculations for two different paths connecting the given states.

56. A pound of air initially at 16 psia and 90°F undergoes a non-flow process which takes place in such a way that $pv^{1.3}$ remains constant. If the final pressure is 50 psia, by how much does the internal energy of the air change? Assume that c_v and c_p for air are constant and equal to 0.172 and 0.241 Btu per lb deg F respectively. *Ans.* 28.4 Btu per lb.

30. Steady-Flow Processes. As explained in the preceding chapter, there are two types of processes of particular interest in engineering thermodynamics. These are *non-flow processes,* which were defined in Art. 16 and have been discussed in some detail in the subsequent articles, and *flow processes,* which were also defined in Art. 16 but have not been discussed further up to this point. The purpose of the present article is to define and illustrate a special kind of flow process known as a "steady-flow" process.

A **steady-flow process** is defined as a flow process during which one or more fluids flow through any kind of apparatus under the following conditions:

(*a*) The state and the velocity of each fluid are uniform and remain constant across each section where the fluids enter or leave the apparatus.

(*b*) The state and the velocity of the fluids either remain constant at each point within the apparatus, or they return periodically to the same values.

(*c*) The rates at which heat and work enter or leave the apparatus remain constant.

Let us consider the following example: Suppose that an insulated tank is provided with an orifice at one end and is connected to a supply of high pressure air at the other end, as shown in Fig. 20. Suppose also that the flow of air from the supply line into the tank can be controlled by a valve and that the pressure and temperature of the air in the supply line remain constant. With the valve initially closed, the pressure within the tank is atmospheric, and no air flows through the orifice. If the valve is now opened, air will flow from the high pressure supply line into the tank, the pressure within the tank will rise, and air will begin to flow out of the tank through the orifice. The mass rates at which the air flows into and out of the tank will not immediately be equal. Instead, the mass rate at which air enters will at first be greater than the mass rate at which it leaves; consequently, the pressure within the tank will continue to rise. As the pressure within the tank rises, the mass rate of flow into the tank may remain constant * or may decrease, but the mass rate of flow out of the tank will increase. Therefore, after a period of time the pressure within the tank will reach an equilibrium value, and the mass rates of flow into and out of the tank will be equal. The air flowing through the valve, the tank, and the orifice is then undergoing a *steady-flow* process. Note that, after equilibrium conditions have been reached, (*a*) the pressure and temperature of the air are uniform and constant both entering the valve and leaving the orifice, and the velocities across these two sections are also approximately uniform and constant, and that (*b*) the pressure, temperature, and velocity of the air at each point within the valve, the tank, and the orifice remain constant. Since the tank is assumed to be insulated, no heat enters or leaves the apparatus. Also, no work enters or leaves.

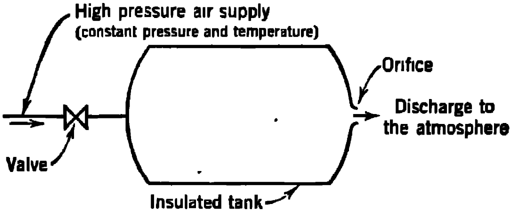

High pressure air supply
(constant pressure and temperature)
Orifice
Discharge to the atmosphere
Valve
Insulated tank

FIG. 20.

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* If the pressure on the downstream side of the valve is less than a certain "critical" value, changes in the tank pressure will not affect the mass rate of flow through the valve. This phenomenon is discussed further in Art. 80 in connection with the flow of air through a tube of constant cross-sectional area.

Let us consider next a steady-flow process in which both heat and work are involved. Suppose a water-jacketed air compressor takes in air from the atmosphere and discharges it into a receiver, as shown in Fig. 21. Suppose further that the receiver is connected through a check valve to a discharge line in which the pressure remains constant at some value above atmospheric pressure. If the compressor is not in operation initially, and if the pressure in the receiver is atmospheric, the check valve will be closed. If the compressor is now started and caused to run at constant speed, the pressure in the receiver will begin to rise. The check valve will remain closed, however, until the pressure in the receiver is equal to the pressure in the discharge line. As the pressure in the receiver rises still further, air will start to flow through the check valve and into the discharge line. As in the preceding example, the pressure in the receiver will reach an equilibrium value when the mass rates of flow into and out of the receiver are equal.

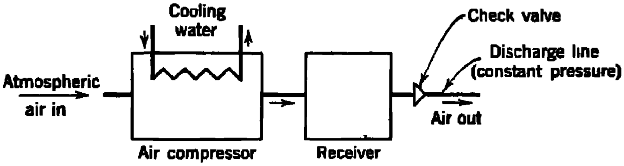

FIG. 21.

If cooling water is flowing through the compressor jacket at a constant rate, the temperature at which it leaves the jacket will at first rise but after a while will also reach an equilibrium value and will undergo no further changes.

The rate at which work must be supplied (that is, the power required) to cause the compressor to operate at constant speed may be relatively high when the compressor is first started because of the inertia of the compressor; but, after the compressor has been brought up to its operating speed, the power required may at first decrease. As the receiver pressure rises, however, the power required to keep the speed constant will continue to increase until the receiver pressure reaches its maximum value. After this the power supplied to the compressor will remain constant.

The temperatures of the various parts of the apparatus will also change at first; but, when the rate at which heat flows from the air and from the cooling water to the outside surfaces of the apparatus is equal to the rate at which heat flows from the outside surfaces to the room, the temperature of each part will remain constant.

When equilibrium operating conditions are reached, the air and the cooling water flowing through the apparatus will be undergoing a *steady-flow process.* Note that (*a*) the pressure and temperature of the air and of the water will be uniform and constant both entering and leaving the apparatus, and the velocities across these sections will be approximately uniform and constant; (*b*) if the compressor is of the centrifugal type, the pressure, temperature, and velocity of the air and cooling water will remain constant at each point within the apparatus, or, if the compressor is of the reciprocating type, they will return periodically to the same values every time the compressor returns to the same point in its cycle of operations; and (*c*) the rate at which heat flows from the apparatus to the room and the rate at which work is supplied to the compressor will remain constant.

In the first of the two foregoing examples, only a single fluid is involved. In the second, two fluids enter and leave the apparatus but do not mix or undergo chemical changes; consequently, the fluids do not change in composition. Examples of steady-flow processes in which two or more fluids flow through an apparatus and undergo changes in composition, due to chemical reactions, to mixing, or to separation, are the processes that take place in internal-combustion engines, air humidifiers, and distillation columns.

It may be noted that, although the velocity of a fluid entering or leaving any actual apparatus is never perfectly uniform across the entire passage, the error introduced by assuming it to be uniform is usually negligible. It may be noted further that, if the velocity and state of each fluid flowing through the apparatus remain constant at each point within the apparatus or return periodically to the same values, the mass rate at which each fluid enters must equal the mass rate at which it leaves. If these rates were not equal, the amount of each fluid within the apparatus would change, and consequently at least one of its properties would have to change.

Although only the special case of steady flow will be considered, this is the most important type of flow process since, except for the initial period of operation, most engineering equipment operates in a manner closely approximating steady-flow operation.

Before considering the application of the First Law of Thermodynamics to steady-flow processes, let us derive expressions for two quantities which enter into this analysis. We shall need (*a*) an expression for the total energy possessed by 1 lb of a fluid which is not subject to any surface effects or to any magnetic or electric effects but is at an elevation z above some horizontal reference plane and has a velocity V. We shall also need (*b*) an expression for the amount of energy needed to force 1 lb of a fluid into or out of any apparatus.

31. The Energy Possessed by an Elevated Fluid in Motion. Consider 1 lb of a fluid which is subject to no surface effects nor to any magnetic or electric effects but which is at the level of some arbitrarily chosen horizontal reference plane and is at rest. As explained in Art. 28, the energy residing within the fluid under these conditions is called its *internal energy* and is denoted by the symbol u. Suppose that work is done on the fluid, not to compress or stir it, but to elevate it the vertical distance z above the reference plane and to give it the velocity V. Its internal energy u remains constant since, from the point of view of an observer who moves with the fluid, its state remains unchanged. The total energy it possesses is increased, however, by the amount of work done to elevate and accelerate it.

The work done on 1 lb * of fluid to elevate it z feet is z foot-pounds, since a vertical force of 1 lb must act on it through the vertical distance z. The work done on 1 lb of the fluid to accelerate it from a state of rest to the velocity V can be determined as follows: Consider first the work required to increase its velocity by the amount dV. To undergo this increase, the fluid must be acted upon by some force F, it must move the distance ds, and the time dt must elapse. From the laws of motion, the force F acting on the fluid is equal to the pounds mass of the fluid divided by g and multiplied by its acceleration dV/dt. Hence, we may write for the work required to increase the velocity of 1 lb of the fluid by the amount dV

$$F\,ds = \frac{1}{g}\frac{dV}{dt}\,ds = \frac{V\,dV}{g}.$$

Therefore, the work required to accelerate it from a state of rest to the velocity V, determined by integrating the last of the foregoing expressions from zero to V, is $V^2/2g$.

Summarizing, we find that the total energy (in British thermal units per pound) possessed by any fluid which is subject to no surface effects nor to any magnetic or electric effects but which is a vertical distance z above some horizontal reference plane and is moving with the velocity V is

$$u + \frac{z}{J} + \frac{V^2}{2gJ},$$

---

* By one pound of any substance is meant a pound *mass* of the substance. If the acceleration of gravity is identical with the acceleration g that a force of one pound gives to a mass of one pound, then a one-pound mass will *weigh* one pound. For the sake of simplicity, it will be assumed that the acceleration of gravity is constant and equal to g, even though it actually varies slightly with elevation and latitude.

where u = the internal energy of the fluid, in British thermal units per pound,

z = the elevation of the fluid above the horizontal reference plane, measured in feet,

V = the velocity of the fluid, in feet per second, and

g = 32.2 ft per sec per sec.

32. The Work Required to Force a Fluid into or out of Any Apparatus. Consider next the amount of work required to force 1 lb of fluid into any apparatus if the pressure at the section where the fluid enters is uniform and remains constant. Let us denote the pressure at the entering section by p_1 and the cross-sectional area by a_1. The force that the fluid about to enter the apparatus exerts on the fluid already within the apparatus is p_1a_1, as shown on Fig. 22. Therefore, to force a small element of volume dv_1 (having the thickness ds in the direction of flow) into the apparatus, the fluid approaching the apparatus must do the work $p_1a_1\,ds = p_1\,dv_1$ on the fluid already within the apparatus. Since the work $p_1\,dv_1$ is required to force the volume dv_1 into the apparatus, and since p_1 remains constant, it follows that the work p_1v_1 is required to force the volume v_1 into the apparatus. Therefore, if v_1 is the specific volume of the fluid at section 1, the work p_1v_1 is done on the fluid already within the apparatus every time 1 lb of fluid enters.* Similarly, if the pressure p_2 at the leaving section remains uniform and constant, the fluid within the apparatus must do the work p_2v_2 on the fluid that has already left in order to force 1 lb of fluid having the specific volume v_2 out of the apparatus. Note that, if the pressure p is expressed in pounds per square foot absolute and if the specific volume v is expressed in cubic feet per pound, the units of the quantity pv are foot-pounds per pound.

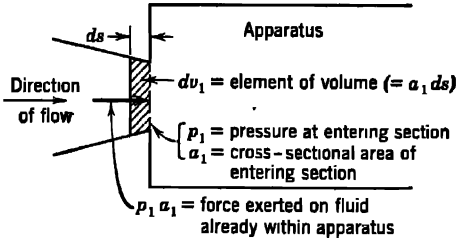
ds
Apparatus
Direction of flow
dv₁ = element of volume (= a₁ ds)
p₁ = pressure at entering section
a₁ = cross-sectional area of entering section
p₁ a₁ = force exerted on fluid already within apparatus

FIG. 22.

33. The Steady-Flow Energy Equation. Now let us consider the application of the First Law of Thermodynamics to steady-flow processes. Before considering the general case where several streams are involved, let us consider a steady-flow process in which only a single stream of fluid enters and leaves the apparatus. The apparatus may be a compressor, a reciprocating engine, a turbine, a valve, a section of pipe, or any other device through which flow can take place. Whatever the apparatus may be, it can be represented diagrammatically by a rectangle, as shown in Fig. 23. Let section 1 be the section where the fluid enters the apparatus, and section 2, the section where it leaves. For a steady-flow process, the state and velocity of the fluid are uniform and remain constant at the entering and leaving sections. Let the properties of the fluid, its elevation above some horizontal reference plane, and its velocity at these two sections be indicated by the subscripts 1 and 2. Also, let the heat *supplied to each pound of fluid* as it flows through the apparatus be denoted by Q; and let the work that *leaves the apparatus* by means of a shaft or other mechanical connection, per pound of fluid flowing through the apparatus, be denoted by W'. The quantity W' will be called the **shaft-work** to distinguish it from the work pv done on each pound of fluid to force it into or out of the apparatus.

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* The quantity pv is sometimes called the *flow-work.*

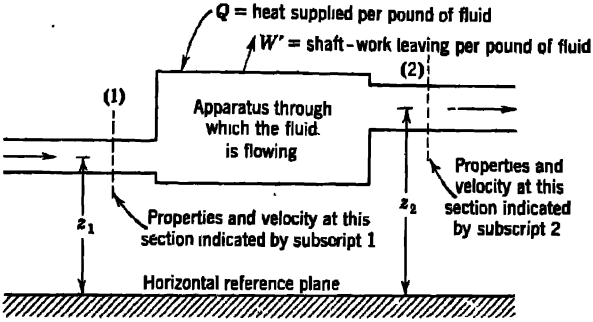
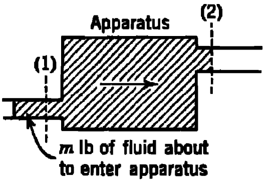

Q = heat supplied per pound of fluid
W' = shaft-work leaving per pound of fluid
(1)
(2)
Apparatus through which the fluid is flowing
Properties and velocity at this section indicated by subscript 1
Properties and velocity at this section indicated by subscript 2
z_1
z_2
Horizontal reference plane

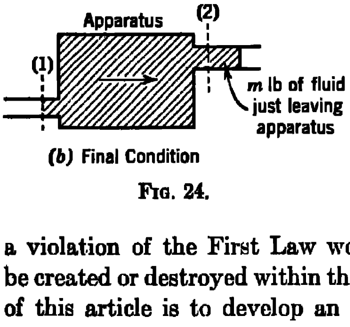
FIG. 23. Diagrammatic sketch of an apparatus through which steady flow is taking place.

Note that Q and W' may be either positive or negative, or they may be zero. For example, if the apparatus is a boiler, heat is supplied to the fluid flowing through it; so Q is positive. If it is a condenser, heat is absorbed from the fluid, and Q is negative. Or, if the apparatus is an insulated section of pipe, heat is neither added to nor removed from the fluid, and Q is zero. Similarly, if the apparatus is an engine, shaft-work leaves it; so W' is positive. If it is a compressor, shaft-work is supplied to it, and W' is negative. Or, if the apparatus is a valve, shaft-work is neither supplied to nor done by the fluid, and W' is zero.

Recall now that the First Law of Thermodynamics states that, when any system undergoes a process during which energy is added to or removed from it, none of the energy added is destroyed within the system, and none of the energy removed is created within the system. The system referred to must be finite. Therefore, in order to apply the First Law to a steady-flow process, it is necessary that we consider the system to be only some finite portion of the fluid flowing through the apparatus. Let us consider the system to be the fluid within the apparatus at any instant *plus* m additional pounds that are about to enter; and let the process this system undergoes be that portion of the steady-flow process during which the m pounds flow past section 1. As explained in Art. 30, while m pounds of the fluid enter the apparatus at section 1, m pounds leave at section 2. Thus, the positions of the system at the beginning and at the end of the process are as shown schematically in Fig. 24. When the system undergoes this process, *the difference between the energy added to it and the energy removed from it must equal the increase in the amount of energy it possesses.* If this were not true, a violation of the First Law would result since energy would have to be created or destroyed within the system. The purpose of the remainder of this article is to develop an equation that expresses this statement in mathematical form.



(a) Initial Condition



(b) Final Condition

FIG. 24.

First, let us write expressions for the energy added to and removed from the system while the m pounds of fluid flow past section 1 into the apparatus. The energy added to the system is the sum of the heat supplied to it and the work the fluid approaching the apparatus does on the fluid within the apparatus to force the m pounds past section 1. Making use of the expression developed in Art. 32, we may write for the energy added to the system during the process

$$m\left(Q + \frac{p_1v_1}{J}\right).$$

The energy removed from the system is the sum of the shaft-work done by the system plus the work the fluid within the apparatus does on the fluid that has already left to force m pounds past section 2. Thus, the energy leaving the system during the process is

$$m\left(\frac{W'}{J}+\frac{p_2v_2}{J}\right).$$

Next, let us write expressions for the total energy possessed by the system in its initial and final states. If we let E' denote the total energy possessed by the fluid *within the apparatus* in its initial state, and if we make use of the expression developed in Art. 31 for the total energy possessed by 1 lb of a fluid that is a vertical distance z above some horizontal reference plane and is moving with the velocity V, then for the total energy possessed by the system in its initial state we may write

$$E'+m\left(u_1+\frac{z_1}{J}+\frac{V_1^2}{2gJ}\right)^*.$$

Similarly, if we let E'' denote the total energy possessed by the fluid *within the apparatus* in the final state, then for the energy possessed by the system in its final state we may write

$$E''+m\left(u_2+\frac{z_2}{J}+\frac{V_2^2}{2gJ}\right).$$

Equating now the difference between the energy added to and removed from the system to the increase in the amount of energy possessed by the system, we obtain

$$m\left(Q+\frac{p_1v_1}{J}\right)-m\left(\frac{W'}{J}+\frac{p_2v_2}{J}\right)$$
$$=\left[E''+m\left(u_2+\frac{z_2}{J}+\frac{V_2^2}{2gJ}\right)\right]-\left[E'+m\left(u_1+\frac{z_1}{J}+\frac{V_1^2}{2gJ}\right)\right].$$

But, if the steady-flow process is of such nature that the state and velocity of the fluid at each point within the apparatus remain constant at all times, the condition of the fluid *within the apparatus* in the final state is in every respect identical with its condition in the initial state. Therefore, it follows that $E'=E''$. Or, if the steady-flow process is of the type in which the state and velocity of the fluid at each point within the apparatus return periodically to the same values, and if m is taken as the number of pounds of fluid entering the apparatus during one of these periods of operation, the condition of the fluid within the apparatus after m pounds have entered and m pounds have left will again be identical with its condition initially. As before, it follows that $E' = E''$. Introducing this equality into the preceding equation and rearranging, we obtain

---

* It is assumed for the sake of simplicity that the entire m pounds of fluid which are about to enter the apparatus are at the elevation z_1 and are moving with the velocity V_1. This assumption is permissible since, from the point of view of an observer who is watching the stream enter the apparatus at section 1, the elevation and the velocity of the entire m pounds *are* z_1 and V_1 respectively. Similarly, the entire m pounds of fluid which leave the apparatus may be assumed to be at the elevation z_2 and to be moving with the velocity V_2.

$$Q + u_1 + \frac{p_1v_1}{J} + \frac{z_1}{J} + \frac{V_1^2}{2gJ} = \frac{W'}{J} + u_2 + \frac{p_2v_2}{J} + \frac{z_2}{J} + \frac{V_2^2}{2gJ}.$$

The quantity $u + pv/J$ is one that appears frequently in thermodynamics. For this reason it is convenient to represent it by a single symbol and to assign a name to it. The symbol used for this purpose is h, and the quantity is called the **enthalpy**. Note that, since the enthalpy is made up of the properties u, p, and v, it is itself a property. Just as equations and tables, based on experimental data, are available for determining changes in internal energy of most of the systems of importance in engineering, equations and tables for determining changes in enthalpy are also available. These will be discussed in the next chapter.

Introducing enthalpies into the preceding equations, we obtain, finally,

$$Q + h_1 + \frac{z_1}{J} + \frac{V_1^2}{2gJ} = \frac{W'}{J} + h_2 + \frac{z_2}{J} + \frac{V_2^2}{2gJ} \quad (13)$$

where Q = the heat supplied to the fluid, in British thermal units per pound,

W' = the shaft-work leaving the apparatus, in foot-pounds per pound,

h = enthalpy $(= u + pv/J)$, in British thermal units per pound,

z = elevation, in feet,

V = velocity, in feet per second, and

g = 32.2 ft per sec per sec.

This equation is a statement in mathematical form of the First Law of Thermodynamics applied to a steady-flow process in which a single stream of fluid enters and leaves the apparatus. It will be called the **steady-flow energy equation.**

If the fluid enters and leaves the apparatus at the same elevation, the two z/J terms in Eq. 13 are equal. Even though the fluid enters and leaves the apparatus at different elevations, the difference between these terms is likely to be quite small compared to some of the other terms in the equation. This is nearly always true for the applications considered in engineering. Consequently, these terms may be dropped, and the equation may be written in the somewhat simpler form,

$$Q + h_1 + \frac{V_1^2}{2gJ} = \frac{W'}{J} + h_2 + \frac{V_2^2}{2gJ}. \tag{13a}$$

It is convenient to note that, for purposes of slide-rule computations, the product $2gJ$ is very nearly equal to 50,000.

If more than one stream of fluid enters or leaves the apparatus, it is simpler to write the steady-flow energy equation on a rate basis. Letting $\bar{Q}$ represent the *rate* at which heat enters the apparatus, $\bar{W}'$ represent the rate at which shaft-work leaves it, and w represent the mass rate of flow of any stream, by an analysis exactly like the foregoing we can show that

$$\bar{Q} + \sum\nolimits_1 \left[w\left(h + \frac{z}{J} + \frac{V^2}{2gJ}\right)\right] = \frac{\bar{W}'}{J} + \sum\nolimits_2 \left[w\left(h + \frac{z}{J} + \frac{V^2}{2gJ}\right)\right], \tag{14}$$

where $\sum_1$ and $\sum_2$ indicate the summation of the product $w(h + z/J + V^2/2gJ)$ for all the streams entering and leaving the apparatus. Since this equation is simply an expanded form of Eq. 13, it will also be called the **steady-flow energy equation.** Just as the z/J terms may usually be dropped from Eq. 13, they may usually be dropped from Eq. 14.

34. The Continuity Equation. Another relation frequently used in the solution of steady-flow problems is the so-called "continuity equation." To obtain this relation, consider a stream of fluid flowing through any passage. The product of the velocity of the fluid times the cross-sectional area of the passage, both quantities taken at the same section, is equal to the volume rate of flow past the section. If this volume rate of flow is divided by the specific volume of the fluid at the section, the mass rate of flow is obtained. Thus,

$$w = \frac{Va}{v},$$

where w = the mass rate of flow of the fluid, in pounds per second,
V = the velocity of the fluid, in feet per second,
a = the cross-sectional area of the passage, in square feet, and
v = the specific volume of the fluid, in cubic feet per pound.

For a steady-flow process, as explained in Art. 30, the mass rates of flow into and out of the apparatus are equal. Therefore, if only a single stream of fluid enters and leaves the apparatus,

$$\boxed{w = \frac{V_1 a_1}{v_1} = \frac{V_2 a_2}{v_2},} \qquad (15)$$

where subscripts 1 and 2 again denote the entering and leaving sections respectively. This relation is called the **continuity equation** for steady-flow.

ILLUSTRATIVE PROBLEMS

57. A steam turbine uses 10,000 lb per hr of steam while delivering 750 kw at the turbine shaft. If the enthalpies of the steam entering and leaving the turbine are 1250 and 960 Btu per lb respectively, and if the velocities of the steam entering and leaving the turbine are 300 and 800 ft per sec respectively, calculate the rate at which heat is lost from the turbine casing and bearings.

Solution. Since there are 3413 Btu per kwhr, by Eq. 13*a*,

$$Q + 1250 + \frac{(300)^2}{50{,}000} = \frac{750 \times 3413}{10{,}000} + 960 + \frac{(800)^2}{50{,}000}.$$

Solving for Q, we get

$$Q = -23.0 \text{ Btu per lb.}$$

The negative sign indicates that the direction of heat flow is away from the apparatus; that is, the heat is *lost* from the turbine casing and bearings. The rate at which heat is lost is 23.0 × 10,000 = 230,000 Btu per hr.

Note that, if the inlet pipe were, say, 10 ft above the exhaust pipe, and if this difference in elevation were taken into account by using Eq. 13, the heat loss per pound of steam would be increased by 10/778 = 0.013 Btu per lb. The error caused by neglecting this difference in elevation is less than 0.1%.

58. A two-stage air compressor has water-jacketed cylinders and a water-cooled intercooler. The compressor takes in 500 cfm of "free" air at 14.7 psia and 70°F and discharges it into a receiver in which the pressure remains constant. Cooling water flows through the jackets and the intercooler at the rate of 35 lb per min, the water entering at 60°F and leaving at 105°F. Assuming that the heat loss from the compressor to its surroundings is negligible, calculate the power supplied to the compressor if the enthalpy of the air increases by 41.0 Btu per lb, if the enthalpy of the cooling water increases by 45.0 Btu per lb, and if the velocities of the air and the water entering and leaving the compressor are small.

Solution. Since two streams enter and leave the apparatus, the problem can be solved by Eq. 14. The mass rate of air flow w_a can be determined after the specific volume of the entering air has been calculated. From $pv = RT$,

$$v = \frac{53.3 \times 530}{14.7 \times 144}$$

$$= 13.3 \text{ cu ft per lb.}$$

Hence,

$$w_a = \frac{500}{13.3}$$

$$= 37.6 \text{ lb per min.}$$

Letting subscript *a* refer to the air and subscript *w* to the cooling water, we may write Eq. 14, after dropping out the z/J and $V^2/2gJ$ terms,

$$\bar{Q} + w_a h_{a,1} + w_w h_{w,1} = \frac{W'}{J} + w_a h_{a,2} + w_w h_{w,2},$$

or

$$\bar{Q} = \frac{W'}{J} + w_a(h_2 - h_1)_a + w_w(h_2 - h_1)_w.$$

Hence,

$$0 = \frac{W'}{J} + 37.6 \times 41.0 + 35 \times 45.0,$$

and

$$\frac{W'}{J} = -3115 \text{ Btu per min.}$$

The negative sign indicates that the work is supplied *to* the apparatus. Since there are 2545 Btu per hp-hr, the horsepower supplied to the compressor is

$$\text{hp} = \frac{3115 \times 60}{2545}$$

$$= 73.5$$

Since the air and the cooling water are not mixed as they flow through the compressor, the problem can also be solved by Eq. 13*a*. Considering the air to be the fluid undergoing the steady-flow process, and taking the specific heat of water as 1.00 Btu per lb deg F, we find that heat is absorbed from the air (by the cooling water) at the rate of 35 × 1.00 (105 − 60) = 1575 Btu per min. Hence, by Eq. 13*a*,

$$-\frac{1575}{37.6} = \frac{W'}{J} + 41.0$$

or

$$\frac{W'}{J} = -83.0 \text{ Btu per lb.}$$

Therefore, the power supplied *to* the compressor is

$$\text{hp} = \frac{83.0 \times 37.6 \times 60}{2545}$$

$$73.5$$

59. Water at 80°F flows at low velocity into an insulated tank (an open-type feedwater heater) at the rate of 10,000 lb per hr. Steam having an enthalpy of 1195 Btu per lb and a specific volume of 9 cu ft per lb also flows into the tank. The rate at which the steam flows is 2000 lb per hr; and the pipe through which it flows has an internal cross-sectional area of 1.50 sq in. The two streams come into direct contact within the tank, causing the steam to condense. The condensate mixes with the water entering the tank, and the two leave the tank as a single stream of heated water flowing at low velocity. If the enthalpy of water is approximately equal to $(t - 32)$ Btu per lb, where t denotes the temperature of the water in degrees Fahrenheit, determine the temperature of the stream of water leaving the tank.

Solution. Let subscript 1 refer to the water entering, subscript s to the steam entering, and subscript 2 to the water leaving. The velocity V_s with which the steam enters the tank can be calculated by Eq. 15:

$$V_s = \frac{wv}{a}$$

$$= \frac{2000 \times 9 \times 144}{3600 \times 1.50}$$

$$= 480 \text{ ft per sec.}$$

Since V_1 and V_2 are small, and since $\bar{Q}$ and $\bar{W}'$ are both zero, Eq. 14 may be written

$$w_1h_1 + w_s\left(h_s + \frac{V_s^2}{2gJ}\right) = w_2h_2$$

why? $V_1 = 0$

Hence,

$$10{,}000(80 - 32) + 2000\left(1195 + \frac{480^2}{50{,}000}\right) = 12{,}000(t_2 - 32).$$

Solving for the temperature t_2 of the water leaving the tank, we get

$$t_2 = 272°\text{F}.$$

PROBLEMS

60. Steam enters a small turbine at a velocity of 250 ft per sec and with an enthalpy of 1187.0 Btu per lb. It leaves at a velocity of 500 ft per sec and with an enthalpy of 1118.5 Btu per lb. If heat is lost from the turbine at the rate of 20,000 Btu per hr, at what rate must steam flow through the turbine in order for the turbine to deliver 100 hp? *Ans.* 4250 lb per hr.

61. An air compressor compresses 200 cfm of air from 14.2 psia and 50°F to a pressure of 65 psia. If the enthalpy of the air increases by 45.0 Btu per lb, and if 30 hp are supplied to operate the compressor, at what rate is heat being removed from the machine? Assume that the velocities of the air in the intake and discharge lines are approximately equal. *Ans.* 595 Btu per min.

62. Steam at 120 psia and 420°F enters a tube having a cross-sectional area of 0.75 sq in. It flows through the tube at the rate of 45 lb per min and leaves at 115 psia with a quality of 50%. If its enthalpy decreases by 485.9 Btu per lb, at what rate is it losing heat?

63. A De Laval nozzle consists of a converging passage followed by a diverging passage (see Fig. 116). Air at 50 psia and 90°F will enter such a nozzle at a velocity of 120 ft per sec and will leave at 15 psia and −70°F. It will flow through the nozzle approximately adiabatically, and its enthalpy will decrease by 38.6 Btu per lb. What must be the cross-sectional area of the nozzle at the section where the air leaves if the air is to flow through the nozzle at the rate of 20 lb per min? *Ans.* 0.331 sq in.

64. A stream of air at 15 psia and 60°F flows at a velocity of 200 ft per sec through a duct having a cross-sectional area of 2.5 sq ft. A second stream at 15 psia and 160°F flows at a velocity of 300 ft per sec through a duct having a cross-sectional area of 1.5 sq ft. The two streams mix, and the resulting stream flows at a velocity of 100 ft per sec. If the air in the first stream has an enthalpy of 14.5 Btu per lb and the air in the second an enthalpy of 38.6 Btu per lb, what is the enthalpy of each pound of air in the final stream?

GENERAL PROBLEMS

65. A pound of steam initially at 100 psia undergoes a non-flow process which takes place in such a way that the steam is always saturated. If in its final state the steam is at 40 psia, and if its internal energy has decreased by 13.2 Btu, how much heat has it absorbed?

66. Air flows through a long pipe at the rate of 30 lb per min. It enters the pipe at 20 psia and 60°F and leaves at 16 psia and 180°F. If its enthalpy increases by 28.9 Btu per lb, and if the pipe has a cross-sectional area of 1.50 sq in., at what rate is heat being supplied to the air? *Ans.* 1040 Btu per min.

67. An insulated tank is divided into two equal compartments by a partition. One compartment is completely evacuated, and the other contains nitrogen at 100 psia and 75°F. If the partition is removed and the nitrogen allowed to reach a state of equilibrium, what will be the final pressure and temperature in the tank? Assume that the values of c_v and c_p for nitrogen are constant and equal to 0.178 and 0.249 Btu per lb deg F respectively. (Note that both Q and W are zero for the process.)

68. Steam at 300 psia and 500°F enters a De Laval nozzle (Fig. 116). It enters with a negligible velocity and flows through the nozzle adiabatically. It leaves at 100 psia with a quality of 98%. If its enthalpy decreases by 88.2 Btu per lb, and if the cross-sectional area of the nozzle at the section where the steam leaves is 0.20 sq in., how many pounds of steam are flowing through the nozzle per minute? *Ans.* 40.4 lb per min.

69. One pound of air initially at 45 psia and 70°F undergoes a non-flow process which takes place in such a way that

$$p = Cv^2,$$

where C is a constant and p and v are expressed in pounds per square inch absolute and in cubic feet per pound respectively. If the final pressure is 60 psia, what is the average specific heat of the air for the process? Assume that c_v and c_p for air are constant and equal to 0.172 and 0.241 Btu per lb deg F respectively.

70. Air at 60 psia and 90°F is supplied to an air motor at the rate of 5 lb per min. Its enthalpy decreases by 19.3 Btu per lb as it flows through the motor, and it leaves at 14.7 psia and 10°F. If the cross-sectional areas of the inlet and discharge passages are 0.40 and 0.60 sq in. respectively, and if the motor operates adiabatically, how many horsepower does it deliver? *Ans.* 2.17 hp.

71. A pound of air initially at 200 psia and 120°F undergoes a non-flow process during which the pressure varies linearly with the volume (see Illustrative Problem 51). If 100 Btu of heat are added to the air during the process, and if the air does 70,000 ft-lb of work, what is the final pressure? Assume that c_v and constant and equal to 0.172 and 0.241 Btu per lb deg F respectively.

SYMBOLS

a	cross-sectional area
E	total energy of a system
F	force

g	acceleration that a force of 1 lb gives to a mass of 1 lb
h	enthalpy per unit mass
J	mechanical equivalent of heat
m	mass
p	pressure
Q	heat added to system; Q_{cycle} = net heat added to system during one cycle; $\bar{Q}$ = rate at which heat is added to a system in steady flow
R	gas constant
s	distance
t	temperature
dt	small interval of time
T	absolute temperature
u	internal energy per unit mass
v	specific volume
V	velocity
w	mass rate of flow
W	work done by system; W_{cycle} = net work done by system during one cycle; W' = shaft-work done by a system in steady flow; $\bar{W}'$ = rate at which shaft-work is done by a system in steady flow
	elevation

Chapter 4

THE INTERNAL ENERGY AND ENTHALPY OF FLUIDS

In the preceding chapter equations were developed which express the relation between the various kinds of energy involved when any system undergoes either a non-flow or a steady-flow process. The first of these (Eq. 12) involves the difference $u_2 - u_1$ between the internal energies of the system in its initial and final states, and the second (Eq. 13) involves the difference $h_2 - h_1$ between the enthalpies of the fluid entering and leaving the apparatus through which steady flow is taking place. Before these equations can be used in the solution of problems, it is necessary to have some means of evaluating $u_2 - u_1$ and $h_2 - h_1$.

For any actual substance the difference between its internal energies at any two states must be determined by experiment. One way of doing this is to let the system change by *any* convenient non-flow process from one state to another, and to measure the heat added and the work done during the process. The difference between the internal energies of the system at the two states can then be calculated by subtracting the work done from the heat added. Differences in the internal energies at any two states of most systems of importance in engineering have been determined by such experiments or other experiments, and the results are available in the form of equations, tables, and graphs.

The enthalpy h of any system has been defined as $u + pv/J$. Therefore, if the difference between the internal energies at any two states of the system has been determined by experiment, and if the pressures and specific volumes of the system at these states are known, the difference between the enthalpies at the two states can be calculated by the relation

$$h_2 - h_1 = (u_2 - u_1) + \frac{p_2v_2}{J} - \frac{p_1v_1}{J}.$$

Values of $h_2 - h_1$ can also be determined experimentally in several other ways and, like differences in internal energy, are available in the form of equations, tables, and graphs.

The purpose of the present chapter is (*a*) to develop equations for calculating differences in internal energy and in enthalpy for perfect gases, and (*b*) to explain how to determine these values for liquids, vapors, and liquid-vapor mixtures by means of tables, such as the *Steam Tables.*

PERFECT GASES

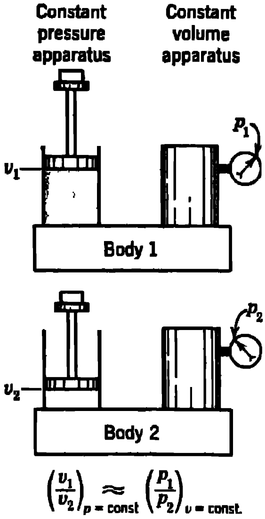

Constant pressure apparatus
Constant volume apparatus
P_1
v_1
Body 1
P_2
v_2
Body 2
$\left(\frac{v_1}{v_2}\right)_{p=\text{const}} \approx \left(\frac{P_1}{P_2}\right)_{v=\text{const}}$

FIG. 25.

35. Joule's Law. Up to this point a "perfect gas" has been defined, in effect, as one which obeys the same p-v-T relation that actual gases obey at pressures approaching zero. A second part which has to do with the internal energy of a perfect gas will now be added to the definition. Before considering it, let us review briefly the material, discussed in Art. 7, leading to the first part of the definition.

Suppose that 1 lb of any actual gas is kept at constant pressure and another pound of the same gas is kept at constant volume. If the two masses of gas are brought into thermal equilibrium first with some body 1 and then with some body 2, as shown in Fig. 25, the relation between the volumes assumed by the first pound and the pressures exerted by the second is found to be

$$\left(\frac{v_1}{v_2}\right)_{p=\text{const.}} \approx \left(\frac{p_1}{p_2}\right)_{v=\text{const.}}.$$

If the pressure of the gas is very low, it is found, further, that these two ratios are exactly equal: that is, for any actual gas at pressures approaching zero,

$$\left(\frac{v_1}{v_2}\right)_{p=\text{const.}} = \left(\frac{p_1}{p_2}\right)_{v=\text{const.}} \tag{1}$$

A *perfect gas* was defined in Art. 7 as one that obeys Eq. 1 at *all* pressures. The ratio of the temperature T_1 of body 1 to the temperature T_2 of body 2 was next defined as

$$\frac{T_1}{T_2} = \left(\frac{v_1}{v_2}\right)_{p=\text{const.}} = \left(\frac{p_1}{p_2}\right)_{v=\text{const.}}, \tag{2}$$

where the volume ratio or the pressure ratio is that which would be obtained if a perfect gas (or an actual gas at pressures approaching zero) were brought into thermal equilibrium with the two bodies. The scale of temperature defined in this way is known as the *absolute perfect gas scale of temperature.* Recall also that, by means of Eq. 2, it was shown in Art. 10 that the p-v-T relation for a perfect gas is

$$pv = RT, \tag{3}$$

where R is a constant for each gas.

Let us now consider how the internal energy of actual gases depends upon the pressure, specific volume, and temperature. It is found by experiment that, for any actual gas, the difference between the internal energy u_1 at one state and the internal energy u_2 at any other state is approximately zero if the temperatures of the gas at the two states are the same—even though the pressures and specific volumes at the two states may be quite different. Thus, for any actual gas,

$$u_2 - u_1 \approx 0, \qquad \text{if } t_1 = t_2.$$

As an illustration of this behavior, let us calculate the difference between the internal energies of a pound of nitrogen at 14.7 psia and 70°F and at 50 psia and 70°F. This can be done by calculating the difference between the heat added to and the work done by the gas when it changes from the given initial state to the given final state by *any* convenient non-flow process. Let the process consist of a constant volume change from state 1 to state x followed by a constant pressure change from state x to state 2, as shown in Fig. 26. The heat added to the gas during this process can be calculated from the values of c_v and c_p for nitrogen after the temperature at state x has been determined. Assuming that the nitrogen obeys Eq. 3, we may write

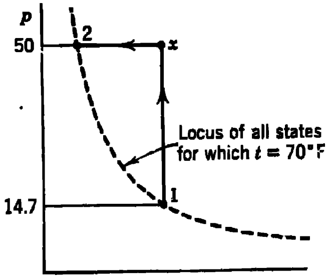


FIG. 26.

$$\frac{p_1 v_1}{T_1} = R = \frac{p_x v_x}{T_x},$$

whence

$$T_x = T_1 \left(\frac{p_x}{p_1}\right)$$

$$= 530 \times \left(\frac{50}{14.7}\right)$$

$$= 1800\text{°F abs.}$$

The values of c_v and c_p for nitrogen have been found (by experiment) to be approximately constant and equal to 0.178 and 0.249 Btu per lb deg F respectively. Hence, for the process 1 to x to 2,

$$Q = 0.178(1800 - 530) + 0.249(530 - 1800)$$
$$= -90.1 \text{ Btu per lb.}$$

For the constant volume portion of the process the work done by the gas is zero. For the constant pressure portion,

$$W = \int p\,dv = p(v_2 - v_x),$$

or, if we make use of Eq. 3, since $p = p_2 = p_x$,

$$W = R(T_2 - T_x).$$

Hence, the work done by the gas during the process 1 to x to 2 is

$$\frac{W}{J} = \frac{55.1 \times (530 - 1800)}{778}$$
$$= -90.1 \text{ Btu per lb.}$$

The value of $u_2 - u_1$ for the nitrogen is, therefore,

$$u_2 - u_1 = Q - \frac{W}{J}$$
$$= -90.1 + 90.1$$
$$= 0.$$

The fact that the internal energy of actual gases remains approximately constant if the temperature of the gas is kept constant was first observed by Joule in 1843. Joule performed the following experiment: He submerged in a water bath two vessels connected by a stopcock. One of the vessels was filled with air under pressure, and the other was evacuated. He measured the temperature of the water initially and then opened the stopcock and allowed the gas to expand into the evacuated vessel. After equilibrium had been reestablished, he again measured the temperature of the water and found it unchanged. He reasoned that no heat had been added to or removed from the air because the temperature of the water had remained unchanged. Also, the air had done no work during the expansion. Hence, the change $u_2 - u_1$ in the internal energy of the air was zero. Since the pressure and specific volume of the air had changed, but its temperature (which had to be the same as that of the water bath) had not, he concluded that for any gas the internal energy depends only upon the temperature. It is now known that, had he been able to measure the temperature of the water more accurately, he would have found that its temperature did not remain absolutely constant and that the internal energy of actual gases is therefore not dependent only upon the temperature.

The fact that the internal energy of any actual gas remains approximately constant if its temperature is kept constant may be stated: the internal energy of an actual gas is approximately a function only of its temperature. Thus, for actual gases,

It has been found by experiment that actual gases obey this relation *exactly* if the pressure of the gas is very low; that is, for any actual gas at pressures approaching zero.

$$u = f(t). \tag{16}$$

The second part of the definition of a perfect gas is similar to the first in that both parts require a perfect gas to obey *at all pressures* the relations that actual gases obey at pressures approaching zero. Thus, the first part of the definition is that a perfect gas is one obeying Eq. 1 at all pressures, and the second part is that a perfect gas is one obeying Eq. 16 at all pressures.* Equation 16 is known as **Joule's Law.** It may be stated: *the internal energy of a perfect gas is a function only of the temperature of the gas.*

As has been explained, any actual gas becomes a perfect gas at pressures approaching zero because it then obeys both Eqs. 1 and 16. Even at higher pressures, in particular at pressures of the magnitude ordinarily encountered in engineering, actual gases obey these relations closely enough that, for most purposes, they may be considered perfect gases. It is for this reason that equations for calculating the difference between the internal energies and enthalpies at any two states of a perfect gas will be derived. Before considering these derivations, however, let us first see how the specific heats at constant volume and at

---

* Instead of defining a perfect gas in this way, an alternative procedure is to define it as one which obeys the relation

$$pv = R\theta,$$

where θ is the temperature of the gas measured on the *thermodynamic scale of temperature* (discussed in Art. 65). If this is done, it can be proved mathematically that the gas will also obey the relation

$$u = f(\theta).$$

constant pressure for a perfect gas depend upon the pressure, specific volume, and temperature of the gas.

36. c_v and c_p for Perfect Gases. The specific heat at constant volume (c_v) for any system was defined in Art. 25 as the amount of heat added to each pound of the system per degree rise in its temperature when the system undergoes a non-flow process during which (*a*) its volume remains constant and (*b*) no work is done on or by it. It follows from this definition that, during any infinitesimal part of such a process, the heat dQ added to the system is $c_v\,dt$ and the work dW done by the system is zero. Hence, for this part of the process the non-flow energy equation (Eq. 12) takes the form

$$dQ = du + \frac{dW}{J},$$

or

$$c_v\,dt = du + 0.$$

Solving for c_v, we obtain

$$c_v = \frac{du}{dt}\cdot$$

or, expressing in mathematical form the fact that the volume must remain constant, we may write

$$c_v = \left(\frac{\partial u}{\partial t}\right)_v$$

But, if the system is a perfect gas, its internal energy u is a function only of its temperature (Eq. 16). Hence, $(\partial u/\partial t)_v$ is also a function only of the temperature, and therefore

$$c_v = f(t).$$

This means that, to raise the temperature of a perfect gas from any temperature t_1 to any other temperature t_2 by a constant volume, non-flow process involving no work, the heat required per pound of the gas is the same regardless of what the pressure or specific volume of the gas may be.

The specific heat at constant pressure (c_p) for any system was defined in Art. 25 as the amount of heat added to each pound of the system per degree rise in its temperature when the system undergoes a non-flow process during which (*a*) its pressure remains constant and (*b*) the only work done by or on the system is equal to the integral of $p\,dv$. It follows from this definition that, during any infinitesimal part of such a process, the heat dQ added to the system is $c_p\,dt$ and the work dW

done by the system is $p\,dv$. Hence, for this part of the process, the non-flow energy equation takes the form

$$dQ = du + \frac{dW}{J},$$

or

$$c_p\,dt = du + \frac{p\,dv}{J}.$$

Solving for c_p and expressing in mathematical form the fact that the pressure must remain constant, we obtain

$$c_p = \left(\frac{\partial u}{\partial t}\right)_p + \frac{p}{J}\left(\frac{\partial v}{\partial t}\right)_p.$$

As before, if the system is a perfect gas its internal energy u is a function only of the temperature; and therefore $(\partial u/\partial t)_p$ is also a function only of the temperature. Moreover, for a perfect gas $pv = RT$. Differentiating with respect to T, keeping p constant, we get

$$p\left(\frac{\partial v}{\partial T}\right)_p = R.$$

Hence, the second part of the foregoing expression for c_p is a constant. It follows that, for a perfect gas,

$$c_p = f(t).$$

This means that, if a perfect gas undergoes a constant pressure, non-flow process during which the only work involved is done on or by a slowly moving boundary (such as a piston), the heat required to raise each pound of the gas from any temperature t_1 to any other temperature t_2 is the same regardless of what the pressure or specific volume of the gas may be.

It may be noted at this point that, whereas the specific heats c_v and c_p for perfect gases may depend upon the temperature but not upon the pressure of the gas, c_v and c_p for actual gases are found to be dependent upon both temperature and pressure. For example, the dependence upon temperature and pressure of c_v and c_p for dry air is shown in Fig. 27. For pressures such as those involved in most engineering applications of thermodynamics, however, these specific heats may be considered independent of the pressure. If the temperature varies over only a moderate range, they may also be considered to be independent of the temperature; that is, for many engineering calculations, c_v and

c_p for air (and most other gases) may be assumed to remain constant. This assumption will usually be made in this text.

Values of c_v and c_p for certain gases at zero pressure and at temperatures from 0° to 4000°F are given in Table III. Approximate values, which may be considered to remain constant over the ranges of pressure and temperature involved in most problems, are given in Table IV.

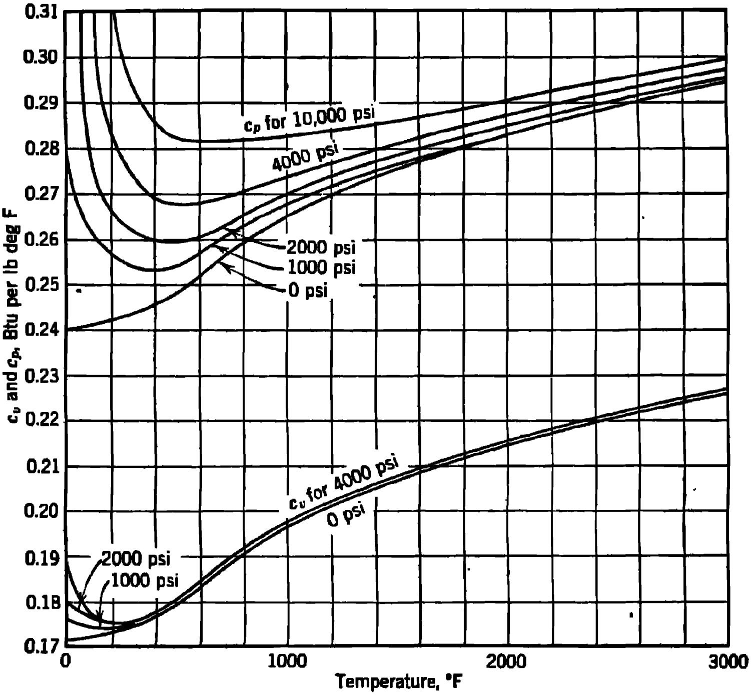


FIG. 27. The effect of temperature and pressure on c_v and c_p for dry air. (From "The Specific Heats of Certain Gases over Wide Ranges of Pressures and Temperatures," by F. O. Ellenwood, N. Kulik, and N. R. Gay, *Cornell Univ. Eng. Exp. Sta. Bull.* 30, Oct., 1942.)

37. $u_2 - u_1$ for a Perfect Gas. An equation for calculating the difference between the internal energy u_1 at any one state of a perfect gas and the internal energy u_2 at any other state will now be derived. As an aid in visualizing certain steps in the derivation, let the two states be represented by points 1 and 2 on a pressure-volume diagram, as shown in Fig. 28. Note that, since states 1 and 2 are *any* two states, the pressures, specific volumes, and temperatures at the two states are in general different. Consider next two other states x and y, chosen so that $t_1 =$

TABLE III

VALUES OF c_v AND c_p FOR CERTAIN GASES AT ZERO PRESSURE

(In British thermal units per pound degree Fahrenheit)

Temperature, °F	Air		O_2		N_2	
	c_v	c_p	c_v	c_p	c_v	c_p
0	0.171	0.240	0.156	0.218	0.178	0.249
500	0.179	0.248	0.173	0.235	0.183	0.254
1000	0.196	0.265	0.191	0.253	0.199	0.269
2000	0.215	0.283	0.209	0.271	0.223	0.294
3000	0.227	0.295	0.220	0.282	0.236	0.306
4000	0.235	0.304	0.229	0.291	0.242	0.313

Temperature, °F	CO_2		CO		H_2	
	c_v	c_p	c_v	c_p	c_v	c_p
0	0.144	0.189	0.177	0.248	2.42	3.40
500	0.201	0.246	0.185	0.256	2.52	3.50
1000	0.234	0.279	0.202	0.273	2.56	3.54
2000	0.267	0.312	0.226	0.297	2.81	3.79
3000	0.284	0.329	0.238	0.309	3.09	4.07
4000	0.290	0.335	0.243	0.314	3.28	4.26

(From "The Specific Heats of Certain Gases over Wide Ranges of Pressures and Temperatures," by F. O. Ellenwood, N. Kulik, and N. R. Gay, *Cornell Univ. Eng. Exp. Sta. Bull.* 30, Oct., 1942.)

TABLE IV

APPROXIMATE VALUES OF c_v AND c_p FOR CERTAIN GASES AT MODERATE PRESSURES AND TEMPERATURES

	c_v Btu per lb deg F	c_p Btu per lb deg F
Air	0.172	0.241
Carbon dioxide	0.165	0.210
Carbon monoxide	0.178	0.249
Helium	0.753	1.250
Hydrogen	2.49	3.48
Nitrogen	0.178	0.249
Oxygen	0.156	0.218
Steam	0.365	0.475

t_x, $t_2 = t_y$, and $v_x = v_y$, as also shown in Fig. 28. By Joule's Law (Eq. 16), since the temperature at states 1 and x and at states 2 and y are the same, $u_1 = u_x$ and $u_2 = u_y$. Hence,

$$u_2 - u_1 = u_y - u_x.$$

The problem of deriving an expression for $u_2 - u_1$ is thus reduced to one of determining an expression for $u_y - u_x$, where states x and y are at the same specific volume. Such an expression can be readily obtained by determining the difference between Q_{x-y} and W_{x-y} for a constant volume, non-flow process connecting states x and y. For such a process, $Q_{x-y} = \int_{t_x}^{t_y} c_v\,dt$, and $W_{x-y} = 0$. Hence,

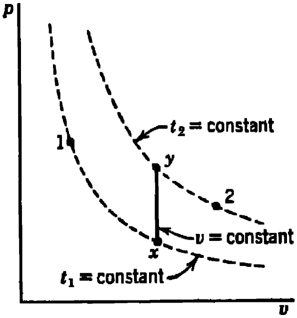


FIG. 28.

$$\begin{aligned} u_2 - u_1 &= u_y - u_x \\ &= Q_{x-y} - \frac{W_{x-y}}{J} \\ &= \int_{t_x}^{t_y} c_v\,dt. \end{aligned}$$

But $t_1 = t_x$ and $t_2 = t_y$; so that, finally, for any two states of a perfect gas,

$$u_2 - u_1 = \int_{t_1}^{t_2} c_v\,dt. \tag{17}$$

Note that this expression for $u_2 - u_1$ is not dependent upon what the specific volume at states x and y may be. This is true because c_v for a perfect gas is a function of the temperature only, and therefore Q_{x-y} is the same regardless of what the specific volume of the gas is during the constant volume process from x to y.

As explained in the preceding article, for moderate ranges of temperature, c_v for a perfect gas may usually be assumed to remain constant. With this assumption, Eq. 17 reduces to

$$u_2 - u_1 = c_v(t_2 - t_1). \tag{17a}$$

This equation will be used frequently in the solution of problems involving perfect gases.

ILLUSTRATIVE PROBLEMS

72. In Illustrative Problem 51 on page 69, the difference between the internal energies of a pound of air at 50 psia and 300°F and at 30 psia and 100°F was determined. Assuming that c_v for air is constant, recalculate this difference by Eq. 17*a*, and compare with the value previously determined.

Solution. From Table IV, c_v for air is approximately 0.172 Btu per lb deg F. Hence, by Eq. 17*a*,

$$u_2 - u_1 = 0.172(100 - 300)$$
$$= -34.4 \text{ Btu per lb.}$$

This was also the value calculated in Illustrative Problem 51.

73. One pound of nitrogen is compressed at a constant temperature of 70°F from a volume of 10 cu ft to a volume of 5 cu ft. If the process is non-flow, and if no work is done on the gas except by a moving piston, how much heat is added to or removed from the gas during the process?

Solution. Since the initial and final temperatures are the same, by Joule's Law (or by Eq. 17*a*) $u_1 = u_2$. Hence, the non-flow energy equation reduces to

$$Q = \frac{W}{J}.$$

Since $W = \int p\,dv$ for the process, and since $p = RT/v$,

$$Q = \frac{1}{J}\int p\,dv$$
$$= \frac{RT}{J}\int \frac{dv}{v}$$
$$= \frac{RT}{J}\ln\left(\frac{v_2}{v_1}\right)$$
$$= \frac{55.1 \times 530}{778}\ln\left(\frac{5}{10}\right)$$
$$= -26.0 \text{ Btu per lb.}$$

The negative sign indicates that the heat is removed from the gas during the process.

74. Three pounds of air initially at 90°F are confined inside a cylinder equipped with a piston. If the air undergoes an adiabatic expansion during which it does 40,000 ft-lb of work, what will be its temperature at the end of the process?

Solution. For an adiabatic process, the non-flow energy equation becomes

$$0 = u_2 - u_1 + \frac{W}{J}.$$

Assuming that c_v for air remains constant, by Eq. 17*a*, we get

$$0 = c_v(t_2 - t_1) + \frac{W}{J}$$
$$= 0.172(t_2 - 90) + \frac{40{,}000}{3 \times 778},$$

whence

$$t_2 = -10°\text{F}.$$

75. In Illustrative Problem 29 on page 53 a fluid initially at 200 psia and having a volume of 3 cu ft undergoes a non-flow, reversible process during which $pV^{1.3}$ remains constant. The work done by the fluid in expanding to a pressure of 40 psia is found to be 89,200 ft-lb. If the fluid is 2 lb of air, how much heat is added to it during the process?

Solution. The heat added can be calculated by the non-flow energy equation if the change in the internal energy of the air can be determined. The latter can be calculated by Eq. 17*a*, after the initial and final temperatures of the air have been found.

To determine T_1 we can make use of the relation $pv = RT$. Thus,

$$T_1 = \frac{200 \times 144 \times \frac{3}{2}}{53.3}$$

$$= 810^\circ\text{F abs.}$$

To determine T_2, it is necessary first to calculate V_2 from the relation $p_1V_1^{1.3} = p_2V_2^{1.3}$. Rearranging this equation, we get

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{1/1.3}$$

$$= 3\left(\tfrac{200}{40}\right)^{1/1.3}$$

$$= 10.34 \text{ cu ft.}$$

Hence, from $pv = RT$,

$$T_2 = \frac{40 \times 144 \times 10.34}{2 \times 53.3}$$

$$= 559^\circ\text{F abs.}$$

Finally, per pound of air,

$$Q = u_2 - u_1 + \frac{W}{J}$$

$$= c_v(T_2 - T_1) + \frac{W}{J}$$

$$= 0.172(559 - 810) + \frac{89{,}200}{2 \times 778}$$

$$= 14.3 \text{ Btu per lb.}$$

The total heat added to the 2 lb of air during the process is therefore 28.6 Btu.

PROBLEMS

In solving problems involving air, assume that the values of c_v and c_p are constant and equal to 0.172 and 0.241 Btu per lb deg F respectively.

76. A pound of air undergoes a non-flow process during which its temperature rises from 60° to 190°F. If 8500 ft-lb of work are done on the air, what is the average specific heat for the process? *Ans.* 0.088 Btu per lb deg F.

77. Eight pounds of air initially at 75°F undergo an adiabatic, non-flow process. If the air does 75,000 ft-lb of work, what is its final temperature?

78. A pound of air undergoes a non-flow process which takes place in such a way that the temperature of the air remains constant at 80°F. If the pressure rises from 14.7 to 100 psia, how much heat is added to the air? *Ans.* −71.0 Btu.

79. A quantity of air undergoes a constant pressure, non-flow process during which it absorbs 40 Btu of heat. How much work does the air do?

80. Three pounds of air undergo a non-flow process which takes place in such a way that the pressure varies linearly with the volume. If the pressure decreases from 100 to 60 psia and the volume increases from 6 to 12 cu ft, how much heat is added to the air? *Ans.* 145.1 Btu.

81. One pound of air initially at 20 psia and 80°F undergoes a non-flow process which takes place in such a way that the quantity $pv^{1.25}$ remains constant. If the final pressure of the air is 50 psia, what is the average specific heat for the process?

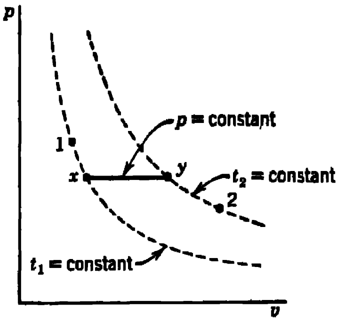


FIG. 29.

38. $h_2 - h_1$ for a Perfect Gas. An equation for calculating the difference between the enthalpy h_1 at any one state of a perfect gas and the enthalpy h_2 at any other state is derived in this article.

Let *any* two states of a perfect gas be represented by points 1 and 2 on a pressure-volume diagram, as shown in Fig. 29. The difference between the enthalpies of the gas at these two states is, from the definition of enthalpy,

$$h_2 - h_1 = \left(u_2 + \frac{p_2v_2}{J}\right) - \left(u_1 + \frac{p_1v_1}{J}\right).$$

Consider now two other states x and y, chosen so that $t_1 = t_x$, $t_2 = t_y$, and $p_x = p_y$, as also shown in Fig. 29. Since $t_1 = t_x$, it follows from Joule's Law that $u_1 = u_x$. Also, from the relation $pv = RT$, it follows that $p_1v_1 = p_xv_x$. Similarly, since $t_2 = t_y$, it follows that $u_2 = u_y$ and $p_2v_2 = p_yv_y$. The foregoing expression for $h_2 - h_1$ may therefore be written

$$h_2 - h_1 = \left(u_y + \frac{p_yv_y}{J}\right) - \left(u_x + \frac{p_xv_x}{J}\right),$$

or, since $p_x = p_y$,

$$h_2 - h_1 = (u_y - u_x) + \frac{p(v_y - v_x)}{J},$$

where $p = p_x = p_y$. The right side of this equation, however, is equal to the heat Q_{x-y} which would be added to the gas if it were to change from state x to state y by a constant pressure, non-flow process during which the only work involved is done on or by a slowly moving piston. Hence,

$$h_2 - h_1 = Q_{x-y}.$$

But, from the definition of c_p, we may also write

$$Q_{x-y} = \int_{t_x}^{t_y} c_p\, dt = \int_{t_1}^{t_2} c_p\, dt.$$

Therefore, finally,

$$h_2 - h_1 = \int_{t_1}^{t_2} c_p\, dt. \tag{18}$$

Note that this expression for $h_2 - h_1$ for a perfect gas is not dependent upon the pressure at states x and y because c_p for a perfect gas is a function of the temperature only: that is, Q_{x-y} has the same value regardless of what the pressure may be during the constant pressure process from x to y.

If c_p may be considered approximately constant, Eq. 18 reduces to

$$h_2 - h_1 = c_p(t_2 - t_1). \tag{18a}$$

Since the assumption that c_p for a perfect gas remains constant will usually be made, Eq. 18a will be used in the solution of many problems involving perfect gases.

ILLUSTRATIVE PROBLEMS

82. Forty pounds per minute of air at 65 psia and 80°F enter a long pipe having a constant cross-sectional area of 3 sq in. If the air leaves the pipe at 15 psia and 150°F, at what rate is heat being supplied to it?

Solution. Since the shaft-work is zero, the steady-flow energy equation (Eq. 13a) may be written

$$Q = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2gJ};$$

or, since $h_2 - h_1 = c_p(t_2 - t_1)$ by Eq. 18a,

$$Q = c_p(t_2 - t_1) + \frac{V_2^2 - V_1^2}{2gJ}.$$

The velocities V_1 and V_2 can be calculated by the continuity equation (Eq. 15) after the specific volumes of the entering and leaving air have been determined from the relation $pv = RT$. Thus,

$$v_1 = \frac{53.3 \times 540}{65 \times 144} = 3.07 \text{ cu ft per lb},$$

$$V_1 = \frac{wv_1}{a} = \frac{40 \times 3.07 \times 144}{60 \times 3} = 98 \text{ ft per sec},$$

$$v_2 = \frac{53.3 \times 610}{15 \times 144} = 15.05 \text{ cu ft per lb},$$

and

$$V_2 = \frac{40 \times 15.05 \times 144}{60 \times 3} = 481 \text{ ft per sec.}$$

Substituting these values into the steady-flow equation, we get

$$Q = 0.241(150 - 80) + \frac{481^2 - 98^2}{50{,}000}$$
$$= 21.3 \text{ Btu per lb.}$$

The rate at which heat is being supplied to the air is therefore 40 × 21.3 = 852 Btu per min.

83. Oxygen at 50 psia and 70°F enters a nozzle with a negligible velocity. It expands adiabatically as it flows through the nozzle and leaves with a velocity of 980 ft per sec. What is its temperature as it leaves the nozzle?

Solution. Since the heat added and the shaft-work are both zero, and since the entering velocity is negligible, the steady-flow energy equation may be written

$$h_1 = h_2 + \frac{V_2^2}{2gJ};$$

or, since $h_2 - h_1 = c_p(t_2 - t_1)$ by Eq. 18*a*,

$$0 = c_p(t_2 - t_1) + \frac{V_2^2}{2gJ}.$$

Therefore,

$$0 = 0.218(t_2 - 70) + \frac{980^2}{50{,}000}$$

whence

$$t_2 = -18.1\text{°F}.$$

84. An air compressor takes in 80 lb per min of air at 14 psia and 60°F and discharges it at 70 psia and 165°F. If 3200 Btu per min of heat are removed by the cooling water, how many horsepower are being supplied to drive the compressor? Assume that the velocities of the air entering and leaving the compressor and the heat loss to the room are all negligible.

Solution. Since the entering and leaving velocities are assumed to be negligible, the steady-flow energy equation may be written

$$Q + h_1 = \frac{W'}{J} + h_2;$$

or, since $h_2 - h_1 = c_p(t_2 - t_1)$ by Eq. 18*a*,

$$\frac{W'}{J} = Q - c_p(t_2 - t_1).$$

Hence,

$$\frac{W'}{J} = -\frac{3200}{80} - 0.241(165 - 60)$$
$$= -65.3 \text{ Btu per lb.}$$

Therefore, since there are 2545 Btu per hp-hr, the power *supplied* to the compressor is

$$\frac{65.3 \times 80 \times 60}{2545} = 123 \text{ hp.}$$

PROBLEMS

85. An air motor operates adiabatically and uses 15 lb per min of air to deliver 5 hp. If the air enters the motor at 50 psia and 70°F, and if the velocities of the air entering and leaving are approximately equal, at what temperature does the air leave? *Ans.* 11.3°F.

86. Air enters a converging-diverging nozzle (see Fig. 116) at 80 psia and 120°F with a velocity of 100 ft per sec. It flows through the nozzle adiabatically and leaves at 30 psia and −15°F. If the cross-sectional area of the discharge end of the nozzle is 0.20 sq in., how many pounds of air flow through the nozzle per minute?

87. A centrifugal compressor takes in 1000 cfm of air at 14.2 psia and 70°F and discharges it at 20.5 psia and 100°F. If the cross-sectional areas of the intake and discharge passages are 25 and 5 sq in. respectively, and if 20 hp are supplied to drive the compressor, at what rate is heat being removed from the machine? *Ans.* 159 Btu per min.

88. A stream of air at 14.5 psia and 90°F flows at the rate of 500 lb per min through a duct having a cross-sectional area of 0.50 sq ft. A second stream also at 14.5 psia but at 220°F flows at the rate of 400 lb per min through a duct having a cross-sectional area of 0.75 sq ft. If the two streams are mixed adiabatically, and if the resulting stream flows with a velocity of 70 ft per sec, what is its temperature?

39. The Relation between c_p, c_v, and R for a Perfect Gas. A relation between c_p, c_v, and R for any perfect gas may be derived as follows: The enthalpy of the gas at any state is, by definition,

$$h = u + \frac{pv}{J},$$

or, since $pv = RT$ for a perfect gas,

$$h = u + \frac{RT}{J}.$$

If the gas undergoes any infinitesimal change of state, the relation between the change in its enthalpy, internal energy, and temperature, obtained by differentiating the above equation, is

$$dh = du + \frac{R}{J}\,dT.$$

But, for an infinitesimal change of state, Eq. 17 may be written $du = c_v\,dt = c_v\,dT$. Likewise, Eq. 18 may be written $dh = c_p\,dt = c_p\,dT$. Introducing these expressions for du and dh into the foregoing equation, we obtain

$$c_p\,dT = c_v\,dT + \frac{R}{J}\,dT,$$

or

$$c_p = c_v + \frac{R}{J},$$

or

$$c_p - c_v = \frac{R}{J}. \tag{19}$$

Thus, at any temperature, the difference between c_p and c_v for a perfect gas is equal to the gas constant R divided by J. This equation is sometimes useful in the solution of problems but is of particular value in simplifying certain later derivations.

The values of c_p and c_v listed in Table III for certain gases at zero pressure may be checked by Eq. 19. Since these gases become perfect gases at zero pressure, the difference between c_p and c_v at any temperature must be equal to R/J for the gas. These differences for air, for example, at the temperatures listed are all either 0.068 or 0.069 Btu per lb deg F, and R/J for air is $53.3/778 = 0.0685$ Btu per lb deg F. Hence, for the number of significant figures given in the table, the values of c_p and c_v are consistent with Eq. 19.

PROBLEMS

89. Show that, if the value of c_v for a perfect gas is constant,

$$J(u_2 - u_1) = \frac{p_2v_2 - p_1v_1}{k - 1}, \tag{17b}$$

where $k = c_p/c_v$.

90. Show that, if the value of c_p for a perfect gas is constant,

$$J(h_2 - h_1) = \frac{k}{k - 1}(p_2v_2 - p_1v_1), \tag{18b}$$

where $k = c_p/c_v$.

LIQUIDS, VAPORS, AND LIQUID-VAPOR MIXTURES

It was explained at the beginning of this chapter that differences in the internal energies and enthalpies at any two states of most systems of importance in engineering have been determined by experiment. For liquids and vapors these differences can be represented by equations, but the equations—unlike those for perfect gases—are so complex that their use for ordinary engineering calculations is impractical. For this reason, tables based on the equations have been prepared.

The Keenan and Keyes *Steam Tables* were used in Chapter 1 to illustrate how values of pressure, specific volume, or temperature can be determined from tables of thermodynamic properties when two of the properties of the system are known. These tables will now be used to illustrate how values of internal energy and enthalpy at any state can be determined from a table of properties. Excerpts from the *Steam Tables* are given on pages 29, 30, and 31. The method of using these tables to determine enthalpies will be considered first.

40. Determination of Enthalpies of Liquids, Vapors, and Liquid-Vapor Mixtures by Means of Tables of Properties. The values of enthalpy listed in tables of thermodynamic properties are the *differences* between the enthalpy at each state of the fluid and the enthalpy at some arbitrarily chosen reference state. The values listed in the *Steam Tables*, for example, are the differences between the enthalpy at each state and the enthalpy of saturated water at 32°F; that is, the enthalpy of saturated water at 32°F is arbitrarily set equal to zero. This in no way affects the usefulness of the tables since they are always used (*a*) to determine the difference between the enthalpies at any two states, or (*b*) to determine a second state having the same enthalpy as some initial state. It is evident that neither of these uses would be affected if a constant were added to or subtracted from each of the values of enthalpy listed in the tables.

The difference between the enthalpy at any state and the enthalpy at the reference state (that is, the value listed in the tables) will be referred to simply as the "enthalpy" at the given state. The method of finding enthalpies by means of the tables is exactly the same as that of finding specific volumes, explained in Art. 15. It will be reviewed briefly.

The enthalpies of saturated water, denoted by h_f, and of saturated steam, denoted by h_g, are listed in both Tables 1 and 2 of the *Steam Tables*. They are listed at integral values of the saturation temperature in Table 1 and at integral values of the saturation pressure in Table 2. Hence, values of the enthalpy of saturated water and of saturated steam may be read directly from these tables if either the saturation temperature or saturation pressure is known.

For convenience, the difference between h_g and h_f, denoted by h_{fg}, is also listed in Tables 1 and 2. It is useful to note that h_{fg} is equal to the **heat of vaporization.** The latter quantity is defined as the amount of heat required to change 1 lb of saturated water into saturated steam by a non-flow, constant pressure process during which the only work involved is equal to the integral of $p\,dv$. That this is true is evident from the fact that, by the definition of h_{fg} and the definition of enthalpy itself,

$$
\begin{aligned}
h_{fg} &= h_g - h_f \\
&= \left(u_g + \frac{pv_g}{J}\right) - \left(u_f + \frac{pv_f}{J}\right) \\
&= (u_g - u_f) + \frac{p(v_g - v_f)}{J}
\end{aligned}
$$

By the non-flow energy equation, the right side of the last equation is equal to the heat added during a constant pressure process in which the initial state is saturated water and the final state is saturated steam; that is, h_{fg} is equal to the heat of vaporization.

If the quality and either the temperature or pressure of a mixture of saturated water and saturated steam are known, the enthalpy of the mixture can be determined by finding the sum of the enthalpies of the water and steam. Thus, since 1 lb of mixture consists of x pounds of saturated vapor and $(1 - x)$ pounds of saturated water, the enthalpy of a pound of the mixture is

$$h = xh_g + (1 - x)h_f. \tag{20}$$

The form of this equation is exactly the same as that of Eq. 5 on page 30, but the $(1 - x)h_f$ term, unlike the $(1 - x)v_f$ term in Eq. 5, may not be neglected. Somewhat more convenient forms of Eq. 20 can be derived readily by making use of the fact that, by definition, $h_{fg} = h_g - h_f$. These alternate forms are

$$h = h_f + xh_{fg} \tag{20a}$$

and

$$h = h_g - (1 - x)h_{fg}. \tag{20b}$$

Note that these equations require that only a single multiplication be made. For purposes of slide-rule calculations, it is more nearly accurate to use Eq. 20*a* if the quality x is less than 0.5 and to use Eq. 20*b* if x is greater than 0.5. As an illustration of the use of these equations, the enthalpy of a mixture of saturated water and saturated steam at 50 psia and having a quality of 90% is, by Eq. 20*b*,

$$
\begin{aligned}
h &= 1174.1 - 0.1 \times 924.0 \\
&= 1081.7 \text{ Btu per lb.}
\end{aligned}
$$

The enthalpies of superheated steam are listed in Table 3 of the *Steam Tables* and can be read directly if any two of the three properties, pressure, specific volume, or temperature are known. For example, the enthalpy of superheated steam at 52 psia and 400°F is 1234.8 Btu per lb, and the enthalpy of superheated steam having a specific volume of 9 cu ft per lb at a pressure of 53 psia is 1214.6 Btu per lb.

In determining values of the enthalpy of subcooled water, little error is made at moderate pressures if, the effect of pressure being neglected, the enthalpy of the subcooled water is assumed to be the same as the enthalpy of saturated water at the same temperature. For example, the enthalpy of subcooled water at 400 psia and 200°F is approximately equal to 167.99 Btu per lb, the value of h_f at 200°F (from Table 1). If the exact value is needed, it can be found by Table 4. Thus, the difference between the enthalpy of subcooled water at 400 psia and 200°F and the enthalpy of saturated water at 200°F is, from Table 4,

$$h - h_f = +0.88 \text{ Btu per lb.}$$

Therefore, the exact enthalpy of the subcooled water is

$$\begin{aligned} h &= h_f + 0.88 \\ &= 167.99 + 0.88 \\ &= 168.87 \text{ Btu per lb.} \end{aligned}$$

In the solution of many problems, the enthalpy and either the pressure or temperature at a state are known. Before any other property can be found, it is necessary to determine whether the fluid is a subcooled liquid, a mixture of saturated liquid and saturated vapor, or a superheated vapor. This can be done by comparing the given value of the enthalpy with the values of h_f and h_g at the given pressure or temperature. If the enthalpy is less than h_f, the fluid is a subcooled liquid; if it is between h_f and h_g, the fluid is a mixture; and, if it is greater than h_g, the fluid is superheated vapor.

For example, suppose that the specific volume of steam at 52 psia and an enthalpy of 1000 Btu per lb is to be determined. At a pressure of 52 psia, the value of h_f for saturated water is 252.63 Btu per lb, and the value of h_g for saturated steam is 1174.8 Btu per lb. Since the given enthalpy is between these values, the steam is a mixture. Before its specific volume can be determined, its quality must be calculated. This can be done by Eq. 20*a*. Thus, since h_{fg} at 52 psia is 922.2 Btu per lb,

$$1000 = 252.63 + x922.2,$$

whence

$$x = 0.810.$$

The specific volume of the mixture, calculated by Eq. 5*a*, is therefore

$$\begin{aligned} v &= 0.810 \times 8.208 \\ &= 6.65 \text{ cu ft per lb.} \end{aligned}$$

If the pressure is kept at 52 psia but the enthalpy is increased from 1000 to 1250 Btu per lb, the steam becomes superheated since this enthalpy is larger than h_g. The specific volume must now be determined by Table 3. Interpolating between 9.914 cu ft per lb (corresponding to an enthalpy of 1244.6 Btu per lb) and 10.154 cu ft per lb (corresponding to an enthalpy of 1254.5 Btu per lb), all at a pressure of 52 psia, we find that the specific volume of the superheated steam is 10.045 cu ft per lb.

If the enthalpy is reduced to 100 Btu per lb while the pressure is kept at 52 psia, the fluid is changed to subcooled water since this enthalpy is less than h_f. If we assume that the water is incompressible (or that h for the subcooled water is equal to h_f for saturated water at the same temperature), its other properties can be found in either Table 1 or 2. Its temperature, for example, is about 132.1°F.

ILLUSTRATIVE PROBLEMS

91. Water at 200 psia and 150°F enters a boiler operating in a steady-flow manner. The steam leaving the boiler is at 190 psia and has a quality of 98%. If the velocities of the water entering and the steam leaving are both low, how much heat is required to generate each pound of steam?

Solution. Since the shaft-work is zero, and since the entering and leaving velocities are negligible, the steady-flow energy equation (Eq. 13*a*) may be written

$$Q = h_2 - h_1.$$

The exact value of h_1, the enthalpy of the water entering the boiler, can be determined by means of Tables 1 and 4. Thus, from Table 1, h_f at 150°F is 117.89 Btu per lb, and, from Table 4, $h_1 - h_f$ at 200 psia and 150°F is +0.48 Btu per lb (by interpolation). Hence, $h_1 = 117.89 + 0.48 = 118.37$ Btu per lb. The value of h_2, the enthalpy of the steam leaving the boiler, can be calculated by Eq. 20*b*. Determining the values of h_g and h_{fg} at 190 psia from Table 2, we find that

$$h_2 = 1197.6 - 0.02 \times 846.8$$

$$= 1180.7 \text{ Btu per lb.}$$

Therefore, the heat Q required to generate each pound of steam is

$$Q = 1180.7 - 118.37$$

$$= 1062.3 \text{ Btu per lb.}$$

Note that only a small error (0.48 Btu per lb) is introduced if the enthalpy of the water is taken simply as h_f at 150°F.

92. If the steam leaving the boiler in the preceding problem flows into a superheater, and if it leaves the superheater at a pressure of 186 psia and at a temperature of 600°F, how much heat is supplied to it in the superheater?

Solution. If we assume that the velocities of the steam entering and leaving the superheater are negligible, we may write the steady-flow energy equation in the form

$$Q = h_3 - h_2.$$

The enthalpy h_3 of the steam leaving the superheater is, from Table 3, 1323.1 Btu per lb. Therefore, the heat Q added to each pound of steam flowing through it is

$$Q = 1323.1 - 1180.7$$

$$= 142.4 \text{ Btu per lb.}$$

93. A small steam turbine delivers 100 hp at the turbine shaft when the steam flow is 1650 lb per hr. The steam enters the turbine at 150 psia and 440°F with a velocity of 80 ft per sec. It leaves the turbine at 3 in. Hg abs with a velocity of 300 ft per sec. If the turbine may be assumed to operate adiabatically, what is the quality of the steam in the exhaust pipe?

Solution. The final quality of the steam can be determined after the enthalpy h_2 of the steam leaving the turbine has been calculated by the steady-flow energy equation. The enthalpy h_1 of the steam entering the turbine is 1242.0 Btu per lb (from Table 3). Hence,

$$Q + h_1 + \frac{V_1^2}{2gJ} = \frac{W'}{J} + h_2 + \frac{V_2^2}{2gJ},$$

or

$$0 + 1242.0 + \frac{(80)^2}{50,000} = \frac{100 \times 2545}{1650} + h_2 + \frac{(300)^2}{50,000},$$

whence

$$h_2 = 1086.0 \text{ Btu per lb.}$$

The quality x_2 can now be calculated by Eq. 20*b*. Since $h_g = 1111.6$ Btu per lb and $h_{fg} = 1028.6$ Btu per lb at 3 in. Hg abs (from Table 2),

$$1086.0 = 1111.6 - (1 - x_2)1028.6,$$

whence

$$(1 - x_2) = 0.025,$$

or

$$x_2 = 0.975.$$

PROBLEMS

94. Complete the following table for H_2O:

State	p, psia	t, °F	v, cu ft per lb	h, Btu per lb
(*a*)	90	440		
(*b*)	600	300		
(*c*)	135		2.572	
(*d*)	440		1.264	
(*e*)	65			236.8
(*f*)	200			1000
(*g*)	360			1263
(*h*)		425	1.350	
(*i*)		460	4.200	
(*j*)		160		200
(*k*)		500		1265

95. Steam enters a condenser at 1½ in. Hg abs with a quality of 85%. The condensate leaving is at 75°F. If the steam is flowing into the condenser at the rate of 10,000 lb per hr, at what rate is heat being removed? Assume that the velocities of the steam entering and the condensate leaving are low.

96. A turbine is to be designed to deliver 7500 kw at its shaft. Steam will be supplied to the turbine at 415 psia and 600°F and will leave at 2½ in. Hg abs. It is estimated that 68,500 lb per hr of steam will be required. If the steam will enter the turbine with a velocity of 120 ft per sec and will leave with a velocity of 300 ft per sec, what must be the cross-sectional area of the exhaust end of the turbine? Assume that the heat loss from the turbine will be negligible. *Ans.* 14.4 sq ft.

97. Steam at 40 psia and 300°F will enter a converging-diverging nozzle (Fig. 116) with a velocity of 225 ft per sec. It will expand adiabatically as it flows through the nozzle and will leave at 15 psia with a quality of 97%. What must be the cross-sectional area of the discharge end of the nozzle if the steam is to flow at the rate of 15 lb per min?

98. Superheated steam at 600 psia and 800°F is to flow into an insulated chamber at the rate of 80,000 lb per hr. Water at 180°F is to be sprayed into the chamber at a rate such that the steam leaving at 600 psia will be saturated. If the velocity of each of the streams is low, at what rate must the water be supplied?

Ans. 15,500 lb per hr.

41. Determination of Internal Energies of Liquids, Vapors, and Liquid-Vapor Mixtures by Means of Tables of Properties. Whereas values of enthalpy are listed in Tables 1, 2, 3, and 4 of the *Steam Tables*, values of internal energy are listed only in Table 2. The reason for this is that most engineering problems involve steady-flow rather than non-flow processes, and consequently values of enthalpy are needed more frequently than are values of internal energy. The internal energy at any state can be calculated readily, however, from the pressure, specific volume, and enthalpy at that state since, from the definition of enthalpy,

$$u = h - \frac{pv}{J}.$$

For saturated water, saturated steam, or for mixtures of the two, the internal energy can also be determined from the values of u_f (the internal energy of saturated water) and u_g (the internal energy of saturated steam) listed in Table 2. For a mixture, the internal energy is the sum of the internal energies of the saturated water and saturated steam. Hence, for 1 lb of mixture,

$$u = xu_g + (1 - x)u_f, \tag{21}$$

or, since by definition $u_{fg} = u_g - u_f$,

$$u = u_f + xu_{fg} \tag{21a}$$

and

$$u = u_g - (1 - x)u_{fg}. \tag{21b}$$

It is preferable for slide-rule calculations to use Eq. 21*a* if x is less than 0.5 and to use Eq. 21*b* if x is greater than 0.5.

If the internal energy and either the pressure or temperature at any state are known, the condition of the fluid can be determined by comparing the value of the internal energy with the values of u_f and u_g at the given pressure or temperature. If the internal energy is less than u_f, the fluid is subcooled liquid; if it is between u_f and u_g, the fluid is a mixture; and, if it is greater than u_g, the fluid is superheated vapor.

ILLUSTRATIVE PROBLEMS

99. A rigid tank contains 5 cu ft of steam at 200 psia and 600°F. How much heat must be removed from the steam in order to lower its pressure to 150 psia?

Solution. Since the steam will undergo a constant volume, non-flow process during which no work is done, the non-flow energy equation (Eq. 12) may be written

$$Q = u_2 - u_1.$$

In its initial state the steam is superheated. From Table 3, its enthalpy is 1322.1 Btu per lb, and its specific volume is 3.060 cu ft per lb. Hence,

$$\begin{aligned} u_1 &= 1322.1 - \frac{200 \times 144 \times 3.060}{778} \\ &= 1208.9 \text{ Btu per lb.} \end{aligned}$$

In its final state the steam will still be superheated since its specific volume will remain at 3.060 cu ft per lb, a value larger than v_g at 150 psia. From Table 3 (by interpolation), the enthalpy of superheated steam at 150 psia and having a specific volume of 3.060 cu ft per lb is 1199.7 Btu per lb. Hence,

$$\begin{aligned} u_2 &= 1199.7 - \frac{150 \times 144 \times 3.060}{778} \\ &= 1114.7 \text{ Btu per lb.} \end{aligned}$$

Therefore,

$$\begin{aligned} Q &= 1114.7 - 1208.9 \\ &= -94.2 \text{ Btu per lb,} \end{aligned}$$

the negative sign indicating that the heat must be removed from the steam during the process. Since the weight of steam in the tank is 5/3.060 = 1.633 lb, the total heat which must be removed is 1.633 × 94.2 = 154 Btu.

100. How much heat would have to be removed from the steam in the preceding problem if the final pressure is to be 100 psia instead of 150 psia?

Solution. In its final state the steam would now be a mixture since its specific volume (3.060 cu ft per lb) would be between v_f and v_g at 100 psia. Its quality can be calculated by Eq. 5*a*. Since $v_g = 4.432$ cu ft per lb at 100 psia,

$$3.060 = x_2 4.432,$$

whence

$$x_2 = 0.691.$$

The final internal energy can now be calculated by Eq. 21*b*. Determining the values of u_g and u_{fg} at 100 psia from Table 2, we find that

$$u_2 = 1105.2 - 0.309 \times 807.1$$
$$= 855.8 \text{ Btu per lb.}$$

Therefore,

$$Q = 855.8 - 1208.9$$
$$= -353.1 \text{ Btu per lb.}$$

The total heat which would have to be removed is 1.633 × 353.1 = 576 Btu.

101. How much heat would have to be removed from the steam in Illustrative Problem 99 in order to condense one half of it (by weight)?

Solution. In its final state the steam would be a mixture having a quality of 0.50 and a specific volume of 3.060 cu ft per lb. In order to calculate its internal energy, it is necessary first to determine the values of u_f and u_{fg}. This can be done as follows: The specific volume v_g of the saturated steam present in the final mixture can be calculated by Eq. 5*a*. Thus,

$$3.060 = 0.50 v_g,$$

whence

$$v_g = 6.120 \text{ cu ft per lb.}$$

Therefore, from Table 2, the final pressure is about 71 psia (at which pressure v_g is 6.124 cu ft per lb), and u_f and u_{fg} are approximately 273.37 and 827.0 Btu per lb respectively. By Eq. 21*a*,

$$u_2 = 273.37 + 0.5 \times 827.0$$
$$= 686.9 \text{ Btu per lb.}$$

Therefore,

$$Q = 686.9 - 1208.9$$
$$= -522.0 \text{ Btu per lb,}$$

and the total heat which would have to be removed is 1.633 × 522.0 = 852 Btu.

PROBLEMS

102. Two pounds of steam initially at 300°F and having a quality of 90% are heated at constant volume to 600°F. How much heat is added to the steam?

Ans. 395 Btu.

103. Two pounds of steam initially at 300°F and having a quality of 90% are heated at constant pressure to 600°F. If the process is reversible, how much heat is added to the steam?

104. A pound of steam initially at 65 psia and having a volume of 7.3 cu ft is confined inside a cylinder equipped with a piston. If the steam were allowed to expand adiabatically until its pressure dropped to 10 psia, its volume would increase to 27.5 cu ft. How much work would the steam do during the process?

Ans. 312 Btu.

105. A pound of steam initially at 200 psia and having an initial volume of 2 cu ft undergoes a non-flow process during which it absorbs 250 Btu of heat and does 100,000 ft-lb of work. If its final pressure is 40 psia, what is its final volume?

GENERAL PROBLEMS

106. A quantity of air undergoes a non-flow process during which its pressure increases from 35 to 100 psia and its volume decreases from 16 to 7 cu ft. If the average specific heat for the process is 0.035 Btu per lb deg F, how much work is done on the air? *Ans.* 51.5 Btu.

107. A stream of air is to be forced through a finned-tube heater by a blower. The air entering the blower will be flowing at the rate of 2000 cfm and will be at 14.7 psia and 60°F. If 2 hp will be needed to operate the blower, and if 90,000 Btu per hr of heat will be supplied to the air by the heater, at what temperature will the stream leave the apparatus? Assume that the velocities of the streams entering and leaving the apparatus will be low.

108. Ammonia flows first through a valve and then through a section of pipe. If the ammonia enters the valve at 140 psia and 70°F, and if it leaves the pipe at 30 psia and 10°F, how much heat does each pound absorb in flowing through the apparatus? Assume that the velocities of the streams entering the valve and leaving the pipe are low. *Ans.* 497.3 Btu per lb.

109. A rigid tank contains 3 lb of saturated water and 1 lb of saturated steam at 25 psia. How much heat must be added to evaporate all the water?

110. Three pounds of a gas having a molecular weight of 35 undergo a constant pressure, non-flow process during which the gas absorbs 50 Btu of heat and its temperature rises from 70 to 150°F. How much heat would be required to raise 2 lb of the gas from 70° to 300°F if the volume of the gas were kept constant? *Ans.* 69.5 Btu.

111. A stream of air at 75 psia and 120°F enters a valve which is insulated perfectly against any loss of heat. The air leaves the valve at 30 psia and flows through a bare pipe which has a cross-sectional area of 2.0 sq in. The air leaves this pipe at 14.7 psia and 90°F. If the velocities of the streams entering and leaving the valve are equal, and if the air flows out of the end of the bare pipe at the rate of 400 cfm, at what rate is the air losing heat as it flows through the pipe?

112. Water at 100 psia and 300°F flows through a valve into a region where the pressure is 14.7 psia. If the valve is insulated against any loss of heat, and if the velocities of the streams entering and leaving the valve are low, what fraction of the water will change into steam on flowing through the valve? *Ans.* 0.0923.

113. A pound of steam initially at 100 psia and having a quality of 80% undergoes a non-flow process which takes place in such a way that the pressure varies linearly with the volume. If in its final state the steam is at 50 psia and 300°F, how much heat is added to it during the process?

114. A tank is divided into two compartments by a partition. One compartment has a volume of 10 cu ft and contains air at 14.7 psia and 60°F. The other has a volume of 5 cu ft and contains air at 80 psia and 200°F. If the partition is removed, and if the air is allowed to come to equilibrium adiabatically, what will the final pressure be? *Ans.* 36.5 psia.

115. Air flows through an insulated valve. On the high pressure side the air is at 60 psia and 70°F and flows with a velocity of 150 ft per sec. On the low pressure side it is at 20 psia. If the pipes leading to and from the valve are of the same size, and if the air flows through the valve adiabatically, what is the temperature in the discharge pipe?

116. Superheated steam flows at the rate of 120 lb per min through a long pipe having an inside cross-sectional area of 4.79 sq in. If the steam enters the pipe at

100 psia and 440°F and leaves at 80 psia and 420°F, at what rate is heat lost from the pipe? *Ans.* 804 Btu per min.

117. An insulated tank is divided into two equal compartments by a partition. One compartment contains steam which is at 120 psia and has a quailty of 20%. The other compartment is completely evacuated. If the partition is removed and the system allowed to come to equilibrium, what will the final pressure be?

118. A cylinder 8 in. in diameter is placed with its axis vertical and is provided with a frictionless piston that weighs 10 lb. The cylinder contains 0.05 lb of air at 70°F. If a 500-lb weight resting on the piston is removed, and if the air is allowed to expand adiabatically, what will be the final volume of the air? Assume that atmospheric pressure is 14.70 psia.

SYMBOLS

c_p	specific heat at constant pressure
c_v	specific heat at constant volume
g	acceleration that a force of 1 lb gives to a mass of 1 lb
h	enthalpy per unit mass; h_f = enthalpy per unit mass of saturated liquid; h_g = enthalpy per unit mass of saturated vapor; $h_{fg} = h_g - h_f$
J	mechanical equivalent of heat
p	pressure
Q	heat added to system
R	gas constant
t	temperature
T	absolute temperature
u	internal energy per unit mass; u_f = internal energy per unit mass of saturated liquid; u_g = internal energy per unit mass of saturated vapor; $u_{fg} = u_g - u_f$
v	specific volume; v_f = specific volume of saturated liquid; v_g = specific volume of saturated vapor; $v_{fg} = v_g - v_f$
V	total volume
V	velocity
W	work done by system; W' = shaft-work done by a system in steady flow
x	quality, weight of saturated vapor per unit weight of mixture

Chapter 5

THE ANALYSIS OF SIMPLE PROCESSES

Two types of processes, known as *non-flow* and *steady-flow* processes, have been discussed in Chapters 2 and 3. These general types will now be subdivided into special types in which (*a*) one of the properties of the system remains constant, (*b*) no heat is added to or removed from the system, and (*c*) certain combinations of the properties of the system remain constant. These various special types are of interest because they are close approximations of many of the actual processes involved in the operation of engineering equipment. Some of them have already been considered in the problems in the preceding chapter. In the present chapter all of them will be discussed, and methods of calculating the heat added to and the work done by any system undergoing each of them will be developed. The various special types of non-flow processes will be considered first.

NON-FLOW PROCESSES

42. General Procedure. Suppose that a system undergoes a non-flow process during which (*a*) it exerts the same pressure on both its moving and stationary boundaries and (*b*) it is not stirred.* As shown in Art. 23, the work done by the system during such a process can be calculated by the equation

$$W = \int p\, dv. \tag{7}$$

Before this integration can be performed, the pressure p must be expressed as a function of the specific volume v. If it is possible to do this, and if the change $u_2 - u_1$ in the internal energy of the system can be determined by the methods discussed in the preceding chapter, the heat Q added to the system during the process can be calculated by the non-flow energy equation,

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* It may be recalled that a system undergoing a *non-flow* process is, by definition (see Art. 16), subjected to no surface, magnetic, or electric effects. Therefore, the system can dissipate work only of the mechanical form.

$$Q = u_2 - u_1 + \frac{W}{J}. \tag{12}$$

The foregoing procedure is used in the following articles to derive expressions for the work W and the heat Q added to or removed from any system during a constant volume, constant pressure, constant temperature, or a "polytropic" non-flow process. An alternative procedure is used in Art. 46 to develop a method for finding the work W done during an adiabatic non-flow process. This procedure must be used since, for this type of process, the pressure p cannot be readily expressed as a function of the specific volume v (unless the system is a perfect gas), and therefore Eq. 7 cannot be integrated.

43. The Constant Volume Non-Flow Process. Suppose that a system undergoes a non-flow process while confined inside a rigid container. It is evident that no work can be done on it to compress it and that no work can be done by it in expanding. If, in addition, the system is not stirred during the process,* we may write

$$W = 0.$$

Consequently, by the non-flow energy equation, the heat Q added is

$$Q = u_2 - u_1. \tag{22}$$

Perfect Gases. If the system is a perfect gas, by Eq. 17

$$u_2 - u_1 = \int c_v \, dt.$$

Therefore, for a perfect gas undergoing a constant volume non-flow process during which the gas is not stirred,

$$Q = \int c_v \, dt, \tag{22a}$$

or, if c_v for the gas remains constant,

$$Q = c_v(t_2 - t_1). \tag{22b}$$

Liquids, Vapors, and Liquid-Vapor Mixtures. Equations **22a** and **22b** also follow directly from the definition of c_v (see Art. 25) and are therefore applicable, not only to perfect gases, but to any system for which c_v is known. If the system is a liquid, a vapor, or a liquid-vapor mixture, the specific heat c_v is usually not known. However, if tables of thermodynamic properties are available, the heat Q added to the system can be calculated by Eq. 22. The values of u_1 and u_2 are found from the tables by the methods explained in the preceding chapter. Illustrative Problems 99, 100, and 101 in that chapter illustrate this type of problem. Illustrative Problem 120, which follows, is an additional example.

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* The process might be reversible or internally reversible, but it would not have to be either since none of the properties of the system need remain uniform during the process. All the properties of the system are uniform in the initial and final states, however, since the system is assumed then to be in equilibrium.

ILLUSTRATIVE PROBLEMS

119. A 10-cu-ft tank contains air initially at 50 psia and 300°F. If 60 Btu of heat are removed from the air, what will be its final pressure?

Solution. The specific volume of the air can be calculated from the relation $pv = RT$:

$$v = \frac{53.3 \times 760}{50 \times 144}$$

$$= 5.63 \text{ cu ft per lb.}$$

Therefore, the tank contains 10/5.63 = 1.78 lb of air. If it is assumed that c_v for the air remains constant, the final temperature of the air can be calculated by Eq. 22b:

$$Q = c_v(t_2 - t_1), \tag{22b}$$

or

$$-\frac{60}{1.78} = 0.172(t_2 - 300),$$

whence

$$t_2 = 104°\text{F (or } T_2 = 564°\text{F abs).}$$

The final pressure can now be calculated from the relation $pv = RT$:

$$p_2 = \frac{53.3 \times 564}{5.63}$$

$$= 5340 \text{ psfa } (= 37.0 \text{ psia}).$$

120. A rigid tank contains 2 lb of saturated water and 3 lb of saturated steam at 20 psia. How much heat must be supplied to evaporate all the water?

Solution. Since the volume remains constant, the heat required can be calculated by Eq. 22. Since the quality of the initial mixture is 3/(2 + 3) = 0.60, its internal energy is, by Eq. 21b,

$$u = u_g - (1 - x)u_{fg}, \tag{21b}$$

or

$$u_1 = 1081.9 - 0.40 \times 885.8$$

$$= 727.6 \text{ Btu per lb.}$$

The specific volume of the initial mixture is

$$v = \frac{2 \times 0.01683 + 3 \times 20.089}{5}$$

$$= 12.060 \text{ cu ft per lb.}$$

Hence, in the final state, the tank contains saturated steam having a specific volume of 12.060 cu ft per lb. Interpolating in Table 2 of the *Steam Tables*, we find that the internal energy u_g of this steam is 1089.9 Btu per lb (corresponding to a pressure of about 34.5 psia). Therefore, the heat required per pound of the contents of the tank is

$$Q = 1089.9 - 727.6$$

$$= 362.3 \text{ Btu per lb},$$

or the total heat required is 5 × 362.3 = 1812 Btu.

PROBLEMS

121. A 6-cu-ft tank contains air at 30 psia. How much heat must be added to the air to raise its pressure to 50 psia? *Ans.* 55.7 Btu.

122. Twelve cubic feet of air initially at 75°F are heated at constant volume If 90 Btu of heat are added to the air, and if its final pressure is 60 psia, what is its final temperature?

123. A 10-cu-ft tank contains 1.31 lb of saturated water and 0.66 lb of saturated steam. How much heat is required to raise the temperature to 520°F? *Ans.* 1363 Btu.

124. A quantity of steam initially at 210 psia and 500°F is cooled at constant volume. If the steam begins to condense after 150 Btu of heat have been removed from it, how much more heat must be removed to lower the temperature to 340°F?

44. The Constant Pressure Non-Flow Process. Suppose that a system undergoes a non-flow process during which (*a*) its pressure remains constant, (*b*) it exerts the same pressure on both its moving and stationary boundaries, and (*c*) it is not stirred.* The work done under these conditions can be determined by Eq. 7. Thus,

$$W = \int p\,dv$$

$$= p(v_2 - v_1). \tag{23}$$

The heat added to the system during the process is, by the non-flow energy equation,

$$Q = u_2 - u_1 + \frac{p(v_2 - v_1)}{J},$$

or, since $p = p_1 = p_2$,

$$Q = u_2 - u_1 + \frac{p_2v_2}{J} - \frac{p_1v_1}{J}$$

$$= h_2 - h_1. \tag{24}$$

---

* As in the preceding article, the process might be reversible or internally reversible but would not have to be either since none of the properties of the system, except its pressure, need remain uniform.

Perfect Gases. If the system is a perfect gas, the work done can be calculated by Eq. 23 or by the equations

$$\left.\begin{aligned} W &= R(T_2 - T_1) \\ &= R(t_2 - t_1). \end{aligned}\right\} \tag{23a}$$

These equations follow directly from Eq. 23 since, for a perfect gas, $pv = RT$.

In Art. 38 of the preceding chapter it was shown that the difference between the enthalpies at any two states of a perfect gas is

$$h_2 - h_1 = \int c_p\, dt. \tag{18}$$

Therefore, combining this equation with Eq. 24, we get

$$Q = \int c_p\, dt, \tag{24a}$$

or, if c_p for the gas remains constant,

$$Q = c_p(t_2 - t_1). \tag{24b}$$

Liquids, Vapors, and Liquid-Vapor Mixtures. If the system is a liquid, a vapor, or a liquid-vapor mixture, and if tables of its thermodynamic properties are available, the work W done by the system and the heat Q added can be calculated by Eqs. 23 and 24. The values of v_1, v_2, h_1, and h_2 needed in these equations are taken directly from the tables.

The heat Q added to such a system can also be calculated by Eq. 24*a* or 24*b* if c_p for the system is known. This is permissible since these equations also follow directly from the definition of c_p (see Art. 25) and are therefore applicable to any system. It is usually simpler, however, to calculate Q for a liquid-vapor system by Eq. 24.

ILLUSTRATIVE PROBLEMS

125. One pound of air initially at 70°F is confined inside a cylinder equipped with a frictionless piston. If its pressure is kept constant, how much heat must be added to the air in order to double its volume? How much work is done by the air during the process?

Solution. From the relation

$$\frac{p_1v_1}{T_1} = \frac{p_2v_2}{T_2},$$

it follows that, if $p_1 = p_2$,

$$T_2 = T_1\left(\frac{v_2}{v_1}\right).$$

Therefore, the final temperature of the air is

$$T_2 = 530 \times 2$$
$$= 1060°\text{F abs.}$$

If it is assumed that c_p for air remains constant, the heat Q which must be added to the air is, by Eq. 24*b*,

$$Q = c_p(T_2 - T_1) \tag{24b}$$
$$= 0.241(1060 - 530)$$
$$= 128 \text{ Btu.}$$

The work W done by the air during the process is, by Eq. 23*a*,

$$W = R(T_2 - T_1) \tag{23a}$$
$$= 53.3(1060 - 530)$$
$$= 28{,}200 \text{ ft-lb.}$$

126. One pound of steam at a pressure of 50 psia and having a volume of 5 cu ft is confined inside a cylinder equipped with a frictionless piston. If 500 Btu of heat are added to the steam while its pressure is kept constant, how much work does the steam do?

Solution. The work done by the steam can be calculated by Eq. 23 after its final volume has been determined. To determine this volume, the final enthalpy of the steam must first be calculated by Eq. 24.

In its initial state the steam is a mixture (since its specific volume is between v_f and v_g at 50 psia). Its quality can be determined by Eq. 5*a*:

$$v = xv_g, \tag{5a}$$

or

$$5 = x_1 8.515,$$

whence

$$x_1 = 0.587.$$

Its initial enthalpy can now be calculated by Eq. 20*b*:

$$h = h_g - (1 - x)h_{fg}, \tag{20b}$$

or

$$h_1 = 1174.1 - 0.413 \times 924.0$$
$$= 792.6 \text{ Btu per lb.}$$

The final enthalpy can now be calculated by Eq. 24:

$$Q = h_2 - h_1, \tag{24}$$

or

$$500 = h_2 - 792.6,$$

whence

$$h_2 = 1292.6 \text{ Btu per lb.}$$

Since this value is larger than h_g at 50 psia, the steam in its final state is superheated. Interpolating in Table 3, we find that the final specific volume (corresponding to an enthalpy of 1292.6 Btu per lb at a pressure of 50 psia) is

$$v_2 = 11.530 \text{ cu ft per lb.}$$

Therefore, the work done by the steam is, by Eq. 23,

$$
\begin{aligned}
W &= p(v_2 - v_1) \qquad (23)\\
&= 50 \times 144(11.530 - 5.000)\\
&= 47{,}000 \text{ ft-lb.}
\end{aligned}
$$

PROBLEMS

127. Five pounds of air initially at 60°F undergo a non-flow process during which the pressure remains constant at 200 psia. Calculate the heat added to the air and the work done by it if in its final state the air occupies a volume of 7 cu ft.

Ans. 284 Btu; 62,900 ft-lb.

128. Two pounds of a certain gas initially at 50°F undergo a non-flow process during which the pressure remains constant at 30 psia. In its final state the gas is at 350°F and has a volume of 36 cu ft. If the gas absorbs 320 Btu of heat during the process, by how much does its internal energy change? *Ans.* 123 Btu per lb.

129. A pound of air undergoes a constant pressure, non-flow process during which its volume increases from 4 to 13 cu ft. If the air does 70 Btu of work during the process, how much heat is added to it?

130. A pound of steam undergoes a constant pressure non-flow process during which its volume increases from 4 to 13 cu ft. If the steam does 70 Btu of work during the process, how much heat is added to it?

131. Four pounds of steam initially at 60 psia and 500°F undergo the following non-flow processes: The steam is first cooled at constant volume until it becomes saturated, and it is then heated at constant pressure back to 500°F. Calculate the net amount of heat added to the steam. *Ans.* 110.4 Btu.

132. A 20-cu-ft tank contains steam initially at 15 psia and a quality of 35%. A steam line, in which the pressure remains constant at 30 psia, is connected to the tank, but a check valve prevents steam from flowing from the line into the tank. If heat is supplied *only to the mixture within the tank*, and if only saturated steam leaves the tank, how much heat is required to evaporate all the water?

45. The Isothermal Non-Flow Process. Suppose that a system undergoes a non-flow process during which (*a*) its temperature remains constant, (*b*) it exerts the same pressure on both its moving and stationary boundaries, and (*c*) it is not stirred.* The work done by the system during such a process can be determined by Eq. 7:

$$W = \int p\,dv. \qquad (7)$$

Before the integration can be performed, the pressure p must be expressed as a function of the specific volume v. This function can be obtained from the p-v-T equation of state for the system (see Art. 9) since, with the temperature remaining constant, the equation of state reduces to $p = f(v)$. If the equation is simple (as it is for perfect gases), this procedure can be carried out algebraically; if it is complex (as it is for liquids and vapors) but if a table of thermodynamic properties is available, the integration can be performed graphically.

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* Since both the temperature and pressure of the system are assumed to remain uniform, and since the system is not stirred, the process must be either reversible or internally reversible.

After the work W done by the system has been determined by either of the foregoing methods, the heat Q added to the system can be calculated by the non-flow energy equation,

$$Q = u_2 - u_1 + \frac{W}{J}. \tag{12}$$

Perfect Gases. The p-v-T equation of state for perfect gases is $pv = RT$. Hence, during an isothermal process, $p = RT/v$, where R and T are both constants. Introducing this expression for p into Eq. 7, we get

$$\begin{aligned} W &= RT\int\frac{dv}{v} \\ &= RT\ln\left(\frac{v_2}{v_1}\right). \end{aligned} \tag{25a}$$

We may write alternative forms of this equation by noting that $p_1v_1 = p_2v_2 = RT$. Thus,

$$\begin{aligned} W &= RT\ln\left(\frac{v_2}{v_1}\right) = RT\ln\left(\frac{p_1}{p_2}\right) \\ &= p_1v_1\ln\left(\frac{v_2}{v_1}\right) = p_1v_1\ln\left(\frac{p_1}{p_2}\right) \\ &= p_2v_2\ln\left(\frac{v_2}{v_1}\right) = p_2v_2\ln\left(\frac{p_1}{p_2}\right). \end{aligned} \tag{25b}$$

The heat Q added to and the work W/J done by the gas during an isothermal non-flow process are equal since, by Joule's Law (Eq. 16), $u_1 = u_2$ when $t_1 = t_2$; that is, for a perfect gas undergoing an isothermal non-flow process,

$$Q = \frac{W}{J}. \tag{26}$$

Liquids, Vapors, and Liquid-Vapor Mixtures. The p-v-T equation of state for most liquids, vapors, and liquid-vapor mixtures is too complex to permit the pressure p to be expressed readily in terms of the specific volume v. Consequently, the integration of Eq. 7 cannot be easily carried out mathematically. If a table of properties is available, however, the integration can be performed graphically by plotting the state change on a pressure-volume diagram and measuring the area under the curve obtained. This method is illustrated in Illustrative Problem 134, which follows. After the work W has been determined, the heat Q added to the system during the process can be calculated by the non-flow energy equation. The required values of u_1 and u_2 are obtained from the table of properties.

The foregoing method of calculating the work done by and the heat added to a system during an isothermal, reversible or internally reversible non-flow process is cumbersome because of the graphical calculation required. Another method, which requires no graphical work and is consequently much simpler, will be explained in Art. 74 after the property called "entropy" has been introduced.

ILLUSTRATIVE PROBLEMS

133. How much work is required to compress 3 lb of air from 14.7 to 100 psia by a reversible, non-flow process during which the temperature of the air is kept constant at 70°F?

Solution. The work *done by* each pound of the air can be calculated by the equation

$$W = RT \ln \left(\frac{p_1}{p_2}\right) \qquad (25b)$$

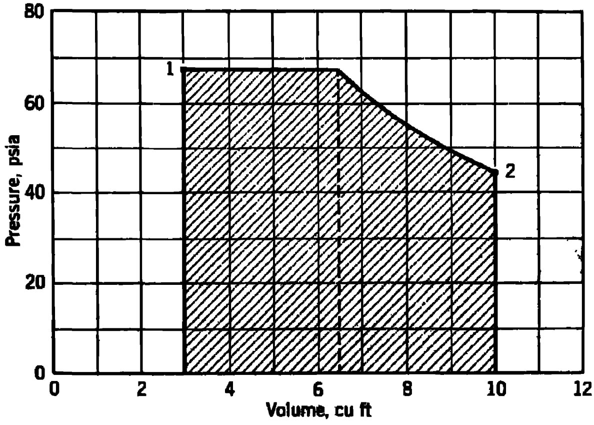
Thus,

$$W = 53.3 \times 530 \ln \left(\frac{14.7}{100}\right)$$

$$= -54{,}000 \text{ ft-lb per lb.}$$

Therefore, the total work which must be *done on* the air is 3 × 54,000 = 162,000 ft-lb. Note that this is also the amount of heat which must be removed from the air during the process in order to keep its temperature constant.

134. One pound of steam having an initial volume of 3 cu ft is expanded at a constant temperature of 300°F to a volume of 10 cu ft. If the process is reversible and non-flow, how much heat must be supplied to the system?



Solution. In its initial state the system is a mixture of saturated water and saturated steam since its volume is between v_f and v_g at 300°F. Its initial pressure, therefore, is 67.013 psia, the saturation pressure corresponding to 300°F. Since the temperature remains constant while the water initially present is evaporated, the pressure also remains constant (at 67.013 psia) during this part of the process. After the water is all evaporated, the volume of the system is 6.466 cu ft (v_g at 300°F). As the expansion continues, the steam becomes superheated. The specific volumes listed in the following table are taken directly from Table 3 of the *Steam Tables* and are used in plotting the accompanying p-v diagram:

t, °F	p, psia	v, cu ft per lb
300	60	7.259
300	55	7.947
300	50	8.773
300	45	9.781
300	44	10.010

The work done by the steam during the expansion is represented by the shaded area shown on the diagram and may be evaluated as follows: During the constant pressure part of the expansion the work done is

$$67.013 \times 144(6.466 - 3.000) = 33{,}400 \text{ ft-lb.}$$

The average pressure during the remainder of the expansion is approximately 54 psia; so the work done during this part of the process is

$$54 \times 144(10.000 - 6.466) = 27{,}500 \text{ ft-lb.}$$

Hence, the total work done by the steam during the entire expansion is 33,400 + 27,500 = 60,900 ft-lb.

Next, the initial and final internal energies of the steam must be determined. Before the initial internal energy can be determined, the initial quality of the steam must be calculated. By Eq. 5a,

$$v = xv_g, \tag{5a}$$

or

$$3 = x_1 6.466,$$

whence

$$x_1 = 0.464.$$

The initial internal energy can now be calculated by Eq. 21a:

$$u = u_f + xu_{fg}, \tag{21a}$$

or

$$u_1 = 269.36 + 0.464 \times 830.2$$

$$= 654.6 \text{ Btu per lb.}$$

The enthalpy and the pressure of the steam in its final state, obtained from Table 3, are approximately 1185.8 Btu per lb and 44.0 psia respectively. Therefore, the internal energy in the final state is

$$u_2 = 1185.8 - \frac{44.0 \times 144 \times 10}{778}$$

$$= 1104.3 \text{ Btu per lb.}$$

Finally, the amount of heat which must be supplied to the system during the process is, by the non-flow energy equation,

$$\begin{aligned} Q &= u_2 - u_1 + \frac{W}{J} \\ &= 1104.3 - 654.6 + \frac{60{,}900}{778} \\ &= 528 \text{ Btu.} \end{aligned}$$

PROBLEMS

135. Three pounds of air initially at 14.7 psia undergo a non-flow process during which the temperature remains constant at 250°F. If in its final state the air has a volume of 13 cu ft, how much heat is added to it during the process? *Ans.* −208 Btu.

136. A pound of air at 25 psia occupies a volume of 8 cu ft. If the air undergoes a constant temperature non-flow process during which 40,000 ft-lb of work are done on it, what will be its final pressure?

137. A quantity of air having an initial volume of 5 cu ft undergoes an isothermal non-flow process during which it absorbs 180 Btu of heat. If in its final state the air is at 20 psia, what is its final volume?

138. A system consisting initially of 1 lb of saturated water at 25 psia undergoes a constant temperature non-flow process. If the final pressure of the system is 15 psia, how much heat is added to it during the process? *Ans.* 995 Btu.

✓**46. The Adiabatic, Reversible Non-Flow Process.** Suppose that any system undergoes an adiabatic non-flow process. By definition of an adiabatic process,

$$Q = 0.$$

It follows from the non-flow energy equation, therefore, that

$$\frac{W}{J} = u_1 - u_2. \qquad (27)$$

Note that this equation is applicable for an adiabatic non-flow process regardless of whether the system exerts the same pressure on both its moving and stationary boundaries and regardless of whether the system is stirred.

Perfect Gases. The change in the internal energy of a perfect gas is, by Eq. 17,

$$u_2 - u_1 = \int_1^2 c_v\, dt.$$

Therefore, if a perfect gas undergoes an adiabatic non-flow process,

$$\frac{W}{J} = -\int_1^2 c_v\, dt, \qquad (27a)$$

or, if c_v remains constant,

$$\frac{W}{J} = c_v(t_1 - t_2). \qquad (27b)$$

The work done by a perfect gas during an adiabatic non-flow process can be calculated by the foregoing equations if the initial and final temperatures of ~~the~~ gas are known. It can also be calculated if the initial and final pressures and specific volumes are known, for then the corresponding temperatures can be determined from the relation $pv = RT$. Frequently, however, two properties of the gas in its initial state are known but only one property in its final state is known. If this is the case, but, if the process, in addition to being adiabatic, is either *reversible* or *internally reversible* and *if* c_v *and* c_p *for the gas are constants*, the other properties of the gas in its final state can be determined from relations derived as follows:

Consider an infinitesimal part of any non-flow process. The non-flow energy equation for this part may be written

$$dQ = du + \frac{dW}{J}.$$

For an *adiabatic* process, $dQ = 0$; for a *perfect gas*, by Eq. 17, $du = c_v\,dT$; and, for either a *reversible* or an *internally reversible* process, by Eq. 7, $dW = p\,dv$. Introducing these three expressions into the foregoing equation, we obtain

$$0 = c_v\,dT + \frac{p\,dv}{J}.$$

But, for a perfect gas, $p = RT/v$. Hence,

$$0 = c_v\,dT + \frac{RT}{J} \times \frac{dv}{v},$$

or

$$0 = c_v\frac{dT}{T} + \frac{R}{J} \times \frac{dv}{v}.$$

Also for a perfect gas, by Eq. 19, $c_p - c_v = R/J$. Hence,

$$0 = c_v\frac{dT}{T} + (c_p - c_v)\frac{dv}{v}.$$

It is convenient at this point to introduce a new symbol to represent the ratio of c_p to c_v. It is customary to use the symbol k for this purpose; that is, by definition, Introducing this symbol into the preceding equation, we obtain

$$0 = \frac{dT}{T} + (k - 1)\frac{dv}{v}.$$

If c_v and c_p for the gas are constants, k is also a constant, and the integration of this equation yields

$$\ln T + (k - 1) \ln v = C_1,$$

or

$$\ln (Tv^{k-1}) = C_1,$$

or

$$Tv^{k-1} = C_2,$$

where C_1 and C_2 are constants. It follows from the last equation that

$$T_1v_1^{k-1} = T_2v_2^{k-1}. \tag{28a}$$

Relations between the initial and final pressures and temperatures and between the initial and final pressures and specific volumes can now be readily derived by combining Eq. 28*a* with the relation

$$\frac{p_1v_1}{T_1} = \frac{p_2v_2}{T_2}.$$

The following equations are obtained:

$$\frac{T_1}{p_1^{(k-1)/k}} = \frac{T_2}{p_2^{(k-1)/k}}, \tag{28b}$$

and

$$p_1v_1^k = p_2v_2^k. \tag{28c}$$

To summarize: if two properties at the initial state and one property at the final state are known, and if c_v and c_p for the gas are constants, the final temperature of a perfect gas undergoing an adiabatic, reversible or internally reversible process can be calculated by Eq. 28*a* or 28*b*. After its final temperature has been determined, the work done by the gas during the process can be calculated by Eq. 27*b*.

An alternative method of determining the work done by a perfect gas during an adiabatic, reversible or internally reversible non-flow process is to make use of the fact that, in accordance with Eq. 28*c*, for such a process

$$pv^k = C,$$

where C is a constant. Solving for the pressure p, we get

$$p = \frac{C}{v^k},$$

and, introducing this expression into Eq. 7, we get

$$W = C\int \frac{dv}{v^k}.$$

Integrating this equation and making use of the fact that $C = p_1v_1^k$ $p_2v_2^k$, we obtain

$$W = \frac{p_1v_1 - p_2v_2}{k - 1}. \qquad (27c)$$

That this equation is equivalent to Eq. 27*b* can be readily shown by making use of the relation $c_p - c_v = R/J$.

Liquids, Vapors, and Liquid-Vapor Mixtures. If the system is a liquid, a vapor, or a liquid-vapor mixture, the work the system does during an adiabatic non-flow process can be calculated by Eq. 27 if two of its properties at the initial state and two at the final state are known. With these data, the initial and final internal energies can be determined from a table of thermodynamic properties, as explained in the preceding chapter. If two of the properties of the system at its initial state but only one at its final state is known, the work done can still be calculated by Eq. 27 if the process, in addition to being adiabatic, is either *reversible* or *internally reversible*. The value of u_1 needed in Eq. 27 can be determined directly from the tables since two properties at this state are known. The value of u_2 can be found by the following method:

Considering again an infinitesimal part of any non-flow process, we may write the non-flow energy equation in the form

$$dQ = du + \frac{dW}{J}.$$

For an adiabatic process, $dQ = 0$; and for a reversible or internally reversible process, $dW = p\,dv$. Introducing these expressions into the foregoing equation, we obtain

$$0 = du + \frac{p\,dv}{J},$$

or

$$\frac{du}{dv} = -\frac{p}{J}. \qquad (29)$$

By means of the last relation, the series of states through which the system passes can be traced on a *u-v* diagram. The procedure is as follows:

(*a*) The values of u_1 and v_1 (corresponding to the two given initial properties) are obtained from the table of properties. These values determine point 1 on the diagram, as shown in Fig. 30.

(*b*) A curve drawn through the series of points representing the states through which the system passes must have, according to Eq. 29, the slope $-p/J$ at the point where the pressure of the system is p. Since the slope of this curve at point 1 must be $-p_1/J$, a straight line having this slope is added to the diagram.

(*c*) Next, several constant pressure lines are plotted. In order to plot these, values of the internal energy u corresponding to several values of the specific volume v are determined for each pressure by means of the table of properties.

(*d*) Short parallel lines intersecting these constant pressure lines and having the slopes $-p/J$ are now added to the diagram, as shown in Fig. 30.

(*e*) Finally, the series of states through which the system passes is determined by drawing a curve through point 1 in such a way that, as it intersects each of the constant pressure lines, its slope is the same as that of the short parallel lines. The final state of the system can be located approximately in this way, and the desired value of u_2 can be read directly from the diagram.

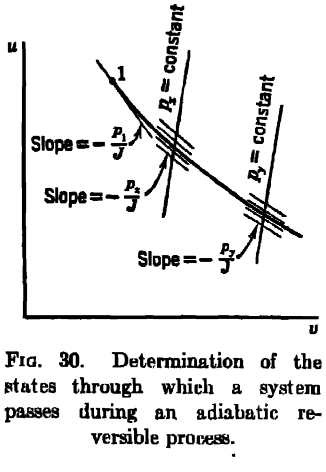


FIG. 30. Determination of the states through which a system passes during an adiabatic reversible process.

The foregoing method of calculating the work done during an adiabatic, reversible non-flow process is illustrated in Illustrative Problem 140, which follows. As in the preceding article, the method is cumbersome because it requires a graphical calculation. A much simpler method of solving this type of problem will be explained in Art. 75 after the property "entropy" has been introduced.

ILLUSTRATIVE PROBLEMS

139. How much work is required to compress 3 lb of air having an initial volume of 40 cu ft from 14.7 to 100 psia by an adiabatic, reversible non-flow process?

Solution. If it is assumed that c_v and c_p for air are constants, the work required can be calculated by Eq. 27*b* after the initial and final temperatures have been determined. The initial temperature can be found from the relation $pv = RT$. Thus,

$$T_1 = \frac{14.7 \times 144 \times 40}{53.3 \times 3}$$

$$= 530°\text{F abs.}$$

The final temperature can now be calculated by Eq. 28*b*:

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k} \qquad (28b)$$

$$= 530\left(\frac{100}{14.7}\right)^{0.4/1.4}$$

$$= 917°\text{F abs.}$$

Therefore, the work *done by* each pound of air is, by Eq. 27*b*,

$$W = 0.172(530 - 917)$$

$$= -66.5 \text{ Btu per lb,}$$

or the total work *done on* the 3 lb of air is 3 × 66.5 = +199.5 Btu.

Alternative Solution. The work required can also be calculated by Eq. 27*c* after the final volume of the air has been determined by Eq. 28*c*. Thus,

$$v_2 = v_1\left(\frac{p_1}{p_2}\right)^{1/k} \qquad (28c)$$

$$= 40\left(\frac{14.7}{100}\right)^{1/1.4}$$

$$= 10.2 \text{ cu ft.}$$

Hence, by Eq. 27*c*,

$$W = \frac{14.7 \times 144 \times 40 - 100 \times 144 \times 10.2}{0.4}$$

$$= -155{,}000 \text{ ft-lb,}$$

or the work *done on* the air is +155,000 ft-lb (=199.5 Btu). Note that the volumes in Eqs. 27*c* and 28*c* may be either specific volumes or total volumes.

140. One pound of steam initially at 100 psia and 400°F undergoes an adiabatic, reversible expansion to 25 psia. If the process is non-flow, how much work does the steam do?

Solution. The work done can be calculated by Eq. 27 after the initial and final internal energies of the steam have been determined. The initial internal energy can be determined from the values of v_1 and h_1 taken from Table 3 of the *Steam Tables*. Thus,

$$u_1 = 1227.6 - \frac{100 \times 144 \times 4.937}{778}$$

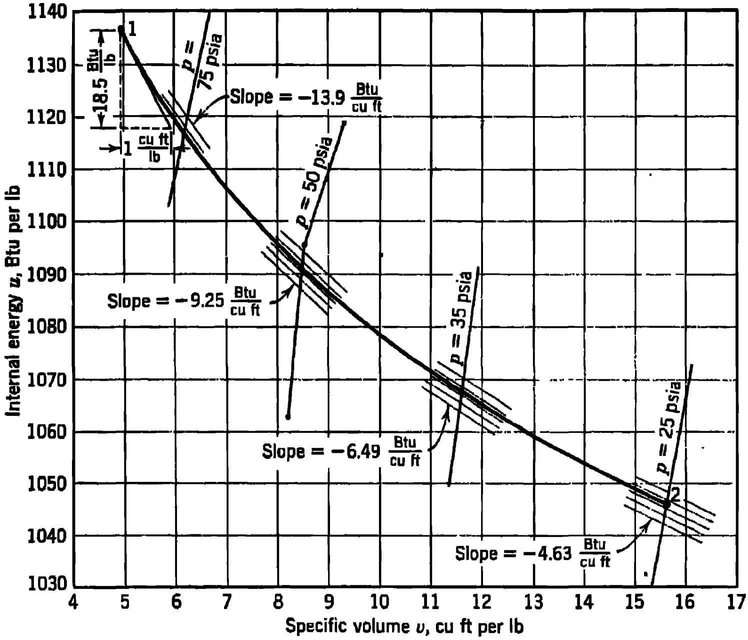
$$= 1136.2 \text{ Btu per lb.}$$

The final internal energy can be determined by constructing a *u-v* diagram, as explained in the preceding article. The steps are as follows:

(*a*) At the initial state, $v_1 = 4.937$ cu ft per lb and $u_1 = 1136.2$ Btu per lb. These values are used to locate point 1 on the *u-v* diagram on the next page.

(*b*) The curve through the series of points representing the states through which the steam passes will have the slope $-p_1/J = -100 \times 144/778 = -18.5$ Btu per cu ft at point 1; so a straight line having this slope is constructed through this point by the method shown on the diagram.

(*c*) Next, constant pressure lines for 75, 50, 35 (all arbitrarily selected), and 25 psia are drawn on the diagram. The values required to plot the 50-psia line, for example,



are determined as follows: At 50 psia, $v_g = 8.515$ cu ft per lb and $u_g = 1095.3$ Btu per lb. These two values determine one point on the line. Another point can be determined by finding the specific volume and internal energy corresponding to 50 psia and, say, $x = 0.96$. Thus, by Eq. 5*a*,

$$v = xv_g \tag{5a}$$

$$= 0.96 \times 8.515$$

$$= 8.175 \text{ cu ft per lb,}$$

and, by Eq. 21*b*,

$$u = u_g - (1 - x)u_{fg} \tag{21b}$$

$$= 1095.3 - 0.04 \times 845.4$$

$$= 1061.5 \text{ Btu per lb.}$$

A third point can be determined by finding the specific volume and internal energy corresponding to 50 psia and, say, 340°F. Thus, from Table 3,

$$v = 9.299 \text{ cu ft per lb,}$$

and

$$u = 1205.0 - \frac{50 \times 144 \times 9.299}{778}$$

$$= 1119.0 \text{ Btu per lb.}$$

The 50-psia line is drawn by connecting these three points with straight lines. (Constant pressure lines on a u-v diagram are straight in the mixture region and are approximately straight over small ranges in the superheat region.)

(*d*) Short parallel lines intersecting these constant pressure lines and having slopes equal to $-p/J$ are now added to the diagram. The slope of the lines intersecting the 50-psia line, for example, is $-50 \times 144/778 = -9.25$ Btu per cu ft.

(*e*) Finally, starting at point 1, a curve is drawn in such a way that, as it intersects each of the constant pressure lines, its slope is the same as that of the short parallel lines. From the diagram, at the intersection of this curve with the $p_2 = 25$-psia line, $u_2 = 1046$ Btu per lb. Therefore, the work done by the steam during the expansion is, by Eq. 27,

$$\frac{W}{J} = u_1 - u_2 \tag{27}$$

$$= 1136.2 - 1046$$

$$= 90 \text{ Btu per lb.}$$

PROBLEMS

The value of k for air may be taken as 1.40.

141. Three pounds of air initially at 14.7 psia and 70°F are compressed adiabatically and reversibly to a volume of 10 cu ft. If the process is non-flow, how much work is done on the air? *Ans.* 202 Btu.

142. A cylinder equipped with a piston contains 10 cu ft of a certain gas at a pressure of 15 psia. If the gas were kept at constant pressure, and if 235 Btu of heat were added to it, the gas would do 47 Btu of work. If, instead, the gas were compressed adiabatically and reversibly to 50 psia, how much work would be required?

143. A pound of air undergoes a non-flow process during which its temperature remains constant at 120°F. If at the end of the process the air were compressed adiabatically and reversibly back to its initial pressure, its temperature would rise to 200°F. How much heat is added to the air during the isothermal process? *Ans.* 18.1 Btu.

144. A pound of steam initially at 200 psia and 500°F undergoes an adiabatic reversible non-flow process. Determine how much work the steam does if its final pressure is (*a*) 100 psia, and (*b*) 50 psia.

145. In the first part of the preceding problem, the steam remains in the superheated state. Recalculate the work done by the steam, *assuming that superheated steam behaves as a perfect gas*. (The value of c_p for superheated steam at 200 psia and 500°F is 0.550 Btu per lb deg F, and the value of R is approximately $1545/18 = 85.8$ ft per deg F. Determine c_v and k from these values.) *Ans.* 55.0 Btu per lb.

✓**47. The Polytropic Non-Flow Process.** Let us consider for a moment the operation of a reciprocating air compressor. With the intake valve open, air is first drawn into the compressor cylinder at approximately atmospheric pressure. Then, with both the intake and discharge valves closed, the volume of the air is decreased, and its pressure is thereby caused to rise. Finally, with the discharge valve open, the high pressure air is forced out of the compressor. During the time that both valves are closed, the air undergoes a non-flow process, the nature of which is affected by two factors: First, since the compressor operates at relatively high speed, the process takes place in a short period of time. Consequently, there is little opportunity for heat to flow from the air to the cylinder walls, and the process tends to be *adiabatic*. Second, since the cylinder walls of all air compressors are provided either with water jackets or with fins, the temperature of the cylinder walls tends to remain near the temperature of the surrounding atmosphere. For this reason, the air loses some heat to the walls during the compression process, and the process tends to be *isothermal*. If these two limiting processes were either reversible or internally reversible, they would appear on a pressure-volume diagram as shown in Fig. 31. As is to be expected, the actual compression curve usually falls somewhere between them. The relation between the pressure p and the specific volume v during an adiabatic reversible or internally reversible compression of air is pv^k = constant (Eq. 28c), and during an isothermal compression it is pv = constant. Hence, it is logical to expect that the relation between these properties during the actual compression process might be approximately pv^n = constant, where n is a constant having a value between unity and k. Experiments show that this is actually true.

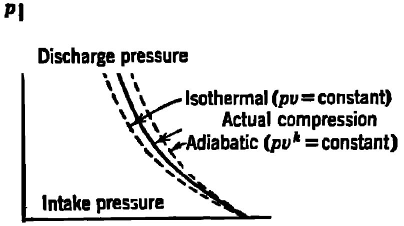


FIG. 31.

It has been found that the relation between the pressure p and the specific volume v during many other actual processes can be represented approximately by the equation pv^n = constant. For example, the relation between p and v during both the compression and expansion strokes of an internal combustion engine and during the expansion stroke of a reciprocating steam engine can all be represented approximately by an equation of this form. Therefore, non-flow processes which take place in this way are of considerable practical interest and have been given the name "polytropic" processes. Thus, a **polytropic process** is defined as one for which the relation between the pressure p and the specific volume v is

$$pv^n = \text{constant}, \tag{30}$$

where n is a constant. Note that n need not fall between unity and k but may have any value.

If the system exerts the same pressure on both its moving and stationary boundaries and is not stirred, the work it does during a polytropic non-flow process can be determined as follows: Since, by definition, $p = C/v^n$ for such a process, by Eq. 7,

$$\begin{aligned} W &= \int p\,dv \\ &= C\int \frac{dv}{v^n} \end{aligned}$$

Integrating this equation and making use of the fact that $C = p_1v_1{}^n = p_2v_2{}^n$, we obtain

$$W = \frac{p_1v_1 - p_2v_2}{n - 1}. \tag{31}$$

After the work done by the system has been determined by this equation, the heat added can be calculated by the non-flow energy equation,

$$Q = u_2 - u_1 + \frac{W}{J}, \tag{12}$$

the value of $u_2 - u_1$ being determined by the methods explained in the preceding chapter.

Perfect Gases. If the system is a perfect gas, the work done can be calculated either by Eq. 31 or by the equation

$$W = \frac{R(t_1 - t_2)}{n - 1}. \tag{31a}$$

Equation 31*a* follows from Eq. 31 since, for a perfect gas, $pv = RT$.

If c_v for the gas remains constant, an equation for the heat added can be derived by making use of the fact that for such a gas, by Eq. 17*a*, $u_2 - u_1 = c_v(t_2 - t_1)$. Substituting in the non-flow energy equation gives

$$Q = c_v(t_2 - t_1) + \frac{R(t_1 - t_2)}{J(n - 1)},$$

and making use of the relation $c_p - c_v = R/J$ (Eq. 19) permits this equation to be reduced to the form

$$Q = \frac{c_p - nc_v}{n - 1}(t_1 - t_2). \tag{32}$$

Relations between the initial and final temperatures and specific volumes and between the initial and final temperatures and pressures of a perfect gas undergoing a polytropic process can be easily derived by combining the equation

$$p_1v_1^n = p_2v_2^n \tag{30}$$

with the equation

$$\frac{p_1v_1}{T_1} = \frac{p_2v_2}{T_2}.$$

The following relations are obtained:

$$T_1v_1^{n-1} = T_2v_2^{n-1}, \tag{30a}$$

and

$$\frac{T_1}{p_1^{(n-1)/n}} = \frac{T_2}{p_2^{(n-1)/n}}. \tag{30b}$$

Note that these relations are identical with Eqs. 28*a*, 28*b*, and 28*c* for a perfect gas undergoing an adiabatic, reversible or internally reversible process, except that k is replaced by n.

If two properties at the initial state and two at the final state are known, and if the process connecting these states is a polytropic process, the value of n can be found by Eq. 30, 30*a*, or 30*b*. For example, if the initial and final pressures and specific volumes are known, the value of n can be determined as follows: Taking the logarithm of both sides of Eq. 30 gives

$$\log p_1 + n \log v_1 = \log p_2 + n \log v_2,$$

whence

$$n = \frac{\log p_2 - \log p_1}{\log v_1 - \log v_2},$$

or

$$n = \frac{\log (p_2/p_1)}{\log (v_1/v_2)}. \tag{33}$$

Similar equations for determining n can be derived from Eqs. 30*a* and 30*b*.

It is interesting to note that, by properly selecting the value of n, the polytropic non-flow process for perfect gases may be reduced to each of the types of processes already discussed. Thus, if $n = 0$, the polytropic process reduces to the constant pressure process; if $n = \infty$, it reduces to the constant volume process; if $n = 1$, it reduces to the isothermal process; and, if $n = k$, it reduces to the adiabatic, reversible process.

Liquids, Vapors, and Liquid-Vapor Mixtures If the system is a liquid, a vapor, or a liquid-vapor mixture, the work done by the system can be calculated by Eq. 31. The heat added to it can then be calculated by the non-flow energy equation, the required values of u_1 and u_2 being obtained from a table of thermodynamic properties. If the initial and final pressures and specific volumes are known, the value of n for the process can be calculated by Eq. 33. Note that Eqs. 30*a*, 30*b*, 31*a*, and 32 are not applicable to any system other than a perfect gas.

ILLUSTRATIVE PROBLEMS

146. One pound of air initially at 14.7 psia and 70°F is compressed to a pressure of 100 psia by a polytropic, reversible non-flow process. If $n = 1.3$ for the process, calculate the work required to compress the air and the heat removed from the air during the process.

Solution. The work *done by* the air can be calculated by Eq. 31*a* after its final temperature has been determined by Eq. 30*b*. Thus,

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(n-1)/n} \tag{30b}$$

$$= 530\left(\frac{100}{14.7}\right)^{0.3/1.3}$$

$$= 825°\text{F abs } (t_2 = 365°\text{F}).$$

Therefore, by Eq. 31*a*,

$$W = \frac{R(t_1 - t_2)}{n - 1} \tag{31a}$$

$$= \frac{53.3(70 - 365)}{0.3}$$

$$= -52{,}400 \text{ ft-lb per lb},$$

or the work *done on* the air is +52,400 ft-lb.

The heat added to the air can now be calculated by the non-flow energy equation. Assuming that c_v for air is a constant, and making use of the relation $u_2 - u_1 = c_v(t_2 - t_1)$, we get

$$Q = u_2 - u_1 + \frac{W}{J}$$

$$= 0.172(365 - 70) - \frac{52{,}400}{778}$$

$$= -16.7 \text{ Btu per lb}.$$

Therefore, the heat which must be *removed from* the air during the process is +16.7 Btu.

The value of Q can also be calculated by Eq. 32. Thus,

$$Q = \frac{c_p - nc_v}{n - 1}(t_1 - t_2) \tag{32}$$

$$= \frac{0.241 - 1.3 \times 0.172}{0.3}(70 - 365)$$

$$= -16.7 \text{ Btu per lb.}$$

147. Steam initially saturated and at 100 psia undergoes a non-flow process during which its pressure drops to 20 psia and its quality becomes 95%. Assuming that the process is polytropic and internally reversible, calculate the amount of heat added to or lost by each pound of the steam during the process.

Solution. The heat added to the steam can be calculated by the non-flow energy equation after the work done has been determined by Eq. 31. The value of n must first be found by Eq. 33. The initial specific volume is 4.432 cu ft per lb (v_g at 100 psia). The final specific volume is, by Eq. 5a,

$$v = xv_g \tag{5a}$$

or

$$v_2 = 0.95 \times 20.089$$

$$= 19.085 \text{ cu ft per lb.}$$

Therefore, by Eq. 33,

$$n = \frac{\log \left(\frac{20}{100}\right)}{\log (4.432/19.085)}$$

$$= 1.10$$

The work done by the steam can now be calculated by Eq. 31:

$$W = \frac{100 \times 144 \times 4.432 - 20 \times 144 \times 19.085}{0.10}$$

$$= 88{,}500 \text{ ft-lb per lb.}$$

The initial internal energy of the steam is 1105.2 Btu per lb (u_g at 100 psia); and the final internal energy is, by Eq. 21b,

$$u = u_g - (1 - x)u_{fg}, \tag{21b}$$

or

$$u_2 = 1081.9 - 0.05 \times 885.8$$

$$= 1037.6 \text{ Btu per lb.}$$

Finally, the heat added to each pound of steam during the process is, by the non-flow energy equation,

$$Q = u_2 - u_1 + \frac{W}{J}$$

$$= 1037.6 - 1105.2 + \frac{88{,}500}{778}$$

$$= 46.2 \text{ Btu per lb.}$$

PROBLEMS

148. Two pounds of a certain gas initially at 15 psia and occupying a volume of 30 cu ft are compressed to 65 psia by a polytropic non-flow process. If $n = 1.2$ for the process, how much work is done on the gas?

149. A pound of steam initially at 300 psia and 560°F undergoes a polytropic non-flow process. If in its final state the steam is at 100 psia and 360°F, how much heat does it absorb during the process? *Ans.* 19.6 Btu.

150. A pound of air initially at 300°F undergoes a non-flow process which takes place in such a way that the quantity $pv^{1.25}$ remains constant. If the air does 40,000 ft-lb of work during the process, and if its final pressure is 20 psia, by how much does its volume increase?

151. Three pounds of air initially at 20 psia and 70°F undergo a polytropic non-flow process. If at the end of the process the air is at 60 psia and 240°F, how much work has it done, and how much heat has it absorbed?

Ans. −80,000 ft-lb; −15.3 Btu.

152. Suppose that a perfect gas having a specific heat at constant volume which remains constant undergoes a non-flow reversible process for which the instantaneous specific heat is constant. Prove that the process must be a polytropic process.

STEADY-FLOW PROCESSES

48. General Procedure. It was shown in Chapter 3 that, when a fluid flows through any apparatus in a steady-flow manner, the heat Q entering and the shaft-work W' leaving the apparatus, each per pound of the fluid, are related to certain properties of the fluid at the sections where the fluid enters and leaves the apparatus. The relation is stated by the equation

$$Q + h_1 + \frac{z_1}{J} + \frac{V_1^2}{2gJ} = \frac{W'}{J} + h_2 + \frac{z_2}{J} + \frac{V_2^2}{2gJ}. \qquad (13)$$

It was also explained that, for most engineering applications of this equation, the difference between the two z/J terms is quite small. For this reason these terms may usually be dropped, and the equation may be written

$$Q + h_1 + \frac{V_1^2}{2gJ} = \frac{W'}{J} + h_2 + \frac{V_2^2}{2gJ}. \qquad (13a)$$

Let us consider how each term in this equation can be determined.

The heat Q added to each pound of fluid can be easily determined if the rate at which heat enters the apparatus and the mass rate of flow w are known. It is necessary only to divide the first of these two rates by the second. Note that in general Q *cannot* be calculated by the equation $Q = \int c\,dt$ since this equation applies only to non-flow processes. This is true because the equation is based on the definition of the specific heat c, and c is defined as the heat added to each pound of a system per degree rise in its temperature when the system undergoes a *non-flow* process (see Art. 25). It is true that, if a perfect gas undergoes a steady-flow process for which W' is zero and V_1 and V_2 are either equal or negligibly small, Eq. 13*a* reduces to the form

$$Q = h_2 - h_1 = \int c_p\, dt.$$

However, this equation follows, not from the definition of c_p, but from the fact that $h_2 - h_1$ for a perfect gas is equal to the integral of $c_p\, dt$, as was proved in Art. 38. The equation cannot be used to calculate Q for any medium other than a perfect gas, nor can it be used even for a perfect gas if the conditions are other than those stated.

The shaft-work W' leaving the apparatus per pound of fluid can be determined if the power delivered by the apparatus (that is, the *rate* at which work leaves) and the mass rate of flow w are known. W' is calculated by dividing the first of these two rates by the second. Note that the shaft-work W' *cannot* be calculated by the equation $W = \int p\, dv$. This integral is equal to the work done by each pound of a fluid only if the fluid undergoes a *non-flow* process (which must take place in the way specified in Art. 23). To illustrate the fact that the shaft-work W' is not equal to this integral, suppose that a fluid flows at constant pressure through a section of pipe. The integral of $p\, dv$ can be easily evaluated $[= p(v_2 - v_1)]$, but it does not equal the shaft-work W' leaving the pipe since the shaft-work is zero. Or, if the specific volume of the fluid flowing through any apparatus remains constant (as is very nearly true for water flowing through a pump), the integral of $p\, dv$ is zero, but the shaft-work is not necessarily also zero.

The enthalpies h_1 and h_2 can be determined by the methods explained in the preceding chapter if two of the properties of the fluid at the sections where the fluid enters and leaves are known. Special equations were developed in the first part of this chapter for calculating Q and W for non-flow processes in which the specific volume, pressure, or temperature of the system remain constant. It is not necessary to develop corresponding equations for steady-flow processes since the constancy of any one of these properties simply supplies us with one of the two properties that must be known at the entering and leaving sections.

If the velocities V_1 and V_2 are not known, they can be determined by the continuity equation,

$$w = \frac{V_1 a_1}{v_1} = \frac{V_2 a_2}{v_2}. \quad \text{lbs/sec} \qquad (15)$$

The mass rate of flow w, the cross-sectional areas a_1 and a_2 of the passages through which the fluid enters and leaves the apparatus, and the specific volumes v_1 and v_2 at these sections must all be known. Or, if the volume rate of flow at either section, instead of the mass rate, is known, the velocity at that section can be determined by dividing the volume rate of flow by the cross-sectional area of the passage.

If sufficient data are known to permit the determination by the foregoing methods of all but one of the terms Q, W', h_1, h_2, V_1, and V_2, the unknown term can be found by means of Eq. 13*a*. Illustrative Problems 82, 83, 84, 91, 92, and 93 in Chapter 4 are examples of this type of steady-flow problem. Another type involves the adiabatic, reversible compression or expansion of a fluid. Problems of this type differ from the foregoing in that only one property at the leaving section is known, and therefore the enthalpy h_2 cannot be immediately determined. Such problems are of particular importance in engineering and are considered separately in the next article. Still another special type of steady-flow process, known as a "throttling process," is discussed in Art. 50.

49. The Adiabatic, Reversible Steady-Flow Process. The flow of a fluid through an apparatus will be approximately adiabatic if (*a*) the apparatus is covered with insulation and there is a negligible flow of heat from one part of the apparatus to another, or if (*b*) the time required for each element of the fluid to pass through the apparatus is very short. Under the latter condition, the flow is approximately adiabatic because there is insufficient time for much heat to be transferred. The flow of a gas or vapor through an ordinary nozzle, for example, is nearly adiabatic because the time required for each element of the fluid to flow through the nozzle is likely to be less than $\frac{1}{1000}$ of a second. Similarly, the flow of steam through a turbine is nearly adiabatic because the parts of the turbine casing that are above room temperature are covered with insulation and the time required for each element of the steam to pass completely through the turbine is short. Since many kinds of engineering apparatus that operate in a steady-flow manner also operate approximately adiabatically, and since these kinds of apparatus would in the ideal case operate reversibly, adiabatic, reversible steady-flow processes are of particular interest.

Before considering steady-flow processes that are both adiabatic and reversible, let us consider the conditions that must be satisfied for a steady-flow process to be reversible. A reversible process was defined in Art. 20 as one during which (*a*) no part of the system or its surroundings ever departs from a state of equilibrium by more than an infinitesimal amount, and (*b*) no friction of any kind occurs. If the process is a steady-flow process, the system referred to must be taken as any small element of the fluid undergoing the process. From the viewpoint of an observer who moves with this element, any work done by it is done in pushing back the fluid that bounds it; and any heat transferred to the element is transferred either from the fluid which bounds it or from those parts of the apparatus with which it comes into contact. Thus, from the viewpoint of such an observer, the element undergoes a *non-flow* process. In order for a steady-flow process to be reversible, therefore, it is necessary first that, from the viewpoint of an observer moving with the element, each element of the fluid must undergo a reversible non-flow process. In addition, there must be no finite differences in velocity between the element and the fluid that bounds it (that is, there must be no turbulence); there must be no friction between the element and any surface of the apparatus across which it may move; and there must be no friction between any of the moving parts of the apparatus.

Now let us consider steady-flow processes that are both adiabatic and reversible. In problems involving this type of process, two properties of the fluid at the section where it enters the apparatus are usually known, but only one property at the section where it leaves is known. A second property at the leaving section can be determined by making use of the fact that, from the viewpoint of an observer moving with the element, each element of the fluid appears to undergo an *adiabatic, reversible non-flow process.* The state of the fluid leaving the apparatus can therefore be determined by the methods developed in Art. 46 for such processes. Thus, if the fluid is a perfect gas, relations between its properties at the entering and leaving sections are expressed by Eqs. 28*a*, 28*b*, and 28*c*. Or, if the fluid is a liquid, vapor, or liquid-vapor mixture, its state leaving the apparatus can be determined by constructing a *u-v* diagram, as explained in the last part of Art. 46. After a second property of the fluid at the leaving section has been determined, its enthalpy can be found by the usual methods.

The student is again reminded that the graphical method used here to determine the properties of any system at the end of an adiabatic, reversible process will be replaced by a much simpler method after the property "entropy" has been introduced.

ILLUSTRATIVE PROBLEMS

153. Two hundred cubic feet of air per minute at 14.7 psia and 70°F are to be compressed to 60 psia. If the compressor were to operate adiabatically and reversibly, and if the velocities of the air entering and leaving the compressor were negligible, how many horsepower would be required to drive it?

Solution. The shaft-work W' supplied to the compressor per pound of air compressed can be calculated by the steady-flow energy equation after the temperature of the air leaving the compressor has been determined. Since the compression is adiabatic and reversible, this temperature can be calculated by Eq. 28*b*. Thus,

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k} \tag{28b}$$

$$= 530\left(\frac{60}{14.7}\right)^{0.4/1.4}$$

$$= 793°\text{F abs } (= 333°\text{F}).$$

Since the process is adiabatic and the entering and leaving velocities are negligible, the steady-flow energy equation takes the form

$$h_1 = \frac{W'}{J} + h_2.$$

Therefore, since $h_2 - h_1 = c_p(t_2 - t_1)$ for a perfect gas (assuming that c_p is a constant),

$$\frac{W'}{J} = c_p(t_1 - t_2)$$

$$= 0.241(70 - 333)$$

$$= -63.4 \text{ Btu per lb},$$

the negative sign indicating that the shaft-work is supplied *to* the compressor.

The number of pounds of air compressed per minute must now be determined. From the relation $pv = RT$,

$$\begin{matrix} 53.3 \times 530 \\ 14.7 \times 144 \end{matrix}$$

$$= 13.33 \text{ cu ft per lb}.$$

Hence, the mass rate of air flow through the compressor is 200/13.33 = 15.0 lb per min.

Finally, since 2545 Btu per hr equal 1 hp, the power required to drive the compressor would be

$$\frac{15 \times 63.4 \times 60}{2545} = 22.4 \text{ hp}.$$

154. Steam enters a nozzle at 100 psia and 400°F and leaves at 25 psia. If the velocity of the steam entering the nozzle is negligible, and if the flow through the nozzle is adiabatic and reversible, at what velocity will the steam leave?

Solution. The velocity V_2 of the steam leaving the nozzle can be calculated by the steady-flow energy equation after the enthalpies of the steam entering and leaving have been determined. From Table 3 of the *Steam Tables*,

$$h_1 = 1227.6 \text{ Btu per lb}.$$

The enthalpy of the steam leaving the nozzle can be found after its final state has been determined by constructing a *u-v* diagram, as explained in Art. 46. The initial pressure and temperature and the final pressure in this problem are the same as those given in Illustrative Problem 140 on page 129. Hence, from the u-v diagram accompanying that problem, $v_2 = 15.6$ cu ft per lb and $u_2 = 1046$ Btu per lb; so

$$h_2 = 1046 + \frac{25 \times 144 \times 15.6}{778}$$

$$= 1118 \text{ Btu per lb.}$$

Since the process is adiabatic, since the shaft-work is zero, and since the velocity of the steam entering the nozzle is negligible, the steady-flow energy equation takes the form

$$h_1 = h_2 + \frac{V_2^2}{2gJ}.$$

Therefore,

$$1227.6 = 1118 + \frac{V_2^2}{50{,}000}$$

or

$$V_2 = 2340 \text{ ft per sec.}$$

155. Steam enters a small turbine at 100 psia and 400°F and exhausts at 25 psia. If the turbine operates adiabatically and reversibly, how many horsepower will it deliver when the steam flow is 3000 lb per hr?

Solution. Assuming that the velocities of the steam entering and leaving the turbine are negligible, we may write the steady-flow energy equation in the form

$$h_1 = \frac{W'}{J} + h_2.$$

The values of h_1 and h_2 are the same as in the preceding problem. Hence,

$$1227.6 = \frac{W'}{J} + 1118,$$

or

$$\frac{W'}{J} = 109.6 \text{ Btu per lb.}$$

Therefore, since 2545 Btu per hr equal 1 hp, the power delivered by the turbine would be

$$\frac{109.6 \times 3000}{2545} = 129 \text{ hp.}$$

PROBLEMS

156. Air initially at 150 psia and 300°F expands adiabatically and reversibly to 50 psia. Calculate the work done per pound of air (*a*) if the process is non-flow, and (*b*) if it is a steady-flow process with equal initial and final velocities.

Ans. (*a*) 35.2 Btu per lb; (*b*) 49.4 Btu per lb.

157. An air motor is to be supplied with 6 lb per min of air at 40 psia and 70°F. If the air were to expand adiabatically and reversibly as it flows through the motor, and if it were to leave at 15 psia, how many horse-power would the motor deliver? The cross-sectional areas of both the inlet and discharge passages are 0.75 sq in.

158. Steam is supplied to a turbine at 400 psia and 600°F. If the steam were to expand adiabatically and reversibly and were to leave the turbine at 30 psia, how many pounds would be required per kilowatthour delivered at the turbine shaft? Assume that the steam enters and leaves the turbine with negligible velocities. (*Note:* By constructing a *u-v* diagram, as explained in Art. 46, it is found that the specific volume and internal energy of the steam leaving the turbine would be 12.6 cu ft per lb and 1016 Btu per lb respectively.) *Ans.* 15.4 lb per kwhr.

159. Air enters a converging-diverging nozzle (see Fig. 116) at 50 psia and 80°F with a velocity of 100 ft per sec. It flows through the nozzle at the rate of 25 lb per min and leaves at a pressure of 14.7 psia. If the air expanded adiabatically and reversibly, what would the cross-sectional area of the discharge end of the nozzle have to be?

50. The Throttling Process. Another type of steady-flow process of special interest is one known as a "throttling" process. A fluid is said to undergo a **throttling process** if (*a*) it flows in steady-flow through a valve, orifice, porous plug, or other constriction, if (*b*) its velocities entering and leaving the constriction are either equal or negligibly small, and if (*c*) it neither receives nor rejects heat. For example, if a fluid flows through a valve which is only slightly opened, the velocities of the fluid entering and leaving will be low; and, if the valve is insulated, there will be no transfer of heat to or from the fluid. Therefore, the fluid will undergo a throttling process.

Let us consider what the energy relations are for a throttling process. Since the process is steady-flow, the steady-flow energy equation applies:

$$Q + h_1 + \frac{V_1^2}{2gJ} = \frac{W'}{J} + h_2 + \frac{V_2^2}{2gJ}. \qquad (13a)$$

Since the process is adiabatic, $Q = 0$; since the flow is through a valve or other constriction, no shaft-work is involved, or $W' = 0$; and, since the entering and leaving velocities V_1 and V_2 are either equal or negligibly small, the two kinetic energy terms also drop out of the equation. Therefore, for a throttling process, the steady-flow energy equation reduces to

$$h_1 = h_2. \qquad (34)$$

One application of Eq. 34 is to the so-called "throttling calorimeter." This device is used to determine the quality of high pressure steam and is arranged as shown in Fig. 32. A sample of the steam whose quality is to be determined is taken from the steam main by means of a sampling tube. This tube, usually closed at the end, is perforated along its length to permit a representative sample to be obtained. The sample flows through a small orifice into an insulated chamber, usually open to the atmosphere. If the quality of the steam in the main is not too low, the sample becomes superheated on being throttled to a lower pressure. Therefore, by measuring the temperature and pressure of the steam in the calorimeter, its enthalpy can be determined from Table 3 of the *Steam Tables*. Since, according to Eq. 34, this is also the enthalpy of the steam in the main, it is necessary only to measure the pressure (or the temperature) of the steam in the main in order to have two properties by means of which its quality in the main can be calculated. The procedure is illustrated in Illustrative Problem 160, which follows.

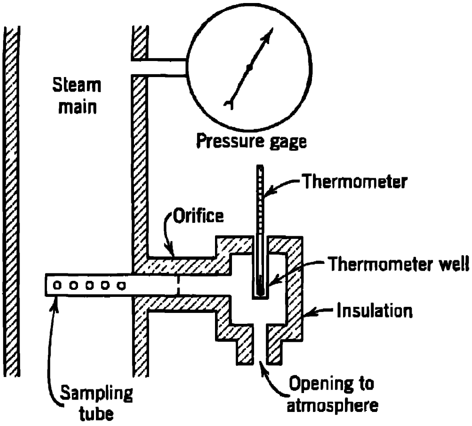


FIG. 32. Throttling calorimeter.

ILLUSTRATIVE PROBLEMS

160. The pressure and temperature in a throttling calorimeter are 14.7 psia and 240°F respectively. If the pressure of the steam in the main is 150 psia, what is its quality?

Solution. From Table 3 of the *Steam Tables*, the enthalpy of the steam in the calorimeter is 1164.2 Btu per lb. By Eq. 34, this is also the enthalpy of the steam in the main. Since this enthalpy is between h_f and h_g at 150 psia, the steam in the main is a mixture. Its quality can be calculated by Eq. 20*b*. Thus,

$$h = h_g - (1 - x)h_{fg}, \tag{20b}$$

or

$$1164.2 = 1194.1 - (1 - x)863.6$$

whence

$$(1 - x) = 0.035.$$

Hence, the quality of the steam in the main is 96.5%.

161. Water at 100 psia and 300°F flows at low velocity through an insulated valve. If the pressure on the downstream side of the valve is 20 psia, how much of the water "flashes" into steam?

Solution. The enthalpy of the water on the upstream side of the valve is approximately 269.59 Btu per lb (h_f at 300°F). By Eq. 34, this is also the enthalpy on the downstream side. Since this value is between h_f and h_g at 20 psia, the fluid leaving the valve is a mixture. Its quality can be calculated by Eq. 20*a*. Thus,

$$h = h_f + xh_{fg}, \tag{20a}$$

or

$$269.59 = 196.16 + x_2 960.1$$

whence

$$x_2 = 0.0765.$$

Hence, 7.65% of the water (by weight) flashes into steam on flowing through the valve.

PROBLEMS

162. The pressure and temperature in a throttling calorimeter are 20 psia and 260°F respectively. What is the quality of the steam in the main if the pressure is 350 psia? *Ans.* 96.0%.

163. The pressure in a throttling calorimeter is 14.7 psia, and the pressure in the steam main to which the calorimeter is attached is 200 psia. The instrument can be used to determine the quality in the main provided that the quality is not too low. Find the minimum quality which can be determined.

164. If water initially at 280 psia and 400°F is throttled to a pressure of 25 in. Hg abs, how much of the water will flash into vapor? *Ans.* 0.209.

165. A steam generator is supplied with feedwater at 2000 psia and 200°F. Steam leaves the generator at 1800 psia and flows through a throttle valve. If the steam leaving the valve is at 400 psia and 600°F, and if it is flowing with a velocity of 300 ft per sec, how much heat is required per pound of steam generated?

GENERAL PROBLEMS

166. Calculate the molar specific heats at constant volume (that is, the amount of heat required to raise the temperature of one mole one degree at constant volume) for each of the diatomic gases listed in Table IV on page 95. On the basis of the results obtained, determine the amount of heat required to raise the temperature of 2 lb of a diatomic gas whose molecular weight is 30 from 70° to 250°F when the volume of the gas is kept constant. *Ans.* 59.8 Btu.

167. Using only the data given in Table 3 of the *Steam Tables*, calculate the value of c_p for superheated steam at 200 psia and 600°F.

168. A pound of superheated steam at 500°F undergoes a constant temperature, non-flow process during which its pressure rises from 100 to 200 psia. (*a*) Determine by a graphical integration the work done on the steam during the process. (*b*) Assuming that superheated steam behaves as a perfect gas, determine by means of Eq. 25*b* the work done on the steam.

169. Four pounds of air initially at 120 psia and 220°F expand adiabatically and reversibly. If the process is non-flow, and if the air does 100 Btu of work, what is the final volume of the air? *Ans.* 15.4 cu ft.

170. Two pounds of air initially at 250°F undergo a polytropic non-flow process. At the end of the process the air is at 100°F, and its volume is double its initial volume. How much work has the air done, and how much heat has it absorbed?

171. Five pounds of steam having an initial volume of 13 cu ft are cooled at constant pressure from 600° to 400°F. If the process is non-flow, how much work does the steam do? *Ans.* −104,000 ft-lb.

172. An air compressor takes in air at 14.2 psia and 50°F and discharges it at 80 psia. If 100 hp are supplied to the machine, and if the air is compressed adiabatically and reversibly, how many cubic feet (measured at inlet conditions) are compressed per minute? Assume that the velocity of the air in the inlet and discharge pipes is negligible.

173. If steam at 300 psia and 500°F is throttled to a pressure of 25 psia, what will be its temperature?

174. A system undergoes an adiabatic reversible process. Show that the states through which it passes can be determined by drawing a curve on an h-p diagram in such a way that (*a*) the curve passes through the point representing the initial state of the system, and (*b*) the curve has at every point a slope equal to v/J.

175. An 8-cu-ft tank contains steam initially at 160 psia and 560°F. How much heat must be absorbed from the steam to cause 1 lb of it to condense? *Ans.* 1040 Btu.

176. Three pounds of steam initially at 400°F and occupying a volume of 12 cu ft undergo a constant pressure, non-flow process. If 200 Btu of work are done on the steam, how much heat is added to it?

177. Two pounds of air at a temperature of 160°F undergo an isothermal, non-flow process. During the process 110 Btu of heat are added to the air, and the pressure of the air decreases by 75 psia. What is the final volume of the air? *Ans.* 16.2 cu ft.

178. A 20-cu-ft tank contains air initially at 100 psia and 80°F. Part of the air is allowed to flow out of the tank through a valve. If the final pressure in the tank is 50 psia, and if no heat flows from the tank to the air, how many pounds of air escape?

179. Three pounds of air initially at 15 psia and 60°F undergo a polytropic non-flow process. If the air rejects 90 Btu of heat during the process, and if its final temperature is 240°F, what is its final pressure? *Ans.* 88.5 psia.

180. Air flows adiabatically and reversibly through a converging-diverging nozzle (see Fig. 116). At the entering section the air is at 100 psia and 70°F, and its velocity is negligible. At the throat the air is at 52.8 psia; halfway between the throat and mouth it is at 25 psia; and at the mouth it is at 15 psia. If the distance between the mouth and throat is 2 in., how long does it take a particle of the air to flow from the throat to the mouth? (*Suggestion:* Integrate graphically.)

181. A system consisting of 1 lb of saturated water and 1 lb of saturated steam is first heated at constant pressure until the water is all evaporated and is then cooled at constant volume until 1 lb of water is again condensed. If both processes are non-flow, and if the final pressure is 45 psia, what is the net amount of heat added to the system? *Ans.* 20.6 Btu.

182. The pressure in a steam main is 20 psia. If the quality in the main is 98%, it cannot be determined by means of a throttling calorimeter unless the pressure in the calorimeter is below a certain maximum permissible value. Determine this maximum pressure.

183. Five pounds of a certain gas initially at 15 psia and 70°F occupy a volume of 118 cu ft. If the gas were heated at constant volume until its pressure rose to 25 psia, 735 Btu of heat would have to be supplied. If, instead, the gas were compressed to 25 psia by an adiabatic, reversible non-flow process, how much work would be required? *Ans.* 137 Btu.

184. A rigid tank contains 1 lb of steam at 150 psia and 540°F. What will be the pressure in the tank after 500 Btu of heat have been absorbed from the steam?

185. Four pounds of a certain gas occupy a volume of 21 cu ft when the gas is at 25 psia and 90°F. If the volume of the gas is kept constant, 82 Btu of heat are required to raise its temperature from 90° to 220°F. How much heat would be required to raise its temperature from 90° to 220°F if the gas were to undergo a constant pressure, non-flow process? *Ans.* 105 Btu.

186. Three pounds of a certain gas are made to undergo the following non-flow process: The gas is first heated at constant pressure from 70° to 350°F and then cooled at constant volume from 350° back to 70°F. During the constant pressure part of the process 170 Btu of heat are supplied to the gas, and during the constant volume part 132 Btu of heat are absorbed from it. If the gas had been made to change from its initial to its final state, not by the foregoing process, but by an isothermal non-flow process, how much heat would have been required?

187. A quantity of air initially at 20 psia and occupying a volume of 12 cu ft is compressed adiabatically and reversibly. If the process is non-flow, and if 45 Btu of work are done on the air, what is the final volume of the air?

188. A quantity of a certain gas undergoes two non-flow processes. During one process the pressure remains constant, and the gas does 21,000 ft-lb of work. During the other process the quantity $pv^{1.35}$ remains constant. If the initial and final temperatures of the gas are the same for both processes, how much work does the gas do during the polytropic process? *Ans.* −60,000 ft-lb.

189. A system consisting initially of 0.6 lb of saturated water and 0.3 lb of saturated steam occupies a volume of 4.40 cu ft. The system is heated first at constant volume until all the water is evaporated and is then heated further at constant pressure. If both processes are non-flow, and if a total of 650 Btu of heat are added to the system, what is the final temperature?

190. Suppose that a perfect gas has a molecular weight of 17 and a specific heat at constant volume which is dependent upon the temperature in accordance with the equation

$$c_v = 0.300 + 0.0001T,$$

where the units of c_v and of T are British thermal units per pound degree Fahrenheit and degrees Fahrenheit absolute respectively. If 1 lb of this gas is initially at 70°F and has an initial volume of 20 cu ft, and if it is compressed to a volume of 10 cu ft by an adiabatic, reversible non-flow process, how much work is required?

191. Suppose that the gas described in the preceding problem enters a compressor at 20 psia and 40°F and leaves at 100 psia. If the gas enters and leaves at a low velocity, and if it is compressed adiabatically and reversibly, how much work is required to compress each pound of it?

192. Six pounds of air initially at 180°F undergo a polytropic non-flow process during which the air does 160 Btu of work. If the air in its final state has a volume double its initial volume, what is its final temperature?

193. A 10-cu-ft tank contains steam initially saturated and at 50 psia pressure. A steam line, in which the pressure remains constant at 50 psia, is connected to the tank. If heat is supplied *only to the steam within the tank,* how much heat is required to raise the temperature within the tank to 600°F? Note that, as the steam within the tank expands at constant pressure, part of it flows out of the tank and into the steam line. (*Suggestion:* Solve by integrating graphically.) *Ans.* 152 Btu.

SYMBOLS

a	cross-sectional area
c	specific heat; c_p = specific heat at constant pressure; c_v = specific heat at constant volume
g	acceleration that a force of 1 lb gives to a mass of 1 lb
h	enthalpy per unit mass; h_f = enthalpy per unit mass of saturated liquid; h_g = enthalpy per unit mass of saturated vapor; $h_{fg} = h_g - h_f$
J	mechanical equivalent of heat
k	ratio of c_p to c_v
n	a constant
p	pressure
Q	heat added to system
R	gas constant
t	temperature
T	absolute temperature
u	internal energy per unit mass; u_f = internal energy per unit mass of saturated liquid; u_g = internal energy per unit mass of saturated vapor; $u_{fg} = u_g - u_f$
v	specific volume; v_f = specific volume of saturated liquid; v_g = specific volume of saturated vapor; $v_{fg} = v_g - v_f$
V	velocity
w	mass rate of flow
W	work done by system; W' = shaft-work done by a system in steady flow
x	quality, weight of saturated vapor per unit weight of mixture
z	elevation

PART II

THE SECOND LAW OF THERMODYNAMICS

Chapter 6

POWER CYCLES AND REFRIGERATION CYCLES

As stated in the preface to Chapter 3, the science of engineering thermodynamics is built upon two basic principles known as the First Law and the Second Law of Thermodynamics. Up to the present, use has been made only of the first of these two laws. It is the purpose of Part II of this text to present the Second Law and to develop certain concepts based upon it. Before considering this law, however, it is necessary to discuss in somewhat greater detail what are known as "cyclic processes" or simply as "cycles."

51. Non-Flow and Steady-Flow Cycles. As explained in Art. 28, if a system undergoes a series of processes such that its final state is exactly the same as its initial state, the system is said to undergo a **cycle.** If each of the processes that the system undergoes in returning to its initial state is a non-flow process, the cycle is called a **non-flow cycle;** if each is a steady-flow process, the cycle is called a **steady-flow cycle.** Let us consider first an example of a non-flow cycle.

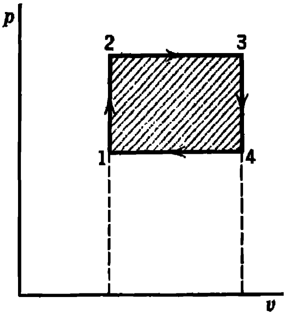
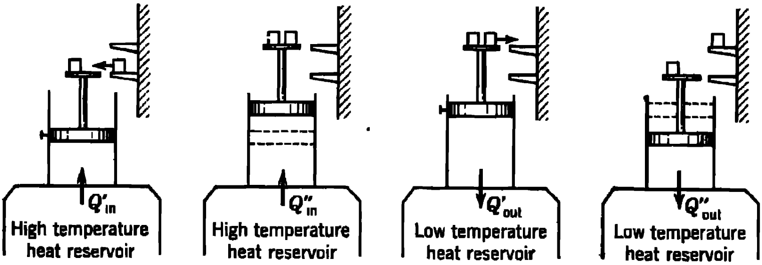


FIG. 33. Pressure-volume diagram for a non-flow cycle.

Suppose that a quantity of air is confined inside a cylinder equipped with a piston and that the initial pressure and volume of the air are represented by point 1 on the pressure-volume diagram shown in Fig. 33. The air can be caused to change from this state to the states represented by points 2, 3, and 4 and then back to its initial state by the series of non-flow processes indicated in Fig. 34.

Process 1–2. With the piston held stationary by a set screw, the cylinder is first brought into contact with a heat reservoir at some temperature higher than that of the air, and heat is allowed to flow into the air. As a result of this constant volume non-flow process, the pressure rises until the air reaches the state represented by point 2 in Fig. 33. An additional weight is now added to the piston, so that, when the set screw is released, the piston remains stationary.

Process 2–3. With the number of weights on the piston left unchanged, the cylinder is again brought into contact with the high temperature heat reservoir, and more heat is allowed to flow into the air. As a result of this constant pressure non-flow process, the volume increases until the system reaches the state represented by point 3. Note that the work done by the air during this process is represented by the total area under the line 2–3 in Fig. 33.


Q'_{in}
High temperature heat reservoir
Q''_{in}
High temperature heat reservoir
Q'_{out}
Low temperature heat reservoir
Q''_{out}
Low temperature heat reservoir

Process 1-2
With the piston held stationary, heat and a weight are added.

Process 2-3
With the piston released, more heat is added.

Process 3-4
With the piston held stationary, heat and a weight are removed.

Process 4-1
With the piston released, more heat is removed.

FIG. 34. A non-flow cycle.

Process 3–4. In order for the air to reach the state represented by point 4, the piston is again held stationary by a set screw while heat is allowed to flow from the air to a reservoir whose temperature is lower than that of the air. After the pressure drops to its initial value, the weight previously added is removed, so that, when the set screw is released, the piston again remains stationary.

Process 4–1. Finally, to complete the cycle, the cylinder is again brought into contact with the low temperature heat reservoir, and more heat is allowed to flow from the air. The work done on the air during this process is represented by the area under the line 4–1 in Fig. 33. Note that, at the completion of the cycle, the net result is that work has been done by the air in raising a weight and that the amount of this work is represented by the shaded area in Fig. 33.

As an example of a steady-flow cycle, let us consider the Rankine cycle—a cycle actually used in practice. The apparatus required consists of a pump, a boiler, a turbine or reciprocating engine, and a condenser; and the system (the "working fluid") is usually water. The component parts of the apparatus are connected as shown in Fig. 35, and the method of operation is as follows: The water flowing out of the condenser at low pressure (section 1, Fig. 35) flows next into the pump which, at the expense of some work, forces it into the boiler. In the boiler, heat flows from a high temperature source (usually some fuel which is burned) to the water and changes it into steam. The steam then flows through the turbine where it expands and does work. Finally, the steam from the turbine exhausts into the condenser where heat flows from it to a low temperature receiver (usually cooling water). As a result, the steam is condensed, and thus the cycle is completed. Since more work is done by the steam as it flows through the turbine than is required to operate the pump, some of the work from the turbine can be used to operate the pump, and the remainder can be delivered to some external system.

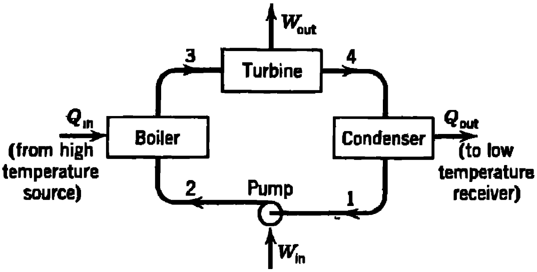


FIG. 35. Flow-diagram for a power plant operating on the Rankine cycle.

52. Power Cycles. It may be noted that both the non-flow and the steady-flow cycles described in the preceding article have the following characteristics in common: (*a*) heat is absorbed by the system from a high temperature reservoir; (*b*) heat is rejected by the system to a low temperature reservoir; and (*c*) regarding work, the *net* result is that work is done *by* the system. Cycles having these characteristics are called **power cycles**, and the engines whose working fluids execute such cycles are called **heat engines**.

Heat engines have as their purpose the conversion of heat into work. The extent to which they are able to accomplish this purpose is indicated by their so-called "thermal efficiency." The **thermal efficiency** η of any heat engine is defined as the net work W_{cycle}/J delivered by the engine to some external system divided by the heat Q_{in} supplied to the engine from a high temperature source; thus,

$$\eta = \frac{W_{\text{cycle}}/J}{Q_{\text{in}}}. \tag{35}$$

Since, by Eq. 10 in Chapter 3,

$$\frac{W_{\text{cycle}}}{J} = Q_{\text{cycle}} \tag{10}$$

$$= Q_{\text{in}} - Q_{\text{out}},$$

Eq. 35 may also be written

$$\eta = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}}. \tag{35a}$$

The thermal efficiencies of the two heat engines described in the preceding article are determined in the following Illustrative Problems.

ILLUSTRATIVE PROBLEMS

194. Calculate the thermal efficiency of a heat engine operating on the non-flow power cycle described in Art. 51 if the temperatures of the air at states 1, 2, and 3 (see Fig. 33) are 100°, 300°, and 500°F respectively.

Solution. The thermal efficiency can be calculated by Eq. 35*a* after the heat supplied to the air from the high temperature reservoir and the heat rejected by the air to the low temperature reservoir have been determined. If it is assumed that c_v and c_p for air are constant, the heat supplied per pound of air can be calculated by Eqs. 22*b* and 24*b*. Thus,

$$\begin{aligned} Q_{\text{in}} &= c_v(t_2 - t_1) + c_p(t_3 - t_2) \\ &= 0.172(300 - 100) + 0.241(500 - 300) \\ &= 82.6 \text{ Btu per lb.} \end{aligned}$$

The heat rejected can also be calculated by these equations after the temperature at state 4 has been determined. To determine this temperature we can make use of the fact that, since $v_1 = v_2$,

$$\frac{p_1}{p_2} = \frac{T_1}{T_2},$$

and, since $v_3 = v_4$,

$$\frac{p_4}{p_3} = \frac{T_4}{T_3}.$$

Since these two pressure ratios are equal, it follows that

$$\frac{T_4}{T_3} = \frac{T_1}{T_2},$$

or

$$\begin{aligned} T_4 &= T_3\left(\frac{T_1}{T_2}\right) \\ &= 960 \times \tfrac{560}{760} \\ &= 707\text{°F abs (or } t_4 = 247\text{°F).} \end{aligned}$$

Hence, for the change from 3 to 4 to 1,

$$Q = c_v(t_4 - t_3) + c_p(t_1 - t_4)$$

$$= 0.172(247 - 500) + 0.241(100 - 247)$$

$$= -78.9 \text{ Btu per lb},$$

or $Q_{out} = +78.9$ Btu per lb. Therefore, by Eq. 35*a*, the thermal efficiency of a heat engine operating on this cycle is

$$\eta = \frac{82.6 - 78.9}{82.6}$$

$$= 0.045.$$

Expressed another way, the engine will convert 4.5% of the heat energy supplied to it into work.

195. Calculate the thermal efficiency of a heat engine operating on the Rankine cycle described in Art. 51 if (*a*) the steam leaving the boiler is saturated and at 500°F, (*b*) the expansion through the turbine is adiabatic and reversible, (*c*) the steam in the condenser condenses at 100°F, and (*d*) the condensate leaving the condenser is saturated water at 100°F.

Solution. The thermal efficiency can be calculated by Eq. 35*a* after the heat supplied to generate each pound of steam in the boiler and the heat rejected by each pound of steam in the condenser have been determined.

If it is assumed that the velocity of the water entering and the velocity of the steam leaving the boiler are both negligible, the steady-flow energy equation applied to the boiler becomes

$$Q_{in} + h_2 = h_3,$$

where the subscripts refer to the section numbers marked on Fig. 35. The enthalpy h_3 of the steam leaving the boiler is 1201.7 Btu per lb (from Table 1 of the *Steam Tables*). To determine the enthalpy h_2 of the water entering the boiler, we must first apply the steady-flow energy equation to the pump. If we assume that the water enters and leaves the pump with negligible velocities and that the pump operates adiabatically, the steady-flow energy equation becomes

$$h_1 = h_2 + \frac{W'}{J}.$$

Thus, the enthalpy h_2 is *greater* than the enthalpy h_1 by the amount of work *supplied to* the pump. Since this amount of work is small, we may for the present assume it to be negligible (a method of evaluating it will be explained in Art. 75) and take $h_2 = h_1 = 67.97$ Btu per lb (from Table 1). Hence, for the boiler,

$$Q_{in} = 1201.7 - 67.97$$

$$= 1134 \text{ Btu per lb}.$$

If we assume that the velocities entering and leaving the condenser are negligible, the steady-flow energy equation applied to the condenser becomes

$$Q + h_4 = h_1.$$

Since the expansion through the turbine is assumed to be adiabatic and reversible, the state of the steam leaving the turbine can be determined by constructing a *u-v* diagram, as explained in Art. 46. By such a graphical calculation, the enthalpy h_4 is found to be 797 Btu per lb. Hence, for the condenser,

$$Q = 67.97 - 797$$

$$= -729 \text{ Btu per lb},$$

or $Q_{out} = +729$ Btu per lb. Therefore, the thermal efficiency of the engine is

$$\eta = \frac{1134 - 729}{1134}$$

$$= 0.357,$$

or 35.7% of the heat supplied is converted into work.

PROBLEMS

196. A pound of air undergoes the following non-flow power cycle: (*a*) The air is first heated at constant volume from 100° to 500°F. (*b*) It is next expanded isothermally and reversibly back to its initial pressure. (*c*) Finally, it is cooled at constant pressure back to its initial state. Calculate the thermal efficiency of a heat engine operating on this cycle. *Ans.* 7.56%.

197. A heat engine using air as its working fluid operates on the following power cycle: (*a*) The air initially at 15 psia and 100°F is compressed adiabatically and reversibly to 30 psia. (*b*) It is then heated at constant pressure to 500°F. (*c*) It is next expanded adiabatically and reversibly back to 15 psia. (*d*) Finally, the air is cooled at constant pressure back to its initial state. Calculate the thermal efficiency of the engine, assuming (1) that the air undergoes a non-flow cycle, and (2) that it undergoes a steady-flow cycle.

198. A steam power plant operates on the Rankine cycle described in Art. 51. Determine what its thermal efficiency would be if (*a*) the steam leaving the boiler were at 100 psia and 500°F, (*b*) the steam expanded through the turbine adiabatically and reversibly, (*c*) the steam condensed in the condenser at a pressure of 3 in. Hg abs, and (*d*) the condensate leaving the condenser were at 100°F. Assume that the work supplied to the pump would be negligible. (*Note:* The specific volume and internal energy of the steam leaving the turbine, determined by constructing a *u-v* diagram, would be 201 cu ft per lb and 921 Btu per lb respectively.) *Ans.* 25.0%.

199. Suppose that a heat engine using air as its working fluid operates on the following non-flow power cycle: (*a*) The air is cooled at constant volume from 500° to 100°F. (*b*) It is then compressed adiabatically and reversibly back to its initial pressure. (*c*) Finally, the air is heated at constant pressure back to its initial state. Determine the thermal efficiency of the engine.

200. A pound of H_2O, initially at 100°F and consisting of 0.99 lb of saturated water and 0.01 lb of saturated steam, undergoes the following non-flow power cycle: (*a*) It is first heated at constant volume to 300°F; (*b*) it is then heated at constant pressure to 500°F; (*c*) it is next cooled at constant volume back to its initial pressure; and, finally, (*d*) it is cooled at constant pressure back to its initial state. Calculate the thermal efficiency of a heat engine operating on this cycle. *Ans.* 5.15%.

201. A pound of air undergoes the following non-flow power cycle: (a) It is first cooled at constant volume from 500° to 100°F; (b) it is then compressed adiabatically and reversibly until its temperature reaches 500°F; and, finally, (c) it is expanded isothermally and reversibly back to its initial state. Calculate the thermal efficiency of a heat engine operating on this cycle.

53. Reversible Cycles. The highest temperature involved in any of the foregoing heat engine problems is 500°F, and the lowest is 100°F. Therefore, the heat supplied to these engines *could* be taken from a heat reservoir (or "source") whose temperature remains constant at 500°F, and the heat rejected *could* be delivered to a heat reservoir (or "receiver") whose temperature remains constant at 100°F. Since the thermal efficiencies of these engines vary over a wide range (from 4.5% to 35.7%), the question logically arises, what is the highest thermal efficiency that a heat engine can have it if receives heat from such a constant temperature source and rejects heat to such a constant temperature receiver? Also, what kind of power cycle is required to realize this maximum possible efficiency?

It will be proved in the next chapter that, *if the temperatures of the source and receiver of heat are both constant, the maximum possible thermal efficiency is obtained by a heat engine operating on a reversible cycle*, a **reversible cycle** being defined as a cycle every process of which is a reversible process. Before considering a cycle that is reversible under the aforenamed conditions (constant temperature source and receiver), let us reexamine the two cycles described in Art. 51 to determine whether they could be reversible under these conditions.

For the non-flow cycle made up of the processes shown in Fig. 33, the temperature of the air is lowest at state 1 and increases continuously during the heat absorption processes from 1 to 2 and from 2 to 3. Therefore, if the heat is taken from a source whose temperature remains constant, the processes cannot be reversible because the temperatures of the air and its surroundings (the source) will differ by a finite amount. Similarly, since the temperature of the air decreases continuously during the processes from 3 to 4 and from 4 back to 1, the temperature difference between the air and the constant temperature receiver will also be finite; so these processes cannot be reversible. This cycle could be reversible only if heat were supplied from an infinite number of sources ranging in temperature from t_1 to t_3, and if heat were rejected to an infinite number of receivers ranging in temperature from t_3 to t_1.

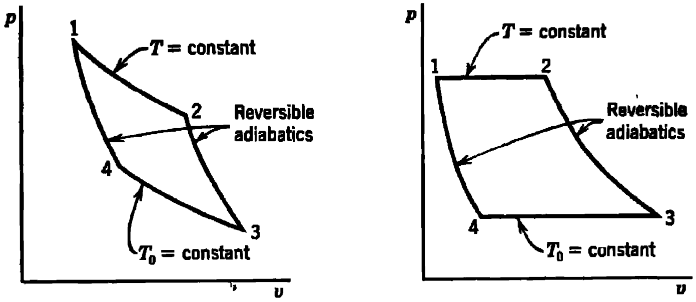
The situation regarding the Rankine cycle is somewhat better in that the temperature of the steam remains constant during the portion of the cycle where heat is rejected (the portion that takes place in the condenser). By having the temperature difference between the steam and the constant temperature receiver infinitesimal, this process could be made reversible. During that part of the heat absorption process where the saturated water is evaporated, the temperature of the system again remains constant; so that, by making the temperature difference between the system and the constant temperature source infinitesimal, this portion of the cycle could also be made reversible. However, the water entering the boiler is very nearly at the condenser temperature and must be raised to the saturation temperature corresponding to the boiler pressure before evaporation can begin. Since there will be a finite difference in temperature between the water and the constant temperature source during this process, this cycle cannot be made reversible either.

A cycle which is reversible under the prescribed condition that the temperatures of the source and receiver both remain constant is the so-called "Carnot cycle." It is described in the next article.

54. The Carnot Power Cycle. If any system is to receive heat only from a source whose temperature remains constant and is to reject heat only to a receiver whose temperature remains constant, the system cannot be made to undergo a reversible cycle unless the following conditions are satisfied: (*a*) the temperature of the system must remain constant and must be only an infinitesimal amount lower than that of the source during the heat reception process; (*b*) the temperature of the system must remain constant and must be only an infinitesimal amount higher than that of the receiver during the heat rejection process; and (*c*) there must be no transfer of heat between the system and the source or between the system and the receiver while the system is changing from one of these temperatures to the other. One series of processes that satisfies these requirements is the following: (*a*) the system initially at the temperature of the source undergoes an isothermal, reversible process during which it receives heat from the high temperature source; (*b*) the system next undergoes an adiabatic, reversible expansion during which its temperature drops from that of the source to that of the receiver; (*c*) the system then undergoes an isothermal, reversible process during which it rejects heat to the low temperature receiver; and, finally, (*d*) the system returns to its initial state by an adiabatic, reversible compression during which its temperature rises from that of the receiver to that of the source. Such a series of processes is called a **Carnot power cycle.**

If the system undergoing a Carnot power cycle is a perfect gas, the states through which it passes may be represented on a pressure-volume diagram by the figure shown in Fig. 36*a*. The process from 1 to 2 is an isothermal, reversible expansion during which heat flows from the source to the gas; the process from 2 to 3 is an adiabatic, reversible expansion during which the temperature of the gas drops from that of the source to that of the receiver; the process from 3 to 4 is an isothermal, reversible compression during which heat flows from the gas to the receiver; and the process from 4 to 1 is an adiabatic, reversible compression during which the temperature of the gas is raised from that of the receiver back to that of the source.

If the system undergoing a Carnot power cycle is initially a saturated liquid at the temperature of the source, and if it changes from this state into a saturated vapor during the isothermal heat absorption part of the cycle, the states through which it passes might be represented on a pressure-volume diagram by the figure shown in Fig. 36*b*. Since the liquid is evaporated at constant temperature during the process from 1 to 2, both its temperature and pressure remain constant. During the adiabatic, reversible expansion from 2 to 3, the temperature and pressure of the system decrease, and either some condensation takes place or the vapor becomes superheated. If the system is a mixture of saturated liquid and saturated vapor at state 3, its pressure will again remain constant during the isothermal rejection of heat from 3 to 4. At state 4 the vapor is not all condensed. Instead, the condensation is stopped just at the point where an adiabatic, reversible compression will return the system to its initial state. The figure representing the Carnot power cycle on a pressure-volume diagram may also take other forms, depending upon the system undergoing the cycle.



(*a*) System is a perfect gas

(*b*) System is a saturated liquid at state 1 and a saturated vapor at 2

FIG. 36. Carnot power cycle.

It should be noted that the distinctive feature about the Carnot power cycle is not just the fact that it is a reversible cycle, but that it is a reversible cycle *under the condition that the temperatures of the source and* *receiver of heat both remain constant.* The thermal efficiencies of heat engines operating on this cycle and using either air or steam as the working fluid are determined in the two Illustrative Problems which follow.

ILLUSTRATIVE PROBLEMS

202. Suppose that a heat engine using air as the working fluid could operate on the Carnot power cycle. If the temperature of the source were 500°F, if the temperature of the receiver were 100°F, and if (referring to Fig. 36*a*) the pressure at 1 were 100 psia and the volume at 2 were twice that at 1, what would be the thermal efficiency of the engine? Assume that the engine would operate on a non-flow cycle.

Solution. The thermal efficiency can be calculated by Eq. 35*a* after the heat supplied to and the heat rejected by each pound of air undergoing the cycle have been determined.

If it is assumed that air is a perfect gas, the heat supplied can be determined by Eqs. 25*a* and 26 in the preceding chapter. Thus,

$$
\begin{aligned}
Q_{\text{in}} &= \frac{RT}{J} \ln \left(\frac{v_2}{v_1}\right) \\
&= \frac{53.3 \times 960}{778} \ln 2 \\
&= 45.6 \text{ Btu per lb.}
\end{aligned}
$$

The heat rejected can be determined by the same equation after the specific volumes of the air at states 3 and 4 have been calculated. To find these, let us first calculate the specific volume at state 1. By the relation $pv = RT$,

$$
\begin{aligned}
v_1 &= \frac{53.3 \times 960}{100 \times 144} \\
&= 3.55 \text{ cu ft per lb.}
\end{aligned}
$$

The specific volume v_2, therefore, is 2 × 3.55 = 7.10 cu ft per lb. Since the expansion from 2 to 3 is adiabatic and reversible, the specific volume at state 3 can now be calculated by Eq. 28*a*. Thus,

$$
v_3 = v_2 \left(\frac{T_2}{T_3}\right)^{1/(k-1)} \tag{28a}
$$

$$
\begin{aligned}
&= 7.10(\tfrac{960}{560})^{1/0.4} \\
&= 27.30 \text{ cu ft per lb.}
\end{aligned}
$$

Similarly, since the compression from 4 back to 1 is also adiabatic and reversible,

$$
v_4 = v_1 \left(\frac{T_1}{T_4}\right)^{1/(k-1)} \tag{28a}
$$

$$
\begin{aligned}
&= 3.55(\tfrac{960}{560})^{1/0.4} \\
&= 13.65 \text{ cu ft per lb.}
\end{aligned}
$$

Hence, for the isothermal process from 3 to 4,

$$Q_{3\text{-}4} = \frac{RT_0}{J} \ln\left(\frac{v_4}{v_3}\right)$$

$$= \frac{53.3 \times 560}{778} \ln\left(\frac{13.65}{27.30}\right)$$

$$= -26.6 \text{ Btu per lb},$$

or $Q_{\text{out}} = +26.6$ Btu per lb.

Finally, the thermal efficiency of the engine would be, by Eq. 35*a*,

$$\eta_{\text{Carnot}} = \frac{45.6 - 26.6}{45.6}$$

$$= 0.417.$$

Note that this efficiency, as proved in the next chapter, is the highest attainable by any heat engine operating between a source of heat at 500°F and a receiver at 100°F. It cannot be realized in actual practice, however, because the reversible processes required cannot be carried out in any finite period of time.

203. Suppose that a heat engine using steam as the working fluid could operate on the Carnot power cycle. If the temperature of the source were 500°F, if the temperature of the receiver were 100°F, and if (referring to Fig. 36*b*) the fluid were saturated water at state 1 and saturated steam at state 2, what would be the thermal efficiency of the engine? Assume that the engine would operate on a steady-flow cycle and that the velocities of the fluid would be negligible when the fluid passed through states 1, 2, 3, and 4.

Solution. The engine could consist of a boiler, a turbine, a condenser, and a compressor. The method of operation would be exactly like that of an engine operating on the Rankine cycle, except that the fluid entering the compressor would be a mixture and the fluid leaving it would be saturated water.

The steady-flow energy equation applied to the boiler reduces to

$$Q_{\text{in}} + h_1 = h_2.$$

From Table 1 of the *Steam Tables*, $h_1 = 487.8$ Btu per lb and $h_2 = 1201.7$ Btu per lb. Hence,

$$Q_{\text{in}} = 1201.7 - 487.8$$

$$= 714 \text{ Btu per lb.}$$

Similarly, the steady-flow energy equation applied to the condenser reduces to

$$Q_{3\text{-}4} + h_3 = h_4.$$

Since the steam flowing through the turbine would undergo an adiabatic, reversible expansion, its state at the end of this process can be determined by constructing a *u-v* diagram, as explained in Art. 46. The enthalpy of the steam leaving the turbine, determined by such a graphical calculation, is $h_3 = 797$ Btu per lb. The state at which the steam would have to enter the compressor can be determined by finding the state which would be reached if saturated water at 500°F *expanded* adiabatically and reversibly to 100°F. By constructing a *u-v* diagram for this process, the enthalpy entering the compressor is found to be $h_4 = 380$ Btu per lb. Hence,

$$\begin{aligned} Q_{3\text{-}4} &= 380 - 797 \\ &= -417 \text{ Btu per lb,} \end{aligned}$$

or $Q_{\text{out}} = +417$ Btu per lb.

Finally, the thermal efficiency of the engine would be, by Eq. 35*a*,

$$\begin{aligned} \eta_{\text{Carnot}} &= \frac{714 - 417}{714} \\ &= 0.417 \end{aligned}$$

Note that the temperature of the source and the temperature of the receiver are the same in both this problem and in the preceding one and that the same value for the thermal efficiency is obtained in both.

PROBLEMS

204. A pound of air undergoes a non-flow Carnot power cycle. Determine what the thermal efficiency of a heat engine operating on this cycle would be if (*a*) the pressure and temperature of the air at the start of the isothermal heat reception process were 150 psia and 500°F, and (*b*) the pressure and temperature at the start of the isothermal heat rejection process were 15 psia and 100°F.

205. A pound of steam undergoes a non-flow Carnot power cycle. Determine what the thermal efficiency of a heat engine operating on this cycle would be if (*a*) the steam were saturated at 500°F at the start of the isothermal heat absorption process, and (*b*) it were saturated at 100°F at the start of the isothermal heat rejection process. (*Note:* By constructing *u-v* diagrams, it is found that at the end of the heat absorption process the specific volume and internal energy of the steam are 64.5 cu ft per lb and 1181 Btu per lb, and at the end of the heat rejection process the specific volume and internal energy are 246 cu ft per lb and 754 Btu per lb.)

55. The Thermal Efficiency of the Carnot Power Cycle. An expression for the thermal efficiency of a heat engine operating on the Carnot power cycle can be readily derived *if the working fluid used in the engine is a perfect gas.* Let us denote the absolute temperature of the source of heat by T and the absolute temperature of the receiver by T_0. The heat transferred from the source to each pound of the gas during the isothermal process from 1 to 2 (see Fig. 36*a*) is, by Eqs. 25*a* and 26,

$$Q_{\text{in}} = \frac{RT}{J} \ln \left(\frac{v_2}{v_1}\right)$$

Similarly, the heat transferred from the receiver to each pound of the gas during the isothermal process from 3 to 4 is

$$Q_{3\text{-}4} = \frac{RT_0}{J} \ln \left(\frac{v_4}{v_3}\right)$$

Hence,

$$Q_{\text{out}} = -\frac{RT_0}{J}\ln\left(\frac{v_4}{v_3}\right)$$

$$= \frac{RT_0}{J}\ln\left(\frac{v_3}{v_4}\right).$$

Therefore, by Eq. 35*a*, the thermal efficiency of the engine is

$$\eta_{\text{Carnot}} = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}} \tag{35a}$$

$$= \frac{\dfrac{RT}{J}\ln\left(\dfrac{v_2}{v_1}\right) - \dfrac{RT_0}{J}\ln\left(\dfrac{v_3}{v_4}\right)}{\dfrac{RT}{J}\ln\left(\dfrac{v_2}{v_1}\right)}.$$

But, for the two adiabatic, reversible processes from 2 to 3 and from 4 to 1, by Eq. 28*a*,

$$\frac{T}{T_0} = \left(\frac{v_3}{v_2}\right)^{k-1} = \left(\frac{v_4}{v_1}\right)^{k-1}. \tag{28a}$$

Therefore,

$$\frac{v_3}{v_2} = \frac{v_4}{v_1},$$

or

$$\frac{v_3}{v_4} = \frac{v_2}{v_1}.$$

By making use of this relation, we can reduce the foregoing expression for the thermal efficiency to

$$\eta_{\text{Carnot}} = \frac{T - T_0}{T}. \tag{36}$$

Note that these temperatures are measured on the *absolute perfect gas scale of temperature.*

In Illustrative Problem 202, the thermal efficiency of a heat engine using air as the working fluid and operating on the Carnot power cycle between a source at 500°F and a receiver at 100°F was found to be 41.7%. This result can also be obtained by Eq. 36. Thus, for these temperatures,

$$\eta_{\text{Carnot}} = \frac{960 - 560}{960}$$

$$= 0.417.$$

56. Refrigeration Cycles. As explained in Art. 52, power cycles are characterized by the facts that (*a*) heat is taken from a reservoir at a high temperature, (*b*) heat is rejected to a reservoir at a low temperature, and (*c*) work is delivered to some external system. In some cycles, exactly the reverse of this takes place: namely, (*a*) heat is taken from a reservoir at a low temperature, (*b*) heat is rejected to a reservoir at a high temperature, and (*c*) work is taken from some external system. Cycles having the latter characteristics are called **refrigeration cycles,** and engines whose working fluids execute such cycles are called **refrigerating machines.** Let us consider two examples of refrigeration cycles.

Suppose that a cylinder equipped with a piston contains air and that the air undergoes the series of processes shown in Fig. 37. At state 1 the temperature of the air is low. During the constant pressure process from 1 to 2, the air absorbs heat, and its temperature rises. The heat absorbed by the air during this process can be taken from any heat reservoir whose temperature is equal to or higher than that of the air at state 2. The air is next compressed adiabatically from 2 to 3, its temperature rising during the process. During the constant pressure process from 3 to 4, the air rejects heat, and its temperature decreases. The heat rejected can be transferred to any heat reservoir whose temperature is equal to or less than that of the air at state 4. To complete the cycle, the air is returned from 4 to 1 by an adiabatic expansion, its temperature decreasing during the process. It is evident that the work done *by* the air in expanding from 4 to 1 to 2 is less than the work done *on* the air in compressing it from 2 to 3 to 4, since the area under the first two curves in Fig. 37 is less than the area under the last two. Hence, the net result is that work must be supplied to the air from some external system.

The air could be made to pass through the states shown in Fig. 37 by a series of steady-flow processes instead of by the series of non-flow processes just described. At one time refrigerating machines were built to operate on such a steady-flow cycle and were known as "air refrigerating machines." The temperature of the air at state 2 (Fig. 37) had to be lower than the refrigeration temperature desired, and the temperature of the air at state 4 had to be higher than the initial temperature of the cooling water used to absorb the heat rejected.

As another example of a refrigeration cycle, let us consider the so-called "vapor compression cycle." This is the cycle used today in most refrigerating machines. It is a steady-flow cycle, and the apparatus required consists of an expansion valve, an evaporator, a compressor, and a condenser. These component parts are connected as shown in

Fig. 38. The refrigerant, leaving the condenser as a liquid at high pressure, flows through the expansion valve and undergoes a drop in pressure. As a result of this throttling process, some of the liquid flashes into vapor (see Illustrative Problem 161). The resulting mixture flows next through the evaporator where it absorbs heat and is usually completely vaporized. The temperature at which this vaporization takes place depends upon the pressure existing in the evaporator and can be lowered by decreasing this pressure. For example, if the refrigerant is ammonia, the saturation temperature will be approximately 0°F if the evaporator pressure is 30 psia, but it will drop to about −27°F if the pressure is lowered to 15 psia. The vapor leaving the evaporator enters the compressor and is compressed to the condenser pressure. The condenser pressure must be high enough that the corresponding saturation temperature is above the temperature of the available cooling water. For example, if the refrigerant is ammonia, and if the cooling water is at 70°F, the ammonia vapor must be compressed to some pressure above 129 psia (the saturation pressure corresponding to 70°F). The cycle is completed when the refrigerant flows through the condenser and changes back into a liquid.

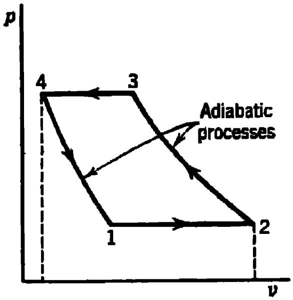


Fig. 37. A refrigeration cycle.

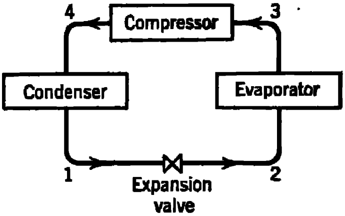


Fig. 38. Flow-diagram for a vapor compression refrigerating machine.

Since the purpose of any refrigerating machine is to absorb heat from some body whose temperature is below atmospheric temperature, and since work must be supplied to the machine in order to accomplish this, it is evident that, the more heat absorbed by the machine for a given amount of work supplied, the better the machine. The ratio of the heat absorbed (that is, the "refrigeration" obtained) to the net work supplied to operate the machine is called its **coefficient of performance.** Thus, if the coefficient of performance is denoted by the symbol c.p.,

$$\text{c.p.} = \frac{Q_{\text{in}}}{W'_{\text{cycle}}/J}, \tag{37}$$

where W'_{cycle} represents the net work *supplied* to the machine. Since, by Eq. 10, $W'_{\text{cycle}}/J = Q_{\text{out}} - Q_{\text{in}}$, Eq. 37 may also be written in the form

$$\text{c.p.} = \frac{Q_{\text{in}}}{Q_{\text{out}} - Q_{\text{in}}}. \tag{37a}$$

The coefficient of performance of refrigerating machines operating on the two cycles described in this article is determined in the following Illustrative Problems.

ILLUSTRATIVE PROBLEMS

206. Suppose that a refrigerating machine uses air as the working fluid and operates on a non-flow cycle consisting of the processes shown in Fig. 37. If the temperatures of the air at states 1, 2, and 4 are −50°, 0°, and 100°F respectively, and if the processes from 2 to 3 and from 4 to 1 are reversible as well as adiabatic, what is the coefficient of performance of the machine?

Solution. The coefficient of performance can be calculated by Eq. 37*a* after the heat added to the air during the process from 1 to 2 and the heat rejected by the air during the process from 3 to 4 have been determined.

Assuming that air is a perfect gas and that c_p for air is a constant, we can determine the heat added to each pound of air during the constant pressure, non-flow process from 1 to 2 by Eq. 24*b*. Thus,

$$\begin{aligned} Q_{\text{in}} &= c_p(t_2 - t_1) \qquad (24b) \\ &= 0.241(0 + 50) \\ &= 12.0 \text{ Btu per lb.} \end{aligned}$$

We can determine the heat added to the air during the constant pressure, non-flow process from 3 to 4 also by Eq. 24*b* after we have found the temperature at state 3. To find this temperature, we can make use of the fact that, since the processes from 2 to 3 and from 4 to 1 are adiabatic and reversible, by Eq. 28*b*

$$\frac{p_3}{p_2} = \left(\frac{T_3}{T_2}\right)^{k/(k-1)} \quad \text{and} \quad \frac{p_4}{p_1} = \left(\frac{T_4}{T_1}\right)^{k/(k-1)} \tag{28b}$$

Since these two pressure ratios are equal, it follows that

$$\frac{T_3}{T_2} = \frac{T_4}{T_1},$$

or

$$\begin{aligned} T_3 &\quad T_2\left(\frac{T_4}{T_1}\right) \\ &\quad 460\left(\tfrac{560}{410}\right) \\ &\quad 628\text{°F abs (or } t_3 = 168\text{°F).} \end{aligned}$$

Hence,

$$Q_{3-4} = c_p(t_4 - t_3) \tag{24b}$$

$$= 0.241(100 - 168)$$

$$= -16.4 \text{ Btu per lb,}$$

or $Q_{\text{out}} = +16.4$ Btu per lb.

Finally, by Eq. 37*a*, the coefficient of performance is

$$\text{c.p.} = \frac{12.0}{16.4 - 12.0}$$

$$= 2.7.$$

Thus, for each Btu of work supplied to the machine, 2.7 Btu of refrigeration are obtained.

207. Suppose that ammonia is used as the refrigerant in the vapor compression refrigerating machine shown in Fig. 38. Calculate the coefficient of performance for the machine if (*a*) the ammonia condenses at 100°F and leaves the condenser as a saturated liquid, (*b*) the ammonia evaporates at 0°F and leaves the evaporator as a saturated vapor, and (*c*) the ammonia is compressed adiabatically and reversibly in the compressor.

Solution. The properties of ammonia can be found from the *Tables of Thermodynamic Properties of Ammonia*, published by the Bureau of Standards (Circular No. 142). An abridgment of these tables is given in the Appendix.

If we assume that the velocities of the ammonia entering and leaving the evaporator are negligible, the steady-flow energy equation applied to the evaporator reduces to

$$Q_{\text{in}} + h_2 = h_3,$$

where the subscripts refer to the sections marked on Fig. 38. To determine the enthalpy h_2 of the ammonia entering the evaporator, we make use of the fact that the flow through the expansion valve is a throttling process, and therefore, by Eq. 34, $h_1 = h_2$. We may read the enthalpy h_1 of saturated liquid ammonia at 100°F directly from the tables. It is equal to 155.2 Btu per lb. We may also read the enthalpy h_3 of saturated ammonia vapor at 0°F directly from the tables. It is equal to 611.8 Btu per lb. Hence, for the evaporator,

$$Q_{\text{in}} = 611.8 - 155.2$$

$$= 457 \text{ Btu per lb.}$$

If we assume that the velocities of the ammonia entering and leaving the condenser are also negligible, the steady-flow energy equation applied to the condenser reduces to

$$Q + h_4 = h_1.$$

To determine the enthalpy at 4, we make use of the fact that, to reach this state, the ammonia enters the compressor as a saturated vapor at 0°F and is compressed adiabatically and reversibly to the condenser pressure. The condenser pressure is equal to 211.9 psia, the saturation pressure corresponding to 100°F. By constructing a *u-v* diagram for this process, as explained in Art. 46, we find that the enthalpy h_4 of the ammonia leaving the compressor is 737 Btu per lb. Hence, for the condenser,

$$Q = 155.2 - 737$$
$$= -582 \text{ Btu per lb,}$$

or $Q_{out} = +582$ Btu per lb.

Finally, by Eq. 37*a*, the coefficient of performance is

$$\text{c.p.} = \frac{457}{582 - 457}$$
$$= 3.66$$

PROBLEMS

208. Suppose that a refrigerating machine using air as its working fluid operates on the following non-flow refrigeration cycle: (*a*) the air initially at −60°F is heated at constant volume to 0°F; (*b*) it is then compressed adiabatically and reversibly to a certain temperature above 100°F; (*c*) it is next cooled at constant volume to 100°F; and, finally, (*d*) it is expanded adiabatically and reversibly back to its initial state. Calculate the coefficient of performance of the machine. *Ans.* 2.50.

209. What would be the coefficient of performance of the refrigerating machine described in the preceding problem if the constant volume heat rejection process (process *c*) were replaced by a constant pressure process?

210. A vapor compression refrigerating machine (Fig. 38) uses ammonia as its refrigerant. Calculate the coefficient of performance of the machine if (*a*) the ammonia leaves the condenser as a saturated liquid at 230 psia, (*b*) it leaves the evaporator as a mixture of saturated liquid and saturated vapor at 30 psia, (*c*) it is compressed adiabatically and reversibly in the compressor, and (*d*) it leaves the compressor as saturated vapor at 230 psia. Assume that the velocity of the ammonia is low throughout the plant. (*Note:* By constructing a *u-v* diagram for the adiabatic reversible compression, it is found that the ammonia must leave the evaporator with a quality of 86%.)

211. A pound of air undergoes the following non-flow refrigeration cycle: (*a*) the air initially at 100°F is expanded adiabatically and reversibly until its temperature drops to −60°F; (*b*) it is next heated at constant pressure to 100°F; and, finally, (*c*) it is compressed isothermally and reversibly back to its initial state. If a refrigerating machine operating on this cycle is to absorb heat from a region at 0°F, what will be its coefficient of performance? (*Note:* The refrigeration produced is equal only to the heat added to the air to heat it at constant pressure from −60° to 0°F. The heat added to heat it from 0° to 100°F must come from the receiver at 100°F.) *Ans.* 2.22.

212. A pound of ammonia undergoes the following non-flow refrigeration cycle: (*a*) the ammonia, initially saturated liquid at 230 psia, is cooled at constant volume until its pressure is reduced to 30 psia; (*b*) it is then heated at constant pressure until it is changed into saturated vapor; (*c*) it is next heated at constant volume until its pressure is raised to 50 psia; (*d*) it is then compressed isothermally to 230 psia; and, finally, (*e*) it is cooled at constant pressure back to its initial state. Determine the coefficient of performance of a refrigerating machine operating on this cycle. [Note that part of the refrigeration produced during process (*b*) must be used to absorb the heat rejected by the ammonia during process (*a*); and part of the heat rejected during process (*d*) can be used to supply the heat added to the ammonia during process (*c*).]

57. The Carnot Refrigeration Cycle. The refrigerating machines described in the preceding problems could all be used to absorb heat from a reservoir whose temperature remains constant at 0°F and to reject heat to a reservoir whose temperature remains constant at 100°F. Since the coefficients of performance of these machines range from 2.22 to 3.66, the question arises, what is the highest possible coefficient of performance that any refrigerating machine can have when operating between two such constant temperature reservoirs? Also, what kind of cycle is required to obtain this maximum possible coefficient of performance?

It will be proved in the next chapter that, *if the temperatures of the source and receiver of heat are both constant, the maximum possible coefficient of performance is obtained by a refrigerating machine operating on a reversible cycle.* Before considering a cycle which is reversible under the given conditions (constant temperature source and receiver), let us reexamine the two cycles described in the preceding article to determine whether they could be reversible.

If a gas undergoes the refrigeration cycle shown in Fig. 37, its temperature changes during the heat reception process from 1 to 2 and also changes during the heat rejection process from 3 to 4. Consequently, the temperature differences between the gas and the constant temperature source and between the gas and the constant temperature receiver cannot be kept infinitesimal. The cycle, therefore, cannot be made reversible. In the vapor compression cycle, the temperature of the refrigerant remains constant during the heat reception process taking place in the evaporator; so this part of the cycle could be made reversible. If the refrigerant leaving the evaporator were a mixture, it could be compressed without becoming superheated. The temperature of the refrigerant would then also remain constant during the heat rejection process taking place in the condenser; so this part of the cycle could also be made reversible. However, the throttling process taking place in the expansion valve necessarily involves friction and therefore is not reversible.

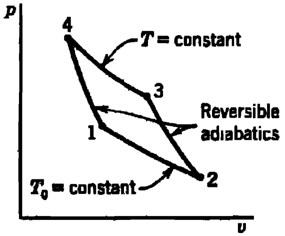
As explained in Art. 54, if any system is to absorb heat only from a source whose temperature remains constant and is to reject heat only to a receiver whose temperature remains constant, the system cannot be made to undergo a reversible cycle unless the following conditions are satisfied: (*a*) the temperature of the system must remain constant and must be only an infinitesimal amount lower than that of the source during the heat reception process; (*b*) the temperature of the system must remain constant and must be only an infinitesimal amount higher than that of the receiver during the heat rejection process; and (*c*) there must be no transfer of heat between the system and the source or between the system and the receiver while the system is changing from one of these temperatures to the other. The foregoing requirements will be satisfied if the system undergoes the following series of processes: (*a*) the system initially at the temperature of the source undergoes an isothermal, reversible process during which it absorbs heat from the low temperature source; (*b*) it next undergoes an adiabatic, reversible compression during which its temperature rises from that of the source to that of the receiver; (*c*) it then undergoes an isothermal, reversible process during which it rejects heat to the high temperature receiver; and, finally, (*d*) it returns to its initial state by an adiabatic, reversible expansion during which its temperature drops from that of the receiver to that of the source. Such a series of processes is called a **Carnot refrigeration cycle.**

It is evident that any system undergoing a Carnot refrigeration cycle passes through the same series of states as when undergoing a Carnot power cycle, except that it passes through these states in the reverse order. Thus, if the system is a perfect gas, the pressure-volume diagram for the cycle looks exactly like Fig. 36*a*, except that the gas passes through these states in the reverse order. If the system is a liquid which evaporates, the pressure-volume diagram might look like Fig. 36*b*, except, again, that the system passes through these states in the reverse order.

In general, any engine operating on a reversible cycle between two heat reservoirs whose temperatures remain constant can be either a heat engine or a refrigerating machine, depending upon the direction in which the working fluid passes around the cycle.

ILLUSTRATIVE PROBLEMS

213. Suppose that a refrigerating machine using air as a working fluid could operate on the Carnot refrigeration cycle. If the temperature of the source were 0°F, if the temperature of the receiver were 100°F, and if (referring to the accompanying figure) the pressure at 1 were 50 psia and the volume at 2 were twice that at 1, what would be the coefficient of performance of the machine? Assume that the machine would operate on a non-flow cycle.



Solution. The coefficient of performance can be calculated by Eq. 37*a* after the heat absorbed and the heat rejected by each pound of air undergoing the cycle have been determined.

If it is assumed that air is a perfect gas, the heat absorbed during the process from 1 to 2 can be determined by Eqs. 25*a* and 26:

$$Q_{\text{in}} = \frac{RT_0}{J} \ln\left(\frac{v_2}{v_1}\right)$$

$$= \frac{53.3 \times 460}{778} \ln 2$$

$$= 21.8 \text{ Btu per lb.}$$

The heat rejected can be determined by the same equation after the specific volumes at 3 and 4 have been found. The specific volume at 1 is

$$v_1 = \frac{53.3 \times 460}{50 \times 144}$$

$$= 3.40 \text{ cu ft per lb.}$$

The specific volume at 2, therefore, is 2 × 3.40 = 6.80 cu ft per lb. Since the process from 2 to 3 is adiabatic and reversible, the specific volume at 3 can be calculated by Eq. 28*a*:

$$v_3 = v_2 \left(\frac{T_2}{T_3}\right)^{1/(k-1)} \tag{28a}$$

$$= 6.80(\tfrac{460}{560})^{1/0.4}$$

$$= 4.15 \text{ cu ft per lb.}$$

Since the process from 4 to 1 is also adiabatic and reversible,

$$v_4 = v_1 \left(\frac{T_1}{T_4}\right)^{1/(k-1)} \tag{28a}$$

$$= 3.40(\tfrac{460}{560})^{1/0.4}$$

$$= 2.08 \text{ cu ft per lb.}$$

Hence, for the isothermal process from 3 to 4,

$$Q_{3\text{-}4} = \frac{RT}{J} \ln\left(\frac{v_4}{v_3}\right)$$

$$= \frac{53.3 \times 560}{778} \ln\left(\frac{2.08}{4.15}\right)$$

$$= -26.6 \text{ Btu per lb,}$$

or $Q_{\text{out}} = +26.6$ Btu per lb.

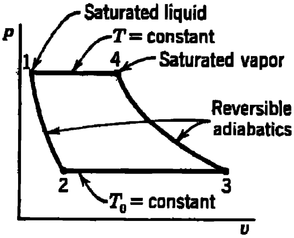
Finally, the coefficient of performance would be, by Eq. 37*a*,

$$(\text{c.p.})_{\text{Carnot}} = \frac{21.8}{26.6 - 21.8}$$

$$= 4.6.$$

As proved in the next chapter, this coefficient of performance is the highest attainable by any refrigerating machine taking heat from a reservoir at 0°F and rejecting heat to a reservoir at 100°F.

214. Suppose that a refrigerating machine using ammonia as the refrigerant could operate on the Carnot refrigeration cycle. If the temperature of the source were 0°F, if the temperature of the receiver were 100°F, and if (referring to the accompanying figure) the ammonia were a saturated liquid at 1 and a saturated vapor at 4, what would be the coefficient of performance of the machine? Assume that the machine would operate on a steady-flow cycle and that the velocities at states 1, 2, 3, and 4 would be negligible.



Solution. The machine could be the same as the one used for the vapor compression cycle (see Fig. 38), except that the expansion valve would have to be replaced by an engine.

The steady-flow energy equation applied to the evaporator reduces to

$$Q_{\text{in}} + h_2 = h_3.$$

To determine the enthalpies at states 2 and 3, we must make use of the fact that these states are reached from states 1 and 4 by adiabatic reversible processes. By constructing u-v diagrams, as explained in Art. 46, we find the enthalpies at 2 and 3 to be 144 and 536 Btu per lb respectively. Hence,

$$\begin{aligned} Q_{\text{in}} &= 536 - 144 \\ &= 392 \text{ Btu per lb.} \end{aligned}$$

The steady-flow energy equation applied to the condenser reduces to

$$Q_{4\text{-}1} + h_4 = h_1.$$

Since the ammonia is a saturated liquid at 1 and a saturated vapor at 4, the values h_1 = 155.2 Btu per lb and h_4 = 633.0 Btu per lb can be taken directly from the tables. Hence,

$$\begin{aligned} Q_{4\text{-}1} &= 155.2 - 633.0 \\ &= -478 \text{ Btu per lb,} \end{aligned}$$

or Q_{out} = +478 Btu per lb.

Finally, by Eq. 37*a*,

$$\begin{aligned} (\text{c.p.})_{\text{Carnot}} &= \frac{392}{478 - 392} \\ &= 4.6. \end{aligned}$$

PROBLEMS

215. A pound of air undergoes a non-flow Carnot refrigeration cycle. Determine what the coefficient of performance of a refrigerating machine operating on this cycle would be if (*a*) the pressure and temperature of the air at the start of the isothermal heat reception process were 40 psia and 0°F, and (*b*) the pressure and temperature at the start of the isothermal heat rejection process were 60 psia and 100°F.

216. A pound of ammonia undergoes a non-flow Carnot refrigeration cycle. Determine what the coefficient of performance of a refrigerating machine operating on this cycle would be if (*a*) the ammonia were saturated vapor at 0°F at the end of the isothermal heat absorption process, and (*b*) it were saturated vapor at 100°F at the end of the isothermal heat rejection process. (*Note:* The following values are obtained by constructing u-v diagrams: At the end of the adiabatic reversible compression, $p = 71.5$ psia, $v = 4.73$ cu ft per lb, and $u = 597.8$ Btu per lb; at the end of the adiabatic reversible expansion, $v = 7.90$ cu ft per lb and $u = 491.8$ Btu per lb.)

58. The Coefficient of Performance of the Carnot Refrigeration Cycle. An expression for the coefficient of performance of a refrigerating machine operating on the Carnot refrigeration cycle can be readily derived *if the working fluid is a perfect gas.* The gas will undergo the processes shown in Fig. 36*a*, except that it will undergo them in the reverse order. If the absolute temperature of the low temperature source of heat is denoted by T_0 and the absolute temperature of the high temperature receiver by T, the heat absorbed by the gas during the isothermal expansion from 4 to 3 is, by Eqs. 25*a* and 26,

$$Q_{\text{in}} = \frac{RT_0}{J} \ln\left(\frac{v_3}{v_4}\right).$$

Similarly, the heat transferred *to* the gas during the isothermal compression from 2 to 1 is

$$Q_{2-1} = \frac{RT}{J} \ln\left(\frac{v_1}{v_2}\right),$$

whence

$$\begin{aligned} Q_{\text{out}} &= -\frac{RT}{J} \ln\left(\frac{v_1}{v_2}\right) \\ &= \frac{RT}{J} \ln\left(\frac{v_2}{v_1}\right). \end{aligned}$$

Therefore, by Eq. 37*a*,

$$(\text{c.p.})_{\text{Carnot}} = \frac{Q_{\text{in}}}{Q_{\text{out}} - Q_{\text{in}}} \tag{37a}$$

$$= \frac{\dfrac{RT_0}{J} \cdot \ln\left(\dfrac{v_3}{v_4}\right)}{\dfrac{RT}{J} \ln\left(\dfrac{v_2}{v_1}\right) - \dfrac{RT_0}{J} \ln\left(\dfrac{v_3}{v_4}\right)}$$

It was shown in Art. 55 that for this cycle

$$\frac{v_3}{v_4} = \frac{v_2}{v_1}.$$

Therefore, by making use of this relation, we may reduce the foregoing expression to

$$(\text{c.p.})_{\text{Carnot}} = \frac{T_0}{T - T_0}. \tag{38}$$

In Illustrative Problem 213, the coefficient of performance of a refrigerating machine using air as the working fluid and operating on the Carnot refrigeration cycle between a source at 0°F and a receiver at 100°F was found to be 4.6. This result can also be obtained by Eq. 38. Thus, for these temperatures,

$$(\text{c.p.})_{\text{Carnot}} = \frac{460}{560 - 460}$$
$$= 4.6$$

ADDITIONAL PROBLEMS

217. A pound of air undergoes the following non-flow power cycle: (*a*) The air is heated at constant pressure from 100° to 500°F; (*b*) it is then expanded adiabatically and reversibly until its temperature drops to 100°F; and, finally, (*c*) it is compressed isothermally and reversibly back to its initial state. Determine what the thermal efficiency of a heat engine would be if it operated on this cycle. *Ans.* 24.8%.

218. A pound of air undergoes the following non-flow refrigeration cycle: (*a*) The air initially at 0°F is compressed adiabatically and reversibly until its temperature reaches 200°F; (*b*) it is then cooled at constant volume to 0°F; and, finally, (*c*) it is expanded isothermally and reversibly back to its initial state. If a refrigerating machine operated on this cycle, and if the machine absorbed heat from a region at 0°F and rejected heat to a region at 100°F, what would be its coefficient of performance?

219. A heat engine using air as its working fluid operates on the following non-flow power cycle: (*a*) The air is first compressed adiabatically and reversibly, its temperature rising from 100° to 300°F during the process; (*b*) the air is then heated at constant volume from 300° to 500°F; (*c*) it is next expanded isothermally and reversibly to its initial pressure; and, finally, (*d*) it is cooled at constant pressure back to its initial state. Determine the thermal efficiency of the engine.

220. It is proposed to build a refrigerating machine to use air as its working fluid and to operate on the following non-flow refrigeration cycle: (*a*) The air initially at 100°F is to be expanded adiabatically and reversibly until its temperature drops to −50°F; (*b*) it is then to be heated at constant volume until its pressure reaches its initial value; and, finally, (*c*) it is to be cooled at constant pressure back to its initial state. If the machine is to be used to absorb heat from a region at 0°F, what will be its coefficient of performance? *Ans.* 0.434.

221. A pound of air undergoes the following non-flow power cycle: (*a*) The air is first heated at constant volume from 100° to 500°F; (*b*) it is then expanded isothermally and reversibly until its volume increases by 25%; (*c*) it is next expanded adiabatically and reversibly to its initial pressure; and, finally, (*d*) it is cooled at constant pressure back to its initial state. Calculate the thermal efficiency of a heat engine operating on this cycle.

SYMBOLS

c_p	specific heat at constant pressure
c_v	specific heat at constant volume
c.p.	coefficient of performance
h	enthalpy per unit mass
J	mechanical equivalent of heat
k	ratio of c_p to c_v
p	pressure
Q	heat added to system; Q_{in} = heat added to system during one cycle; Q_{out} = heat rejected by system during one cycle; Q_{cycle} = net heat added to system during one cycle = $Q_{\text{in}} - Q_{\text{out}}$
R	gas constant
t	temperature
T	absolute temperature
u	internal energy per unit mass
v	specific volume
W	work done by system; W' = shaft-work done by a system in steady flow; W_{cycle} = net work done by a system during one cycle; W'_{cycle} = net work supplied to a system during one cycle
η	thermal efficiency

Chapter 7

THE SECOND LAW OF THERMODYNAMICS

In the preceding chapter it was stated that, of all heat engines taking heat from a constant temperature source and rejecting heat to a constant, lower temperature receiver, the highest possible thermal efficiency is attained by an engine operating on a reversible cycle. It was also stated that, of all refrigerating machines absorbing heat from a constant temperature source and rejecting heat to a constant, higher temperature receiver, the maximum possible coefficient of performance is attained by a refrigerating machine operating on a reversible cycle. The proof of these statements is based on the Second Law of Thermodynamics. The purpose of this chapter is first to state and discuss this law, then to prove the two foregoing statements, and finally to explain why reversible processes are called "reversible."

59. The Second Law of Thermodynamics. *Heat* was defined in Art. 18 as "something" which is transferred between two systems because of a difference in their temperatures. The "something" transferred was later given the name *energy*. It was also stated in Art. 18 that the direction in which the transfer takes place is from the system at the higher temperature to the system at the lower temperature. To determine which of the two is at the higher temperature, any thermometer may be used provided that its scale is such that the temperature of steam condensing at atmospheric pressure is higher than that of melting ice.

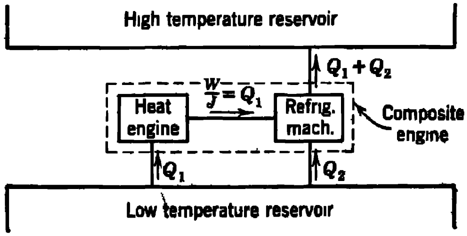
Whenever any system is brought into contact with another at a lower temperature, heat flows *out* of the first system, and certain changes may occur within it. Thus, the system may undergo a decrease in temperature or perhaps a change of phase. It is a matter of experience that these changes characterizing the flow of heat *out* of the system cannot be brought about by bringing the system into contact with another at a higher temperature. Stated another way, when two systems at different temperatures are brought into contact, the direction of heat flow is always from the one at the higher temperature to the one at the lower temperature. Although heat can be caused to flow from the lower to the higher temperature system with the help of a refrigerating machine, this requires that work (or its equivalent) be supplied from some third system to operate the refrigerating machine. The last statement may be expressed as follows: *No machine whose working fluid undergoes a cycle can absorb heat from one system, reject heat to another at a higher temperature, and produce no other effect.* This is one statement of the **Second Law of Thermodynamics.** Like the First Law, the Second Law cannot be proved but is nevertheless believed to be true because no exceptions to it (or to any concepts based upon it) have ever been found.

The preceding statement of the Second Law is known as the "Clausius" statement. Another equivalent statement is given in the following Illustrative Problem.

ILLUSTRATIVE PROBLEM

222. The "Kelvin-Planck" statement of the Second Law of Thermodynamics is: *No engine whose working fluid undergoes a cycle can absorb heat from a single reservoir, deliver an equivalent amount of work, and produce no other effect.* Show that if this statement were false, a violation of the Clausius statement would be possible.

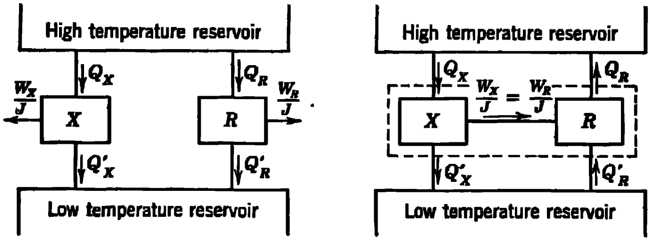
Solution. Let us assume that the Kelvin-Planck statement of the Second Law is false: that is, let us assume that a heat engine can be built which will absorb heat from some reservoir, deliver an equivalent amount of work, and produce no other effect. The work from this engine could drive an ordinary refrigerating machine; and this machine could absorb heat from the same reservoir used to supply heat to the heat engine and could reject heat to another reservoir at some higher temperature, as shown in the accompanying figure. Now, considering the heat engine and the refrigerating machine *together*, we see that the composite machine absorbs heat from one reservoir, rejects heat to another at a higher temperature, and produces no other effect. This is a direct violation of the Clausius statement. Therefore, if the Clausius statement is true, the Kelvin-Planck statement must also be true.



PROBLEM

223. Show that, if the Clausius statement of the Second Law were false, a violation of the Kelvin-Planck statement would be possible. (*Note:* Since it can be shown that a violation of either statement leads to a violation of the other, it follows that the two statements are equivalent.)

60. The Carnot Principle. In the preceding chapter the question was raised, what is the highest possible thermal efficiency that any heat engine can have if it takes heat from a constant temperature source and rejects heat to a constant, lower temperature receiver? It was stated that, under the given conditions, the highest possible efficiency is that obtained by a heat engine operating on a *reversible* cycle, such as the Carnot power cycle. This statement is known as the **Carnot Principle.** If the name **reversible engine** is used to denote an engine operating on a reversible cycle, the principle may be stated: *Of all heat engines receiving heat from the same constant temperature source and rejecting heat to the same constant temperature receiver, none can be more efficient than a reversible engine.* The proof of this principle consists in showing that, if the principle were false, a violation of the Second Law would be possible. The steps are as follows:


High temperature reservoir
Q_X
Q_R
$\frac{W_X}{J}$
$\frac{W_R}{J}$
X
R
Q'_X
Q'_R
Low temperature reservoir
High temperature reservoir
Q_X
Q_R
$\frac{W_X}{J} = \frac{W_R}{J}$
X
R
Q'_X
Q'_R
Low temperature reservoir

(*a*) *X* and *R* both operating as heat engines

(*b*) *R* reversed and driven by *X*

FIG. 39.

Suppose that a reversible heat engine *R* and any other heat engine *X* take heat from the same constant temperature source and reject heat to the same constant, lower temperature receiver, as shown in Fig. 39*a*. Let us assume that the thermal efficiency of engine *X* is greater than that of engine *R*: that is, *by assumption*,

$$\eta_X > \eta_R. \qquad (a)$$

Also, let the amount of heat supplied to each engine be such that both deliver the same amount of work: that is, let

$$\frac{W_X}{J} = \frac{W_R}{J}. \tag{b}$$

The relation between the amounts of heat supplied to the two engines and the relation between the amounts rejected can be determined as follows: Dividing equality (*b*) by inequality (*a*), we get

$$\frac{W_X/J}{\eta_X} < \frac{W_R/J}{\eta_R}.$$

But, from the definition of thermal efficiency, $Q_{\text{in}} = (W/J)/\eta$. Therefore,

$$Q_X < Q_R. \tag{c}$$

Since the working fluids of both engines perform cycles, by Eq. 10 the net heat added to each is equal to the net work done; that is,

$$Q_X - Q'_X = \frac{W_X}{J},$$

or

$$Q'_X = Q_X - \frac{W_X}{J}.$$

Similarly,

$$Q'_R = Q_R - \frac{W_R}{J}.$$

Therefore, subtracting equality (*b*) from inequality (*c*), we get

$$Q_X - \frac{W_X}{J} < Q_R - \frac{W_R}{J},$$

or

$$Q'_X < Q'_R. \tag{d}$$

Now let us suppose that engine *R* is run as a refrigerating machine. This can be done without changing the values of W_R/J, Q_R, and Q'_R—except for sign. Since the work required to drive engine *R* can be obtained from engine *X*, as shown in Fig. 39*b*, the two engines together comprise a machine which neither consumes nor delivers work but which takes heat from the low temperature reservoir (since $Q'_X < Q'_R$) and delivers heat to the high temperature reservoir (since $Q_X < Q_R$). We know that it is impossible for any machine to do this, however, because such a machine would violate the Second Law. We conclude, therefore, that our assumption—that engine *X* is more efficient than reversible engine *R*—is inadmissible; and thus we prove the Carnot Principle.

PROBLEMS

224. Prove Carnot's Principle by showing, as is done in the preceding article, that a violation of the Clausius statement of the Second Law would be possible if the principle were false. Instead of letting $W_X = W_R$, however, let $Q_X = Q_R$.

225. Prove Carnot's Principle by showing that a violation of the Kelvin-Planck statement of the Second Law would be possible if the principle were false. Let $Q'_X = Q'_R$.

61. A Corollary of the Carnot Principle. A corollary of the Carnot Principle is that *all reversible heat engines receiving heat from the same constant temperature source and rejecting heat to the same constant temperature receiver have the same thermal efficiency.* This corollary can be proved as follows:

Suppose that two reversible heat engines R_1 and R_2 receive heat from the same source and reject heat to the same receiver. Since engine R_1 is a reversible engine, it follows from the Carnot Principle that no other engine, such as engine R_2, can have an efficiency higher than that of R_1; that is,

$$\eta_{R_2} \leq \eta_{R_1}.$$

Similarly, since R_2 is also a reversible engine, no other engine, such as engine R_1, can have an efficiency higher than that of R_2; that is,

$$\eta_{R_1} \leq \eta_{R_2}.$$

Therefore, since neither engine can be more efficient than the other, it follows that they must have the same efficiency, and the corollary is proved.

It was shown in Art. 55 that the thermal efficiency of a heat engine operating on the Carnot power cycle and using a perfect gas as the working fluid is

$$\eta_{\text{Carnot}} = \frac{T - T_0}{T}, \tag{36}$$

where T and T_0 are the absolute temperatures of the source and of the receiver, measured on the perfect gas scale. Since such an engine is a reversible engine, it follows from the foregoing corollary that *the efficiency of any reversible engine taking heat from a source at the constant absolute temperature T and rejecting heat to a receiver at the constant absolute temperature T_0 is*

$$\eta_{\text{rev}} = \frac{T - T_0}{T}. \tag{36a}$$

Note that the engine may operate on any reversible cycle—not necessarily the Carnot cycle—and that any substance may be used as the working fluid. For example, the thermal efficiencies of two engines, both operating on the Carnot cycle but one using a perfect gas and the other using steam as the working fluid, were calculated in Illustrative Problems 202 and 203. Both operated between the same source and the same receiver, and both were found to have the same efficiency.

The foregoing corollary of the Carnot Principle is of great importance in that it permits a scale of temperature called the "thermodynamic scale of temperature" to be defined. This scale, which is independent of the thermometric fluid, leads in turn to the concept of the property called "entropy." These concepts will be developed in the following two chapters.

PROBLEMS

226. Suppose that it were possible to build a reversible heat engine. If 50,000 Btu per hr of heat were supplied to the engine from a source at 500°F, and if the engine delivered 10 hp, what would the temperature of the receiver to which it rejected heat have to be? *Ans.* 11°F.

227. Ten pounds of air are at atmospheric pressure and 200°F. If the air were used as the source of heat for a reversible engine, and if the engine rejected heat to the atmosphere, how much work could the engine deliver? Assume that the temperature of the atmosphere remains constant at 60°F. (*Note:* If a reversible engine received and rejected only infinitesimal amounts of heat each time it performed a single cycle, it could operate between a source and a receiver whose temperatures change. This is true because the source and receiver would remain virtually at constant temperature *while the engine performed a single cycle.*)

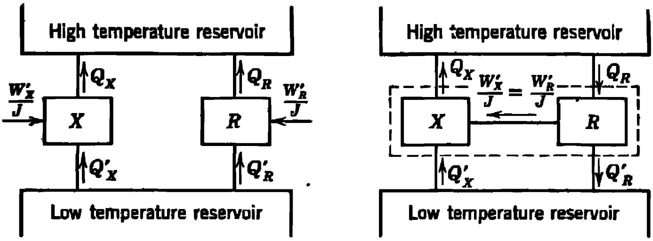
228. Ten pounds of air are at atmospheric pressure and −50°F. If the air were used as the receiver of heat for a reversible engine, and if the atmosphere were used as the source, how much work could the engine deliver? Assume that the temperature of the atmosphere remains constant at 60°F. *Ans.* 32 Btu.

229. Two pieces of copper are of equal mass m, but one is at temperature T_1 and the other is at a lower temperature, T_2. Show that, if the first piece is used as the source of heat for a reversible engine and the second is used as the receiver, the maximum work which the engine can deliver is equal to $mc_p(T_1 + T_2 - 2\sqrt{T_1T_2})$, where c_p denotes the specific heat at constant pressure for copper (assumed to be constant). (*Suggestion:* The maximum work will have been delivered when the two pieces of copper have reached the same temperature. Determine this temperature first.)

62. The Coefficient of Performance of a Reversible Refrigerating Machine. In the preceding chapter the question was also raised, what is the highest possible coefficient of performance that any refrigerating machine can have if it absorbs heat from a constant temperature source and rejects heat to a constant, higher temperature receiver? It was stated that, under the given conditions, the highest possible coefficient of performance is that obtained by a refrigerating machine operating on a *reversible* cycle, such as the Carnot refrigeration cycle.

Such a machine will be called a **reversible refrigerating machine.** The fact that its coefficient of performance is the highest possible can be proved in a manner analogous to that used in proving the Carnot Principle. The steps are as follows:

Suppose that a reversible refrigerating machine R and any other refrigerating machine X absorb heat from the same constant temperature source and reject heat to the same constant, higher temperature receiver, as shown in Fig. 40*a*. Let us assume that the coefficient of performance of X is greater than that of R; that is, *by assumption,*


High temperature reservoir
Q_X
$\frac{W'_X}{J}$
X
Q'_X
Q_R
$\frac{W'_R}{J}$
R
Q'_R
Low temperature reservoir
High temperature reservoir
Q_X
$\frac{W'_X}{J} = \frac{W'_R}{J}$
Q_R
X
R
Q'_X
Q'_R
Low temperature reservoir

(a) X and R both operating as refrigerating machines

(b) R reversed and driving X

FIG. 40.

$$(\text{c.p.})_X > (\text{c.p.})_R,$$

or, from the definition of coefficient of performance,

$$\frac{Q'_X}{W'_X/J} > \frac{Q'_R}{W'_R/J}. \qquad (a)$$

Also, let the same amount of work be supplied to each machine; that is, let

$$\frac{W'_X}{J} = \frac{W'_R}{J}. \qquad (b)$$

Combining inequality (*a*) and equality (*b*), we get

$$Q'_X > Q'_R. \qquad (c)$$

Since the working fluids of both machines perform cycles, $Q - Q' = W'/J$, or $Q = Q' + W'/J$. Hence, adding equality (*b*) to inequality (*c*), we get

$$Q'_X + \frac{W'_X}{J} > Q'_R + \frac{W'_R}{J},$$

or

$$Q_X > Q_R. \qquad (d)$$

Now let us suppose that machine R is run as a heat engine. Since this can be done without changing the values of W'_R/J, Q_R, and Q'_R (except for sign), the work delivered by R can be used to drive X, as shown in Fig. 40*b*. But X and R together now comprise a machine which, without having any work supplied to it, takes heat from the low temperature source (since $Q'_X > Q'_R$) and delivers heat to the high temperature receiver (since $Q_X > Q_R$). We know that it is impossible for any machine to do this, however, because such a machine would violate the Second Law. The assumption that any refrigerating machine X can have a coefficient of performance greater than that of a reversible refrigerating machine is inadmissible, therefore, and the proposition is proved.

Just as it was shown that all reversible heat engines operating between the same high temperature source and low temperature receiver have the same efficiency, it can also be shown that all reversible refrigerating machines operating between the same low temperature source and high temperature receiver have the same coefficient of performance. The proof consists in showing that, since neither can have a higher coefficient of performance than the other, they must have the same coefficient.

In Art. 58 it was shown that the coefficient of performance of a refrigerating machine operating on the Carnot refrigeration cycle and using a perfect gas as the working fluid is

$$(\text{c.p.})_{\text{Carnot}} = \frac{T_0}{T - T_0}, \tag{38}$$

where T_0 is the absolute temperature of the low temperature source and T, the absolute temperature of the high temperature receiver. Since such a machine is a reversible refrigerating machine, and since all reversible machines have the same coefficient of performance, it follows that the coefficient of performance of *any* reversible refrigerating machine operating between two such constant temperature reservoirs is

$$(\text{c.p.})_{\text{rev}} = \frac{T_0}{T - T_0}. \tag{38a}$$

In Illustrative Problems 213 and 214, the coefficients of performance of two reversible refrigerating machines were calculated. Both machines operated on the Carnot refrigeration cycle, but one used air as the refrigerant and the other ammonia. Both operated between the same source and same receiver and were found to have the same coefficient of performance.

PROBLEMS

230. As is done in the preceding article, show that the Second Law could be violated if any refrigerating machine operating between two constant temperature reservoirs had a coefficient of performance higher than that of a reversible machine operating between the same two reservoirs. Instead of letting $W'_X = W'_R$, however, let $Q'_X = Q'_R$.

231. Refrigerating machines are sometimes used to heat buildings by letting the machines absorb heat from the outdoor air and reject heat to the air within the buildings. If the air within a building is to be maintained at 70°F, if the rate of heat loss from the building is 50,000 Btu per hr when the outdoor air is at 0°F, and if electricity costs 1 cent per kwhr, what is the minimum possible cost per day of operating such a plant? *Ans.* $0.465 per day.

232. If 10 lb of air were cooled at constant pressure from 50° to −50°F by a reversible refrigerating machine rejecting heat to the atmosphere at 70°F, how much work would be required to drive the machine?

233. If a reversible refrigerating machine absorbed heat from the atmosphere at 70°F and rejected heat to 20 lb of air whose pressure remained constant, how much work would have to be supplied to raise the temperature of the air from 80° to 150°F? *Ans.* 26.0 Btu.

234. A reversible refrigerating machine absorbs heat from a 10-lb mass of air initially at 50°F and rejects heat to a 20-lb mass of air initially at 80°F. If both masses of air are kept at constant pressure, how much work must be supplied to lower the temperature of the first mass to −50°F?

63. An Alternative Definition of a Reversible Process. As explained in Art. 20, a system is said to undergo a *reversible process* if, during the process, (*a*) no part of the system or its surroundings ever departs from a state of equilibrium by more than an infinitesimal amount, and (*b*) no friction of any kind occurs. In Arts. 21 and 22, processes were described that satisfy these conditions. In these processes a fluid is (*a*) compressed or expanded, and (*b*) heated or cooled. It will now be shown that, at the completion of each of these processes, *both the system and its surroundings can be returned to their initial states without producing a finite change in any auxiliary system.*

Let us consider first the reversible compression of a fluid. As explained in Art. 21, a gas confined inside a cylinder equipped with a frictionless piston could be compressed reversibly by adding infinitesimal weights to the piston one at a time. If these weights were added to the piston by sliding them horizontally and without friction from an infinite number of platforms, each an infinitesimal distance lower than the preceding one, all the weights except the first could be returned to their initial positions by sliding them one at a time off the piston and back onto the platforms. The first one would have to be taken off at the lowest level in order to start the expansion. To return this weight to the top platform, work would have to be taken from some auxiliary system. But, since the weight is infinitesimal in size, only an infinitesimal amount of work would be required to raise it; so no finite change would be produced in the auxiliary system.

Suppose that, instead of a reversible compression, the gas undergoes a reversible expansion. It could be recompressed by taking an infinitesimal amount of work from some auxiliary system in order to raise the first weight from the lowest platform to the level of the piston when the piston is in its highest position. After this is done, all the weights could be returned to the piston by sliding them horizontally off the platforms and back onto the piston one at a time. Again, only an infinitesimal amount of work would be required; so no finite change would be produced in the auxiliary system.

Note that, if the weights added to or removed from the piston were finite instead of infinitesimal in size, the process would not be reversible; and a finite weight would have to be raised from the lowest to the highest level in order to restore the gas and the weights to their initial states. This would require that a finite amount of work be taken from some auxiliary system, and therefore a finite change in the auxiliary system would result.

Let us consider next the reversible heating of a fluid. As explained in Art. 22, a gas could be heated reversibly from one temperature to another by bringing it into contact successively with an infinite series of reservoirs, each at a temperature infinitesimally higher than that of the gas. To return the gas and each of the reservoirs to their initial states, heat must be caused to flow from the gas back to each reservoir. Since the temperature of each reservoir is always infinitesimally *higher* than that of the gas, this flow of heat could be caused to take place only with the help of a refrigerating machine. However, if a reversible refrigerating machine were used, the coefficient of performance of the machine would be infinite since the denominator of Eq. 38*a* would be infinitesimal. Hence, only an infinitesimal amount of work would be required to drive the machine; so again no finite change would be produced in the auxiliary system that supplied this work.

If the gas were cooled reversibly by letting heat flow from it to an infinite series of reservoirs, each at a temperature infinitesimally lower than that of the gas, the gas and the reservoirs could be returned to their initial states with the help again of a reversible refrigerating machine. As before, the work required to drive the machine would be infinitesimal; so no finite change would be produced in the auxiliary system supplying this work.

Note that, if the gas were heated or cooled by letting heat flow from or to a reservoir whose temperature differed from that of the gas by a finite amount, the process would not be reversible; and the gas and the reservoir could be returned to their initial states only at the expense of a finite amount of work supplied by some auxiliary system. This is true because the best possible refrigerating machine that could be used would be a reversible machine, and its coefficient of performance would be finite since the denominator of Eq. 38*a* would be finite.

It may be concluded from the foregoing discussion that, if any system undergoes a reversible process during which it is compressed, expanded, heated, or cooled, both the system and its surroundings can be restored to their initial states without producing more than an infinitesimal change in any auxiliary system. The converse of this statement can also be shown to be true: namely, if at the completion of a process both the system and its surroundings can be restored to their initial states without requiring more than an infinitesimal change in any auxiliary system, then conditions (*a*) and (*b*) listed at the beginning of this article will have been satisfied. Therefore, a **reversible process** may also be defined as follows: *A process is reversible if, at the completion of the process, both the system and its surroundings can be restored to their initial states without producing more than an infinitesimal change in any auxiliary system.** The reason why reversible processes are called "reversible" is now apparent.

SYMBOLS

c.p.	coefficient of performance
J	mechanical equivalent of heat
Q	a quantity of heat
T	absolute temperature
W	a quantity of work
η	thermal efficiency

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* The usual procedure is to define a reversible process in this way and then to show that such a process can involve no fluid friction, mechanical friction, or flow of heat across a finite interval of temperature. If none of these things is to be involved, however, the process must take place in such a way that it satisfies the definition of a reversible process given in Art. 20.

The foregoing procedure has been reversed in this text to simplify the explanation of *how* a reversible process could be made to take place and to make clearer some of the reasons *why* such processes are of interest.

Chapter 8

THE THERMODYNAMIC SCALE OF TEMPERATURE

It was proved in the preceding chapter that all reversible heat engines receiving heat from the same constant temperature source and rejecting heat to the same constant temperature receiver have the same thermal efficiency. This fact can be used to establish a scale of temperature which is independent of the thermometric fluid and is an *absolute* scale; that is, a scale whose zero value is the lowest temperature which can possibly be attained. This scale is of great importance in thermodynamics since it leads to the concept of a property called "entropy." It is discussed in the present chapter.

64. Thermometers and Temperature Scales: A Recapitulation. Let us review briefly the material discussed in Arts. 5, 6, and 7. As explained in these articles, any system may be used as a thermometer if one of its properties, called its *thermometric property*, assumes a unique value when the system is brought into equilibrium with any one of a group of other systems that are not in equilibrium with each other. For example, a given mass of any gas kept at constant pressure may be used as a thermometer because (*a*) the volume of the gas will assume a different value each time the gas is brought into equilibrium with a different one of a group of systems not in equilibrium with each other, and (*b*) the volume of the gas will always return to the same value whenever the gas is brought back into equilibrium with the same one of these systems. After some system is selected as a thermometer, a scale of temperature may next be defined by specifying arbitrarily a relation between the thermometric property of the system and the temperature. For example, if the system selected is a given mass of some gas kept at constant pressure, a scale of temperature may be defined by specifying arbitrarily that

$$t = a + bV,$$

where a and b are constants. The value of these constants may be fixed by specifying further that the temperature of melting ice shall be, say, 32° and the temperature of steam condensing at atmospheric pressure shall be, say, 212°. In effect, this is how the Fahrenheit scale of temperature is defined.

As explained in Art. 6, if several different thermometers are used to measure the temperature of any one system, it is found that the numerical value obtained depends upon (*a*) the kind of thermometer used, (*b*) the kind of thermometric fluid used, and (*c*) the method arbitrarily selected for specifying the relation between the thermometric property of the fluid and the temperature. The thermometers can be made to read the same only by selecting one of them as a standard and making the scales on the others agree with the scale on this standard thermometer.

The absolute perfect gas scale of temperature is of particular interest in the present chapter. For this reason, let us review briefly the definition of this scale. As explained in Art. 7, if one pound of any actual gas is kept at constant pressure and another pound of the same gas is kept at constant volume, the relation between the volumes of the first pound and the pressures of the second, when the two masses are brought into thermal equilibrium with two bodies 1 and 2, is found to be

$$\left(\frac{v_1}{v_2}\right)_p \approx \left(\frac{p_1}{p_2}\right)_v .$$

A *perfect gas* is defined as one for which these two ratios are exactly equal; that is, for a perfect gas,

$$\left(\frac{v_1}{v_2}\right)_p = \left(\frac{p_1}{p_2}\right)_v . \tag{1}$$

The absolute perfect gas scale temperatures of bodies 1 and 2 are denoted by T_1 and T_2, and the ratio of these temperatures is, by definition, equal to either of the foregoing ratios; that is,

$$\frac{T_1}{T_2} = \left(\frac{v_1}{v_2}\right)_p = \left(\frac{p_1}{p_2}\right)_v . \tag{2}$$

The numerical values of T_1 and T_2 could be fixed by selecting arbitrarily some value for the temperature of any one reference system. For example, a value could be assigned to the temperature of melting ice. The numerical value of T_1 could then be determined by (*a*) measuring the ratio of the volumes $(v_1/v_{\text{ice}})_p$ which a perfect gas, kept at constant pressure, would attain when brought into thermal equilibrium first with simply that the direction of heat flow was reversed, a reversible engine operating between a reservoir at any positive temperature T_1 and a reservoir at any negative temperature T_2 would receive heat from *both* reservoirs. The engine would deliver work equivalent to the sum of these two quantities of heat, as shown in Fig. 42*a*. However, since the reservoir at T_2 is at a lower temperature than the reservoir at T_1, heat Q_2 could also flow directly from the high to the low temperature reservoir, as shown in Fig. 42*b*. Since the low temperature reservoir would both gain and lose heat Q_2, it would not be affected and could be dispensed with. In effect, the reversible engine would be supplied with heat Q_1 plus heat Q_2 from the reservoir at T_1 and would convert all this heat into work. Now suppose that this work were used to operate an ordinary refrigerating machine. This machine could absorb heat Q_3 from any reservoir whose temperature is lower than T_1 and could deliver heat $Q_1 + Q_2 + Q_3$ to the reservoir at T_1. The net result would be that the two machines *together* would absorb heat Q_3 from a low temperature reservoir and deliver it to a higher temperature reservoir. This result is impossible, however, since it is a direct violation of the Second Law. The assumption that T_2 can be negative is not permissible, therefore, and the proposition is proved.

The reason why the absolute perfect gas scale of temperature is called an "absolute" scale is now apparent.

68. The International Temperature Scale. As explained in Art. 66, temperatures on the thermodynamic scale can be measured by means of a perfect gas thermometer. Since no perfect gas exists, however, it is necessary to use an actual gas in the thermometer and to take sufficient data to permit extrapolation to zero pressure. Because of this, and because of the necessity of making certain other corrections, the accurate determination of the temperature of any system on the thermodynamic scale is quite difficult and may require months of careful work. The thermodynamic temperatures at which certain substances melt or boil at atmospheric pressure have been accurately determined by such work, however, and these temperatures are now used as basic fixed points.

In order to have a scale of temperature that can be easily used to calibrate industrial and scientific thermometers, the **international temperature scale** was adopted by thirty-one nations at the Seventh General Conference of Weights and Measurements in 1927. By definition, this scale agrees with the thermodynamic scale at the aforementioned basic fixed points. Intervening temperatures are defined by specifying the kind of thermometer and the interpolation formula to be used in determining them. The difference between the international scale and the thermodynamic scale at any such intervening temperatures is negligible for most purposes.

GENERAL PROBLEMS

235. The temperature of melting ice is 0° on the Réaumur scale, and the temperature of steam condensing at atmospheric pressure is 80°. What temperature on the Réaumur scale corresponds to absolute zero on the thermodynamic scale?

Ans. −218.5° Réaumur.

236. If the temperatures of melting ice and of steam condensing at atmospheric pressure were arbitrarily set equal to 20° and 90° respectively on the perfect gas scale, what constant would have to be added to temperatures on this scale to convert them to absolute perfect gas scale temperatures?

237. Suppose that a perfect gas scale of temperature is defined by assigning (*a*) a value of 10° to the temperature of melting ice, and (*b*) a value of 50° to the temperature of a system whose thermodynamic temperature is known by experiment to be 1.200 times that of melting ice. At what temperature on this scale does steam at atmospheric pressure condense? *Ans.* 83.24°.

238. Liquid mercury freezes at −38°F, and a certain form of sulfur melts at 235°F. Suppose that a thermodynamic scale of temperature is defined by specifying that the difference between the temperatures of the melting sulfur and freezing mercury shall be 100°. What temperature on this scale corresponds to 70°F?

SYMBOLS

p	pressure
Q	a quantity of heat
t	temperature
T	absolute temperature on the perfect gas scale
v	specific volume
V	volume
η	thermal efficiency
θ	absolute temperature on the thermodynamic scale

Chapter 9

ENTROPY

The thermodynamic scale of temperature discussed in the preceding chapter is of interest, not only because it is an absolute scale and is independent of the thermometric fluid used, but because it permits an additional property called "entropy" to be discovered. This property is useful in engineering thermodynamics principally because it permits problems involving either isothermal or adiabatic reversible processes to be solved easily. In the general science of thermodynamics, the property entropy is even more useful because it is a criterion of equilibrium. It can be shown that the entropy of any isolated system can never decrease and therefore that, when the system reaches a state where its entropy is a maximum, the system must be in a state of equilibrium. Although this so-called "entropy principle" is not required for the usual engineering applications of thermodynamics, it is discussed briefly in the last article of this chapter, and an application of interest to the engineer is presented.

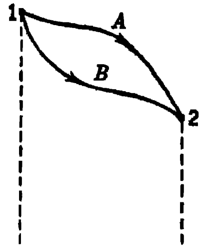

FIG. 43.

69. The Integral of dQ/T. Suppose that a system changes from any state 1 to any other state 2 first by some reversible or internally reversible non-flow process A and then by some other reversible or internally reversible non-flow process B, as shown in Fig. 43. Let us consider how the heat added to the system during process A compares with the heat added during process B. To make this comparison, we need only note that the heat added to the system during any non-flow process is equal to the increase in its internal energy plus the work it does, that is, by the non-flow energy equation,

$$Q = u_2 - u_1 + W \qquad (12)$$

Therefore, since the change $u_2 - u_1$ in its internal energy is the same for both processes but the work done (represented by the areas under the curves A and B) is different, it follows that the heat added to the system must also be different for the two processes. Thus, it is evident that the heat added to any system during any process by which the system may change from one given state to another is not, in general, a constant but depends upon the process.

In view of the foregoing statement, it is rather surprising to learn that the following is true: If a system changes from any given initial state to any given final state by a reversible or internally reversible process, if the heat added during the process is divided into a large number of small parts, if each of the numbers denoting the size of these parts is divided by the thermodynamic temperature of the system at the time that the part is added, and if the sum of these quotients is determined, then *exactly the same total is obtained regardless of the reversible or internally reversible process by which the system changes from the given initial state to the given final state.* Stated mathematically,

$$\frac{dQ_a}{T_a} + \frac{dQ_b}{T_b} + \frac{dQ_c}{T_c} + \cdots = \text{a constant},$$

where dQ_a = the first infinitesimal amount of heat added to the system (a negative number if the heat is absorbed from the system),

T_a = the thermodynamic temperature of the system at the time that heat dQ_a is added,

dQ_b = the second infinitesimal amount of heat added to the system,

T_b = the thermodynamic temperature of the system at the time that heat dQ_b is added,

and so forth. Making use of the calculus, we may write this equation in the simpler form,

$$\int_1^2 \frac{dQ}{T} = \text{a constant}.$$

Before proving this statement, let us consider a numerical example.

Suppose that 1 lb of air initially at 100 psia and 70°F changes to 60 psia and 150°F by the three reversible non-flow processes shown in Fig. 44. Process 1–a–2 consists of a constant pressure expansion followed by a constant volume cooling; process 1–b–2, an isothermal expansion followed by a constant pressure expansion; and process 1–c–2, an adiabatic expansion followed by a constant volume heating. Let us determine the value of the integral of dQ/T for each of these three reversible processes.

Consider first the constant pressure expansion from 1 to a. To perform the integration of dQ/T for this process, we must express the heat dQ added to the system in terms of the temperature T. We can do this by making use of the specific heat at constant pressure. Thus, from the definition of c_p,

$$dQ = c_p\,dT.$$

Therefore,

$$\int_1^a \frac{dQ}{T} = \int_1^a \frac{c_p\,dT}{T}.$$

Assuming for the sake of simplicity that c_p for air is a constant, we may write

$$\int_1^a \frac{dQ}{T} = c_p \int_1^a \frac{dT}{T}$$

$$= c_p \ln\left(\frac{T_a}{T_1}\right).$$

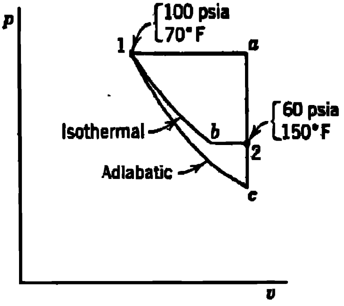

FIG. 44.

To determine the temperature T_a, let us first calculate the specific volumes v_1 and v_2 from the characteristic equation $pv = RT$:

$$v_1 = \frac{53.3 \times 530}{100 \times 144}$$

$$= 1.96 \text{ cu ft per lb},$$

and

$$v_2 = \frac{53.3 \times 610}{60 \times 144}$$

$$= 3.76 \text{ cu ft per lb}.$$

Hence, since $p_a = p_1 = 100$ psi [illegible] $= v_2 = 3.76$ cu ft per lb,

$$T_a = \frac{1\text{[illegible]}4 \times 3.76}{53.3}$$

$$= 1018\text{°F abs.}$$

We can now evaluate the foregoing expression for $\int_1^a dQ/T$:

$$\int_1^a \frac{dQ}{T} = 0.241 \ln\left(\frac{1018}{530}\right)$$

$$= 0.157 \text{ Btu per lb deg F.}$$

Consider next the constant volume process from a to 2. Making use of the specific heat at constant volume and assuming it to be a constant, we may write

$$\int_a^2 \frac{dQ}{T} = \int_a^2 \frac{c_v\,dT}{T}$$

$$= c_v \ln\left(\frac{T_2}{T_a}\right)$$

$$= 0.172 \ln\left(\tfrac{610}{1018}\right)$$

$$= -\ 0.088 \text{ Btu per lb deg F.}$$

Finally, therefore, the integral of dQ/T for the reversible process 1–a–2 is

$$\int_1^2 \frac{dQ}{T} = 0.157 - 0.088$$

$$= 0.069 \text{ Btu per lb deg F.}$$

Now consider the reversible process 1–b–2. It is evident that, for the isothermal part of this process,

$$\int_1^b \frac{dQ}{T} \qquad \frac{\int_1^b dQ}{T}$$

$$\frac{Q_{1-b}}{T}$$

To determine Q_{1-b}, we may use Eqs. 25b and 26:

$$Q_{1-b} = \frac{RT}{J} \ln\left(\frac{p_1}{p_b}\right)$$

$$= \frac{53.3 \times 530}{778} \ln\left(\frac{100}{60}\right)$$

$$= 18.6 \text{ Btu per lb.}$$

Hence,

$$\int_1^b \frac{dQ}{T} = \frac{18.6}{530}$$

$$= 0.035 \text{ Btu per lb deg F.}$$

The integral of dQ/T for the constant pressure expansion from b to 2 is

$$\int_b^2 \frac{dQ}{T} = c_p \ln\left(\frac{T_2}{T_b}\right)$$

$$= 0.241 \ln\left(\tfrac{610}{530}\right)$$

$$= 0.034 \text{ Btu per lb deg F.}$$

Therefore, the integral of dQ/T for the reversible process 1–b–2 is

$$\int_1^2 \frac{dQ}{T} = 0.035 + 0.034$$

$$= 0.069 \text{ Btu per lb deg F.}$$

This is the same value as was obtained for process 1–a–2.

Finally, consider the reversible process 1–c–2. Since the heat added to the air during the adiabatic expansion from 1 to c is zero,

$$\int_1^c \frac{dQ}{T} = 0.$$

The integral of dQ/T for the constant volume process from c to 2 is

$$\int_c^2 \frac{dQ}{T} = c_v \ln\left(\frac{T_2}{T_c}\right).$$

Before this expression can be evaluated, the temperature T_c must be determined. This can be done by Eq. 28a:

$$= T_1 \left(\frac{v_1}{v_c}\right)^{k-1} \tag{28a}$$

$$= 530 \left(\frac{1.96}{3.76}\right)^{0.4}$$

$$= 409\text{°F abs.}$$

Hence,

$$\int_c^2 \frac{dQ}{T} = 0.172 \ln\left(\frac{610}{409}\right)$$

$$= 0.069 \text{ Btu per lb deg F.}$$

Therefore, for the reversible process 1–c–2,

$$\int_1^2 \frac{dQ}{T} = 0 + 0.069$$

$$= 0.069 \text{ Btu per lb deg F.}$$

Thus, the same value is obtained for the integral of dQ/T for all three reversible processes between states 1 and 2.

PROBLEMS

239. In the numerical example presented in the preceding article, the integral of dQ/T is evaluated for three reversible processes by which a pound of air initially at 100 psia and 70°F might change to a final state in which it is at 60 psia and 150°F. Evaluate this integral for each of the following reversible processes by which the air might change from the given initial to the given final state:

(*a*) The air is cooled at constant volume until its pressure drops to 60 psia and is then heated at constant pressure to the given final state.

(*b*) The air is expanded isothermally to its final volume and is then heated at constant volume to the given final state.

(*c*) The air is expanded adiabatically to 60 psia and is then heated at constant pressure to the given final state.

(*d*) The air is compressed adiabatically until its temperature reaches 150°F and is then expanded isothermally to the given final state.

(*e*) The air is expanded isothermally to a state such that an adiabatic compression brings it to the given final state.

240. A pound of saturated steam at 14.7 psia is changed into water at 14.7 psia and 100°F. Evaluate the integral of dQ/T for the following two reversible processes by which the system might change from its initial to its final state:

(*a*) The steam is first condensed and the condensate is then cooled to 100°F, the pressure being kept constant throughout the entire process. Assume that c_p for water is 1.00 Btu per lb deg F.

(*b*) The steam is first expanded adiabatically until its temperature drops to 100°F, it is then condensed at constant pressure, and finally the condensate is compressed isothermally to 14.7 psia. (*Note:* By constructing a *u-v* diagram for the adiabatic reversible expansion it is found that at the end of the process $v = 307$ cu ft per lb and $u = 925$ Btu per lb. For the isothermal compression of the water, Q is virtually zero.)

241. A pound of saturated steam at 340°F is changed into superheated steam at 40 psia and 500°F. Evaluate the integral of dQ/T for the following two reversible processes by which the steam might change from its initial to its final state:

(*a*) The steam is heated at constant pressure to 500°F and is then expanded isothermally to 40 psia.

(*b*) The steam is expanded isothermally to 40 psia and is then heated at constant pressure to 500°F. (*Suggestion:* Determine an average c_p for each of the constant pressure processes by finding $\Delta h/\Delta t$.)

242. Suppose that a perfect gas has a molecular weight of 28 and a specific heat at constant volume which depends upon the temperature in accordance with the equation

$$c_v = 0.200 + 0.3 \times 10^{-6}T^2,$$

where c_v and T have the units British thermal units per pound degree Fahrenheit and degrees Fahrenheit absolute. Suppose further that a pound of this gas initially at 50 psia and 300°F changes to a state in which it is at 20 psia and 100°F. Evaluate the integral of dQ/T for the following two reversible processes by which the gas might change from its initial to its final state:

(*a*) The gas is cooled at constant volume until its pressure drops to 20 psia and is then heated at constant pressure to its final state.

(*b*) The gas is expanded isothermally to its final volume and is then cooled at constant volume to its final state.

70. The Integral of dQ/T (Continued). It was stated in the preceding article that, for all reversible or internally reversible processes by which a system can change from one given state to another,

$$\int \frac{dQ}{T} = \text{a constant.}$$

This statement will now be proved.

Consider any two states 1 and 2 of the system, and let *A* and *B* be two reversible or internally reversible processes by which the system can change from one state to the other. These two states and the processes connecting them might appear on a pressure-volume diagram as shown in Fig. 45*a*.* First, let the two processes be divided into a number of segments by a series of reversible adiabatic lines, as shown in Fig. 45*b*. Then let each of these segments be replaced by a reversible isothermal process that satisfies the following requirements: (*a*) the heat added to the system during the isothermal process is equal to the heat added during the segment it replaces, and (*b*) the temperature during the isothermal process is the same as that of the system at some time during the segment. That it is always possible to replace each segment by such a reversible isothermal process (if *A* and *B* are reversible or internally reversible processes) will be shown at the end of the article.

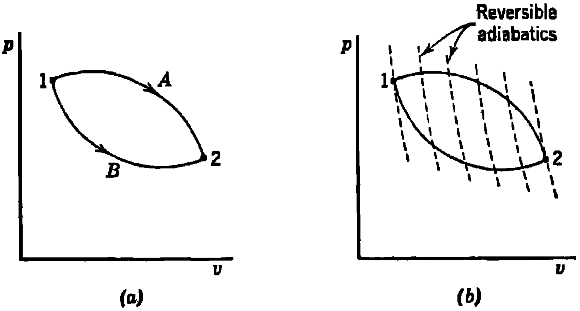

FIG. 45.

Now consider the two zigzag processes between states 1 and 2 shown by the heavy lines in Fig. 45*c*. We need to show that the integrals of dQ/T for these processes are equal. To do this, we note that, if the system were to undergo the cycle *abxya* shown in Fig. 45*d* (process x–y being a reversible isothermal process at any temperature T_x lower than T_a or $T_{a'}$), the system would perform a Carnot cycle. Therefore we may write, by Eq. 40,

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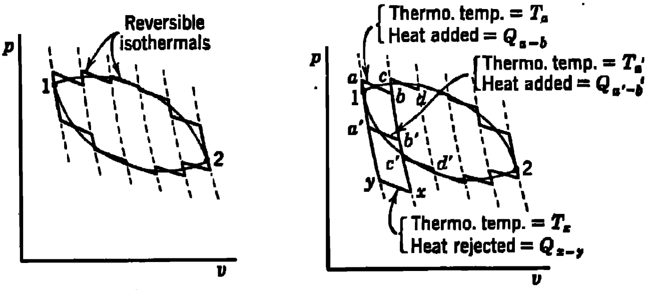
* If the system is flowing, it must be considered from the viewpoint of an observer who moves with it. The system will then appear to undergo a non-flow process, which can be represented by a curve on a pressure-volume diagram.

$$\frac{T_a}{T_x} = \frac{Q_{a-b}}{Q_{x-y}},$$

where T_a and T_x represent the thermodynamic temperatures of the system during the isothermal processes a–b and x–y, and Q_{a-b} and Q_{x-y} represent the amounts of heat added to and rejected by the system during these processes. Similarly, if the system were to undergo the cycle *a′b′xya′*, it would again perform a Carnot cycle, and therefore we may write



(*c*) (*d*)

FIG. 45 (*continued*).

$$\frac{T_{a'}}{T_x} = \frac{Q_{a'-b'}}{Q_{x-y}}.$$

Rearranging the two foregoing equations, we get

$$\frac{Q_{a-b}}{T_a} = \frac{Q_{x-y}}{T_x}, \quad \text{and} \quad \frac{Q_{a'-b'}}{T_{a'}} = \frac{Q_{x-y}}{T_x};$$

whence

$$\frac{Q_{a-b}}{T_a} = \frac{Q_{a'-b'}}{T_{a'}}.$$

It is evident that an equation similar to the last can be derived for each of the pairs of isothermals bounded by adjoining reversible adiabatic lines. If we add all these equations, we obtain

$$\frac{Q_{a-b}}{T_a} + \frac{Q_{c-d}}{T_c} + \cdots = \frac{Q_{a'-b'}}{T_{a'}} + \frac{Q_{c'-d'}}{T_{c'}} + \cdots. \tag{a}$$

Since the heat added to the system during the adiabatic portions of the two zigzag paths is zero, the integral of dQ/T for each of these portions is also zero. Therefore, the left side of Eq. (a) is equal to the integral of dQ/T *for the entire zigzag path replacing process* A, and the right side is equal to this integral *for the entire zigzag path replacing process* B.

Now the two zigzag processes shown in Fig. 45d may be made to approach processes A and B as closely as we please by increasing indefinitely the number of segments into which processes A and B are divided. Moreover, when these segments become infinitesimal, the ratio dQ/T for each isothermal process is equal to the ratio dQ/T for the segment of the original process that it replaces. This is true because (a) the heat dQ added to the system is the same for both these infinitesimal processes, and (b) the temperature of the system during the segment of the original process becomes the same as the temperature T during the isothermal process. Hence, in the limit we may write Eq. (a) in the form

$$ {}_A\int_1^2 \frac{dQ}{T} = {}_B\int_1^2 \frac{dQ}{T}, $$

where subscripts A and B indicate that the integration is to be carried out over processes A and B respectively. Since it can be shown in exactly the same way that the integral of dQ/T for any other reversible or internally reversible process between states 1 and 2 is also equal to ${}_A\int_1^2 dQ/T$, it follows that, for any reversible or internally reversible process between these two states,

$$ \int_1^2 \frac{dQ}{T} = \text{a constant.} $$

Thus, the proposition is proved.

It was assumed in the foregoing proof that any curve representing a reversible or internally reversible process on a pressure-volume diagram can be divided into an infinite number of infinitesimal segments by a series of adiabatic reversible lines. To show that this is always possible, we need only prove that such adiabatic reversible lines can never intersect; for then, by drawing a sufficient number of them, any curve (except a reversible adiabatic, for which the integral of dQ/T is zero) can be divided into infinitesimal segments. To prove that adiabatic reversible lines on a pressure-volume diagram cannot intersect, let us assume that two such lines do intersect, as shown in Fig. 46. The system could then be made to undergo a reversible cycle consisting of an isothermal expansion from 1 to 2, an adiabatic expansion from 2 to 3, and an adiabatic compression from 3 back to 1. Heat would be supplied to the system during the isothermal process from 1 to 2; and, since no heat would be rejected by the system during the rest of the cycle, all the heat supplied to it would be converted into work (see Eq. 10). Now this work could be used to operate a refrigerating machine that absorbed heat from the reservoir used to supply the engine and rejected heat to a reservoir at some higher temperature. But the engine and the refrigerating machine together would constitute a machine, operating in a cycle, that absorbed heat from one reservoir, rejected heat to another at a higher temperature, and produced no other effect. Since this is a direct violation of the Second Law, such a machine is impossible; and the assumption that adiabatic reversible lines cross is therefore not permissible.

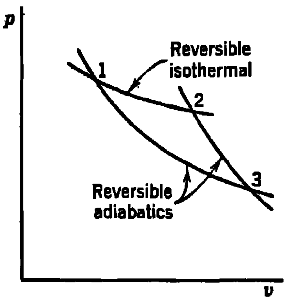
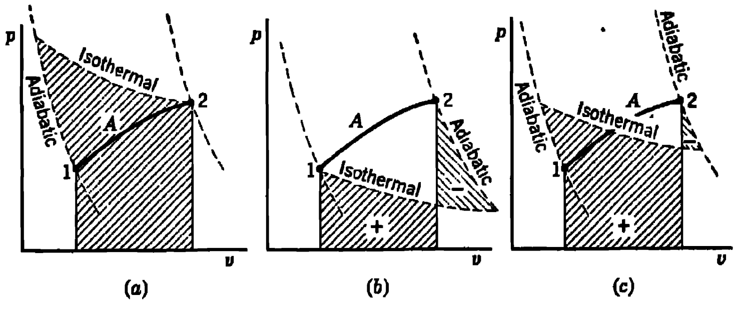


FIG. 46.

It was also assumed in the foregoing proof that a small segment of any reversible or internally reversible process can be replaced by a zigzag process made up of an adiabatic, an isothermal, and another adiabatic process, all either reversible or internally reversible. The isothermal part of the zigzag process must be such that (*a*) the heat added to the system during the isothermal process is equal to the heat added during the segment of the original process, and (*b*) the temperature during the isothermal process is the same as that of the system at some time during the segment of the original process.

To show that such a zigzag process is always possible, let us consider the reversible or internally reversible process A which is shown in Fig. 47*a*. This process connects two states, 1 and 2, which are close together. If the system were compressed adiabatically from state 1 to the temperature t_2 and were then expanded isothermally to state 2, the net work done by the system would be represented by the shaded area shown in Fig. 47*a*. It would be *greater* than the work done during process A (represented by the area under curve A). On the other hand, if the system were expanded isothermally from state 1 and were then compressed adiabatically to state 2, the net work done by the system would be represented by the difference between the shaded areas in Fig. 47*b*. The net work done would be *less* than the work done during process A. It is evident, therefore, that a zigzag process—consisting of an adiabatic, an isothermal, and another adiabatic, as shown in Fig. 47*c*—can be found



(*a*) (*b*) (*c*)

FIG. 47.

for which the net work done by the system is *equal* to the work done during process A. But, if the work done is the same for the zigzag process and for process A, the heat added must also be the same for both. This is true because the change in the internal energy of the system is the same for both, and the heat added is equal to the increase in the internal energy plus the work done; that is, by the non-flow energy equation,

$$Q = u_2 - u_1 + \frac{W}{J}. \tag{12}$$

It is also evident that the isothermal portion of the zigzag process must intersect curve A; that is, the temperature during the isothermal process must be the same as that of the system at some time during process A. Thus, it has been shown that process A can be replaced by a zigzag process satisfying the requirements stated in the preceding paragraph. Reflection will show that this can be done, not only for process A, but for any reversible or internally reversible process connecting two states that are close together.

It has been specified that the process for which the integral of dQ/T is evaluated must be either a reversible or an internally reversible process. This restriction is necessary for the following reasons: First, the temperature of the system must be uniform (but not constant) at all times during the process if the quotient dQ/T is to have any meaning. Second, the work done by the system must equal the integral of $p\,dv$ if each segment of the process is to be replaceable by a zigzag process of the kind specified. A zigzag process for which the work and heat are the same as for process A in Fig. 47, for example, can always be found; but its isothermal part *must* intersect curve A only if the work done during A is equal to the integral of $p\,dv$ (that is, if the work is represented by the area under curve A). It is only during either a reversible or an internally reversible process that *both* the temperature of a system remains constant and the work done by it equals the integral of $p\,dv$. Hence, the process for which the integral of dQ/T is evaluated must be one of these two kinds of processes.

71. Entropy. As explained in Art. 28, a characteristic of any property of a system is the fact that the change in its value between any two states does not depend upon the process by which the system changes from one state to the other. Conversely, any quantity that depends only upon the initial and final states of a system, and not upon the process connecting these states, is the change in a property. *Since the integral of dQ/T is such a quantity, this integral is the change in a property.* We call the property **entropy** and denote it by the symbol s. Thus, the change in the entropy of a system between any two states 1 and 2 is defined as

$$s_2 - s_1 = \int_1^2 \frac{dQ}{T}, \tag{41}$$

where the integral may be evaluated for *any reversible or internally reversible process* connecting states 1 and 2.

It is important to note that, if a system changes from one state to another by a process that is irreversible, the change $s_2 - s_1$ in its entropy *cannot* be calculated by finding the integral of dQ/T for the irreversible process. The value of $s_2 - s_1$ can be determined, however, by evaluating this integral for *any* reversible process by which the system could change from the given initial to the given final state. Note also that $s_2 - s_1$ may be positive or negative, depending upon whether heat is added to or absorbed from the system during the reversible process for which the integral is evaluated. Finally, note that if dQ is expressed in Btu per pound and T is expressed in degrees Fahrenheit absolute, the units of entropy are Btu per pound degree Fahrenheit.

Although a physical concept of entropy cannot be readily obtained, this property is of great value in the science of thermodynamics. It is of particular value to the engineer in solving problems involving isothermal or adiabatic processes that are also either reversible or internally reversible. Before considering how such problems can be solved with the aid of this property, let us determine how the difference between the entropy at any two states of (*a*) a perfect gas, and (*b*) a liquid, vapor, or liquid-vapor mixture, can be found.

72. $s_2 - s_1$ for a Perfect Gas. Consider an infinitesimal part of any reversible or internally reversible non-flow process by which a perfect gas can change from any state 1 to any other state 2. For this infinitesimal part of the process, the non-flow energy equation may be written

$$dQ = du + \frac{dW}{J}. \tag{12a}$$

From Eq. 17 it follows that for a perfect gas $du = c_v\,dT$, and from Eq. 7 it follows that for a reversible or internally reversible non-flow process $dW = p\,dv$. Introducing these expressions for du and dW into Eq. 12*a* gives

$$dQ = c_v\,dT + \frac{p\,dv}{J}.$$

Therefore, the change ds in the entropy of the gas during this infinitesimal part of the process is, by Eq. 41,

$$\begin{aligned} ds &= \frac{dQ}{T} \\ &= c_v\frac{dT}{T} + \frac{p}{T} \times \frac{dv}{J}; \end{aligned}$$

or, since from the characteristic equation of a perfect gas $p/T = R/v$,

$$ds = c_v\frac{dT}{T} + \frac{R}{J} \times \frac{dv}{v}.$$

The difference between the entropy of the gas in its initial and final states can be determined by integrating this equation. *If c_v for the gas is a constant*, the integration yields

$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + \frac{R}{J}\ln\left(\frac{v_2}{v_1}\right). \tag{42}$$

Alternative forms of this equation, in terms of the initial and final temperatures and pressures and in terms of the initial and final pressures and volumes, are stated in Problem 244.

Equation 42, or its alternative forms, may be used to determine the difference between the entropies of a perfect gas for which c_v is a constant at *any* two states 1 and 2. If the volume of the gas at state 2 is the same as the volume at state 1, Eq. 42 reduces to

$$(s_2 - s_1)_v = c_v \ln\left(\frac{T_2}{T_1}\right). \tag{43a}$$

If the pressure of the gas at state 2 is the same as the pressure at state 1, Eq. 42*a* reduces to

$$(s_2 - s_1)_p = c_p \ln\left(\frac{T_2}{T_1}\right). \tag{43b}$$

Or, if the temperature of the gas at state 2 is the same as the temperature at state 1, Eqs. 42 and 42*a* reduce to

$$\left.\begin{aligned}(s_2 - s_1)_T &= \frac{R}{J}\ln\left(\frac{v_2}{v_1}\right)\\ &= \frac{R}{J}\ln\left(\frac{p_1}{p_2}\right).\end{aligned}\right\} \tag{43c}$$

The foregoing special forms can also be derived directly from Eq. 41. Thus, if the initial and final volumes of the gas are the same, the gas could change from one state to the other by a constant volume reversible process. Since $dQ = c_v\,dT$ for such a process, Eq. 41 may be written

$$(s_2 - s_1)_v = \int \frac{c_v\,dT}{T};$$

and therefore, if c_v for the gas is a constant,

$$(s_2 - s_1)_v = c_v \ln\left(\frac{T_2}{T_1}\right).$$

Similarly, if the initial and final pressures of the gas are the same, the gas could change from one state to the other by a constant pressure reversible process. Hence,

$$(s_2 - s_1)_p = \int \frac{c_p\,dT}{T};$$

and, if c_p for the gas is a constant,

$$(s_2 - s_1)_p = c_p \ln\left(\frac{T_2}{T_1}\right).$$

Finally, if the initial and final temperatures of the gas are the same, the gas could change from one state to the other by an isothermal reversible process. For such a process Eq. 41 reduces to

$$(s_2 - s_1)_T = \frac{Q}{T},$$

where Q denotes the heat added to the gas during the process. By Eqs. 25*b* and 26,

$$Q = \frac{RT}{J}\ln\left(\frac{v_2}{v_1}\right) = \frac{RT}{J}\ln\left(\frac{p_1}{p_2}\right),$$

and therefore

$$(s_2 - s_1)_T = \frac{R}{J}\ln\left(\frac{v_2}{v_1}\right) = \frac{R}{J}\ln\left(\frac{p_1}{p_2}\right).$$

Note again that, whereas all the foregoing expressions for $s_2 - s_1$ were derived by assuming that the gas changes from state 1 to state 2 by a reversible or an internally reversible process, these expressions are correct regardless of the nature of the actual process by which the change takes place.

ILLUSTRATIVE PROBLEM

243. One pound of air initially at 40 psia and having an initial volume of 5 cu ft undergoes a non-flow process during which 90 Btu of heat are added to it and 32,000 ft-lb of work are done by it. If at the end of the process the air is at 25 psia, by how much has its entropy changed?

Solution. The difference between the initial and final entropies of the air can be calculated by Eq. 42 (if it is assumed that air is a perfect gas and that its c_v is constant) after the initial and final temperatures and specific volumes of the air have been determined.

The initial temperature can be determined from the characteristic equation $pv = RT$:

$$T_1 = \frac{40 \times 144 \times 5}{53.3}$$

$$= 540\text{°F abs.}$$

The final temperature can be determined by Eq. 17*a*,

$$u_2 - u_1 = c_v(T_2 - T_1), \tag{17a}$$

after the increase in the internal energy of the air has been found by the non-flow energy equation. Thus,

$$u_2 - u_1 = Q - \frac{W}{J} \tag{12}$$

$$= 90 - \frac{32{,}000}{778}$$

$$= 48.9 \text{ Btu per lb.}$$

Hence, by Eq. 17*a*,

$$48.9 = 0.172(T_2 - 540),$$

whence

$$T_2 = 824°\text{F abs.}$$

The final specific volume can now be calculated from the characteristic equation:

$$v_2 = \frac{53.3 \times 824}{25 \times 144}$$

$$= 12.2 \text{ cu ft per lb.}$$

Finally, the change in the entropy of the air is, by Eq. 42,

$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + \frac{R}{J}\ln\left(\frac{v_2}{v_1}\right) \tag{42}$$

$$= 0.172 \ln\left(\frac{824}{540}\right) + \frac{53.3}{778}\ln\left(\frac{12.2}{5.0}\right)$$

$$= 0.133 \text{ Btu per lb deg F.}$$

PROBLEMS

244. For a perfect gas,

$$\frac{p_2v_2}{T_2} = \frac{p_1v_1}{T_1},$$

or

$$\left(\frac{p_2}{p_1}\right)\left(\frac{v_2}{v_1}\right) = \frac{T_2}{T_1}.$$

Hence,

$$\ln\left(\frac{p_2}{p_1}\right) + \ln\left(\frac{v_2}{v_1}\right) = \ln\left(\frac{T_2}{T_1}\right).$$

By combining the last equation with Eq. 42 and making use of the relation

$$c_p - c_v = \frac{R}{J},$$

show that the difference between the entropies at any two states of a perfect gas for which c_v is a constant is equal to

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - \frac{R}{J}\ln\left(\frac{p_2}{p_1}\right) \tag{42a}$$

or

$$s_2 - s_1 = c_v \ln\left(\frac{p_2}{p_1}\right) + c_p \ln\left(\frac{v_2}{v_1}\right). \tag{42b}$$

245. It was shown in Art. 46 that, if a perfect gas having constant values of c_v and c_p undergoes an adiabatic reversible process, the initial and final pressures and volumes of the gas are related by the equation

$$p_1v_1^k = p_2v_2^k, \tag{28c}$$

where $k = c_p/c_v$. Rederive this equation by making use of Eq. 42*b*.

246. Three pounds of air having an initial volume of 15 cu ft undergo a non-flow reversible process during which the pressure remains constant at 50 psia. If 25 Btu of work are done on the air, by how much does its entropy change?

Ans. −0.143 Btu per deg F.

247. Three pounds of a certain gas initially at 80°F are heated at constant volume. If adding 125 Btu of heat to the gas causes its pressure to increase by 50%, by how much does its entropy change?

248. A pound of air undergoes a non-flow reversible process during which its temperature remains constant at 70°F. What must be its initial pressure if during the process its volume increases by 10 cu ft and its entropy increases by 0.060 Btu per deg F?

Ans. 27.4 psia.

249. If air at 150 psia and 80°F flows through an insulated valve into a region where its pressure is 15 psia, by how much does its entropy change? Assume that the air enters and leaves the valve at low velocity.

250. Suppose that a perfect gas has a molecular weight of 39 and a specific heat at constant pressure which depends upon the temperature in accordance with the equation

$$c_p = 0.120 + \frac{10}{T},$$

where the units of c_p and of T are British thermal units per pound degree Fahrenheit and degrees Fahrenheit absolute. If a pound of this gas initially at 20 psia and 90°F changes to a state where its pressure and temperature are 60 psia and 220°F, by how much does its entropy change?

Ans. −0.0271 Btu per lb deg F.

73. Determination of the Entropies of Liquids, Vapors, and Liquid-Vapor Mixtures by Means of Tables of Properties.

Equations for determining the difference between the entropies at any two states of most of the fluids of importance in engineering have been developed either from Eq. 41 or from certain other relations.* These equations are so complex, however, that their use for ordinary engineering calculations is impractical. For this reason tables based on them have been prepared. Just as the Keenan and Keyes *Steam Tables* were used in Chapters 1 and 4 to illustrate how values of p, v, t, h, and u at any state can be determined when any two of these properties are known, the *Steam Tables* will now be used to illustrate how values of the entropy s at any state can be found.

The values of entropy listed in any table of thermodynamic properties are the *differences* between the entropy at each state of the fluid and the entropy at some arbitrarily chosen reference state. The values of entropy listed in the *Steam Tables*, for example, are the differences between the entropy at each state and the entropy of saturated water at 32°F: that is, the entropy of saturated water at 32°F is arbitrarily set equal to zero. Although the values listed are actually differences in entropy, they will be referred to simply as the "entropy" at each state. Since values of the entropy are found in the tables by exactly the same methods as are used in finding values of the specific volume, enthalpy, and internal energy, the method of finding them will be discussed only briefly.

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* The difference between the entropies at any two states of a system can be calculated by means of the so-called *Maxwell relations* if the p-v-T relation for the system is known.

The entropy of saturated water, denoted by s_f, and the entropy of saturated steam, denoted by s_g, are listed in both Tables 1 and 2 of the *Steam Tables*. These values may be read directly from these tables if either the saturation temperature or the saturation pressure is known.

The entropy of a mixture of saturated water and saturated steam can be readily determined by making use of the fact that the entropy of such a mixture is equal to the sum of the entropies of the water and steam present in the mixture. Thus, per pound of mixture,

$$s = xs_g + (1 - x)s_f, \tag{44}$$

where x denotes the quality of the mixture. This equation can be readily transformed into the following more convenient forms by making use of the quantity s_{fg}, which is defined as the difference between s_f and s_g and is listed in the tables:

$$s = s_f + xs_{fg} \tag{44a}$$

and

$$s = s_g - (1 - x)s_{fg}. \tag{44b}$$

The entropies of superheated steam are listed in Table 3 of the *Steam Tables*. They can be easily found if the pressure and temperature, or if either of these together with the specific volume or the enthalpy, are known. For example, the entropy of superheated steam at 400°F and having a specific volume of 9.500 cu ft per lb can be found by following down the 400°F column until the specific volume 9.500 cu ft per lb is reached. By interpolation, the entropy at this state is found to be 1.7283 Btu per lb deg F.

Sufficiently accurate values of the entropy of subcooled water can usually be found by assuming that the entropy of subcooled water is the same as the entropy of saturated water *at the same temperature*. (This is equivalent to assuming that the water is incompressible.) For example, the entropy of subcooled water at 400 psia and 200°F is approximately 0.2938 Btu per lb deg F (the value of s_f at 200°F, from

Table 1). The exact value can be found by means of Table 4. Thus, the difference between the entropy of subcooled water at 400 psia and 200°F and the entropy of saturated water at 200°F is, from Table 4,

$$(s - s_f) \times 10^3 = -0.47,$$

or

$$s = s_f - 0.00047.$$

Therefore, using the value $s_f = 0.29382$ Btu per lb deg F from the top of Table 4, we find that the exact value of the entropy is

$$s = 0.29382 - 0.00047$$

$$= 0.29335 \text{ Btu per lb deg F}.$$

Note that, if the entropy and either the pressure or the temperature at any state are known, it is easy to determine whether the fluid is a subcooled liquid, a mixture of saturated liquid and saturated vapor, or a superheated vapor. It can be done by comparing the given value of the entropy with the values of s_f and s_g at the given pressure or temperature. If the value is less than s_f, the fluid is a subcooled liquid; if it is between s_f and s_g, the fluid is a mixture; and, if it is greater than s_g, the fluid is superheated vapor.

PROBLEMS

251. Find the entropy per pound of H_2O corresponding to each of the following pairs of properties:

State	p, psia	t, °F	v, cu ft per lb	h, Btu per lb	s, Btu per lb deg F
(*a*)	800	600			?
(*b*)	800	300			?
(*c*)	400			1000	?
(*d*)	300			1400	?
(*e*)	200			200	?
(*f*)		400	1.500		?
(*g*)		400	6.000		?

252. Complete the following table for H_2O:

State	p, psia	t, °F	v, cu ft per lb	h, Btu per lb	s, Btu per lb deg F
(*a*)		300			1.4500
(*b*)		600			1.4500
(*c*)	300				0.3000

253. Assuming that c_p for water is 1.00 Btu per lb deg F, calculate (*a*) the value of s_f for saturated water at 14.7 psia, (*b*) the value of s_g for saturated steam at 14.7 psia, and (*c*) the value of s for superheated steam at 14.7 psia and 500°F. Note that, since virtually no heat must be removed when saturated water at 32°F is compressed isothermally to 14.7 psia, the entropy of water at 14.7 psia and 32°F is very nearly the same as that of saturated water at 32°F. Compare your results with the values listed in the *Steam Tables*. (*Suggestion:* Determine an average c_p for the superheated steam by finding $\Delta h/\Delta t$.)

254. A 6-cu-ft tank contains 2 lb of steam initially at 500°F. If the steam is cooled to 300°F, by how much does its entropy change?

Ans. −0.6427 Btu per lb deg F.

255. A pound of H_2O initially at 70°F undergoes a non-flow process during which its pressure remains constant at 60 psia. If 1200 Btu of heat are added to the system, by how much does its entropy increase?

74. The Analysis of Isothermal Reversible Non-Flow Processes.

In Art. 45 a method was developed for calculating the heat Q added to and the work W done by any system during an isothermal reversible or internally reversible non-flow process. The method is cumbersome, however, if the system is any fluid other than a perfect gas because it involves a graphical integration. Now that the property entropy has been introduced, a much simpler method of calculating Q and W for this type of process can be presented.

For an isothermal reversible or internally reversible non-flow process, the integration of Eq. 41 yields

$$s_2 - s_1 = \frac{Q}{T}.$$

Therefore, solving for Q, we get

$$Q = T(s_2 - s_1); \tag{45}$$

and, by the non-flow energy equation, we get

$$\frac{W}{J} = T(s_2 - s_1) - (u_2 - u_1). \tag{46}$$

By means of these equations, the heat added and the work done can be determined by finding only the initial and final entropies and internal energies of the system from a table of thermodynamic properties.

ILLUSTRATIVE PROBLEM

256. In Illustrative Problem 134 following Art. 45, the heat added and the work done were determined for 1 lb of steam undergoing a reversible non-flow expansion from a volume of 3 cu ft to a volume of 10 cu ft at a constant temperature of 300°F. Recalculate these quantities by making use of the entropy of the system.

Solution. Since the initial specific volume of the steam is between v_f and v_g at 300°F, the steam is initially a mixture. Its quality can be determined by Eq. 5*a*:

$$v = xv_g, \tag{5a}$$

or

$$3 = x_1 6.466,$$

whence

$$x_1 = 0.464.$$

The initial entropy of the system can now be calculated by Eq. 44*a*:

$$s = s_f + xs_{fg}, \tag{44a}$$

or

$$\begin{aligned} s_1 &= 0.4369 + 0.464 \times 1.1980 \\ &= 0.992 \text{ Btu per lb deg F.} \end{aligned}$$

Since the final specific volume of the steam is larger than v_g at 300°F, the steam in its final state is superheated. Following down the 300°F column in Table 3 until we reach the specific volume 10 cu ft per lb, we find by interpolation that the final entropy of the steam is

$$s_2 = 1.6877 \text{ Btu per lb deg F.}$$

Therefore, the heat added to the steam during the process is, by Eq. 45,

$$\begin{aligned} Q &= T(s_2 - s_1) \\ &= 760(1.6877 - 0.992) \\ &= 528 \text{ Btu per lb.} \end{aligned}$$

The initial and final internal energies of the steam were determined in Illustrative Problem 134 and were found to be 654.6 and 1104.3 Btu per lb respectively. Hence, the work done by the steam during the process is, by the non-flow energy equation,

$$W = 528 - (1104.3 - 654.6)$$

$$78 \text{ Btu per lb (or 60,600 ft-lb per lb).}$$

PROBLEMS

257. A mixture of saturated steam and saturated water is at 25 psia and has a specific volume of 10 cu ft per lb. If the system undergoes an isothermal reversible non-flow process during which its specific volume increases to 20 cu ft per lb, how much work does each pound of the system do? *Ans.* 45.0 Btu per lb.

258. If the system in the preceding problem were air instead of steam, how much work would each pound of air do?

259. A pound of steam initially at 200 psia and 560°F expands isothermally and reversibly to 40 psia. If the process is non-flow, how much heat does the steam absorb and how much work does it do? *Ans.* 190.3 Btu; 180.7 Btu.

260. A pound of air initially at 70 psia undergoes an isothermal reversible non-flow process. If its entropy increases by 0.105 Btu per lb deg F, and if its final volume is 15 cu ft, how much heat does the air absorb during the process?

261. A pound of steam initially at 20 psia and 340°F undergoes an isothermal reversible non-flow process during which 150,000 ft-lb of work are done on it. How much heat does the steam reject? *Ans.* 555 Btu.

262. A system consisting of 2 lb of saturated water and 3 lb of saturated steam occupies a volume of 7 cu ft. If the system were to undergo an isothermal reversible non-flow process, and if it absorbed 2000 Btu of heat during the process, how much work would it do?

75. The Analysis of Adiabatic Reversible Processes.

In Art. 46 a method was developed for determining the state of any system at the end of an adiabatic reversible or internally reversible non-flow process. For systems other than perfect gases, the method consists in determining the states through which the system passes by drawing a curve on a u-v diagram such that the slope of the curve is at all times equal to minus the pressure of the system (see Eq. 29). This method is cumbersome, however, because it is graphical. A much simpler method can be presented now that the property entropy has been introduced.

If any system undergoes an adiabatic reversible or internally reversible non-flow process, it follows from Eq. 41 that the change in its entropy is zero; that is,

$$s_2 - s_1 = 0.$$

Therefore, if any two properties of the system in its initial state are known, its entropy can be determined; and, since the final entropy is the same as the initial, only one additional property in the final state need be known to determine all the other properties.

Consider next an adiabatic reversible steady-flow process. As explained in Art. 49, if any fluid undergoes such a process, each element of the fluid—from the viewpoint of an observer who moves with the element—undergoes an adiabatic reversible *non-flow* process; and its entropy remains constant. Consequently, if any two properties of the fluid at the section where it enters the apparatus are known, its entropy can be determined; and only one additional property at any other section need be known to determine all the other properties at that section.

If either saturated or subcooled water is compressed adiabatically and reversibly, it will be subcooled in its final state. The enthalpy of the water in its final state can be readily determined from Fig. 3 which accompanies Table 4 in the *Steam Tables*. The abscissa of this figure is entropy, and the ordinate is the difference between the enthalpies of subcooled water and of saturated water *at the same entropy*. To illustrate how the figure is used, suppose that saturated water initially at 150°F is compressed adiabatically and reversibly to a pressure of 2000 psia. The entropy of saturated water at 150°F is 0.2149 Btu per lb deg F (from Table 1). From Fig. 3 the value of $(h - h_f)_s$, corresponding to this entropy and 2000 psia, is about 6.0 Btu per lb. Therefore, since $h_f = 117.89$ Btu per lb at 150°F (or at $s_f = 0.2149$ Btu per lb deg F), the enthalpy of the water at the end of the compression is

$$\begin{aligned} h &= 117.89 + 6.0 \\ &= 123.9 \text{ Btu per lb.} \end{aligned}$$

If any system undergoes a process during which its entropy remains constant, the system is said to undergo an **isentropic** process. For convenience in writing, the term "isentropic" will be used in the remainder of this text to mean that a process is adiabatic and either reversible or internally reversible, although the entropy of a system could remain constant under certain other conditions as well.

ILLUSTRATIVE PROBLEM

263. In Illustrative Problem 140 following Art. 46, the internal energy u_2 of a pound of steam, initially at 100 psia and 400°F, was determined at the end of an adiabatic reversible expansion to 25 psia by constructing a *u-v* diagram for the process. Redetermine the value of u_2 by making use of the property entropy.

Solution. From Table 3 of the *Steam Tables*, the initial entropy of the steam is

$$s_1 = 1.6518 \text{ Btu per lb deg F.}$$

Since the process is adiabatic and reversible, this is also the entropy of the steam at the end of the process. It is between the values of s_f and s_g at 25 psia, and therefore the fluid in its final state is a mixture. Its quality can be determined by either Eq. 44*a* or 44*b*. Since the given value of s is closer to s_g than to s_f, the quality is closer to 1.00 than to zero. Consequently, for slide-rule calculations, a more accurate value of the quality can be determined by using Eq. 44*b*:

$$s = s_g - (1 - x)s_{fg}, \tag{44b}$$

or

$$1.6518 = 1.7139 - (1 - x_2)1.3606,$$

whence

$$(1 - x_2) = 0.0456.$$

The final internal energy of the steam can now be calculated by Eq. 21*b*:

$$u = u_g - (1 - x)u_{fg}, \tag{21b}$$

or

$$\begin{aligned} u_2 &= 1085.1 - 0.0456 \times 876.8 \\ &= 1045.3 \text{ Btu per lb.} \end{aligned}$$

PROBLEMS

264. Steam enters a turbine at 600 psia and 900°F. If it expanded isentropically to 60 psia, how much work would each pound do? Assume that the steam would enter and leave the turbine at low velocities. *Ans.* 259.8 Btu per lb.

265. Steam initially at 160 psia and 460°F expands isentropically until it becomes saturated. How much work will each pound of the steam do if (*a*) the expansion is a non-flow process, and (*b*) it is a steady-flow process and the initial and final velocities of the steam are equal?

266. Two pounds of steam initially saturated at 150 psia undergo the following non-flow processes: (*a*) The steam is cooled at constant volume until its pressure drops to 15 in. Hg abs, and then (*b*) it is compressed isentropically back to 150 psia. How much work is required to compress it? *Ans.* 13.8 Btu.

267. Steam at 150 psia and 420°F enters a converging-diverging nozzle (Fig. 116) with a velocity of 200 ft per sec. It leaves at a pressure of 40 psia. How many pounds flow through the nozzle per minute if the cross-sectional area of the nozzle mouth is 1 sq in. and the steam flows adiabatically and without friction?

268. If the fluid in the preceding problem were air instead of steam, how many pounds would flow through the nozzle per minute? *Ans.* 137 lb per min.

269. A pound of steam initially at 220 psia and 560°F expands isentropically to a volume of 15 cu ft. If the process is non-flow, how much work does the steam do?

270. Suppose that the system in the preceding problem is a pound of air instead of a pound of steam. How much work would the air do? *Ans.* 102.0 Btu.

271. At the start of the expansion stroke, the cylinder of a steam hammer contains 0.015 lb of steam at a pressure of 160 psia and a quality of 97%. At the end of the expansion stroke, the steam is at 40 psia. If the moving parts of the hammer weighed 650 lb and moved without friction, and if the steam expanded isentropically, what would be the velocity of the moving parts at the end of the expansion stroke? Assume that the moving parts are at rest initially.

76. Temperature-Entropy Diagrams. It was stated in Art. 9 that the usual systems considered in engineering thermodynamics are either (*a*) single chemical substances in the form of a liquid, a vapor, or a mixture of a liquid and its vapor, and (*b*) mixtures of various kinds of gases. It was also stated that, if such systems are subject to no surface, electric, or magnetic effects, and if they are at rest, *only two properties are required to specify any equilibrium state of the system.* It is for this reason that such states can be represented by points on a diagram whose coordinates are any two properties of the system. Also, since a system undergoing either a reversible or an internally reversible process passes through an infinite number of equilibrium states, each of these states can be represented by a point on such a diagram, and these points form a continuous curve. Thus, reversible or internally reversible processes can be represented by curves on diagrams whose coordinates are any two properties of the system.

Up to this point the various processes considered have been shown only on pressure-volume diagrams. This particular pair of properties has been used because, as explained in Art. 24, if the process is non-flow, the area under the curve for the process represents the *work* done by the system. Now that the property entropy has been introduced, reversible or internally reversible processes will frequently be represented by curves on temperature-entropy diagrams. This pair of properties is used because, if the ordinate is thermodynamic temperature and the abscissa is entropy, the area under the curve for the process represents the *heat* added to the system. That this is true can be shown as follows: For any infinitesimal part of a reversible or internally reversible process, Eq. 41 may be written

$$ds = \frac{dQ}{T}.$$

Solving for dQ gives

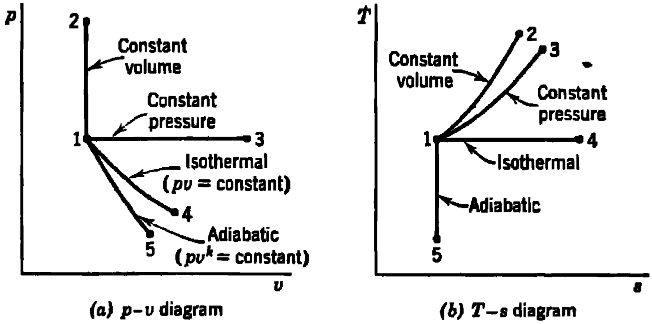
$$dQ = T\,ds;$$

and integrating this equation for the entire process yields

$$Q = \int T\,ds.$$

But the area *under* the curve for the process on a T-s diagram is also equal to the integral of $T\,ds$. Therefore, this area represents the heat added to the system during the process.

77. The Temperature-Entropy Diagram for a Perfect Gas. Let us now consider the appearance on a T-s diagram of the curves for various simple processes if the system is a perfect gas. Suppose first that a perfect gas undergoes a *constant volume* reversible or internally reversible process during which heat is added to the gas. On a p-v diagram the process might be represented by the line 1–2 shown in Fig. 48*a*. To show the process on a T-s diagram, we must first locate the point representing state 1. This point must be somewhere on the horizontal T_1 = constant line, but its position on this line may be selected arbitrarily; that is, any value may be arbitrarily chosen for the initial entropy of the gas. Since heat is assumed to be added to the gas during



(a) p–v diagram (b) T–s diagram

FIG. 48. Pressure-volume and temperature-entropy diagrams for a perfect gas.

the process, the final temperature T_2 will be higher than T_1, and the final entropy s_2 will be larger than s_1 (see Eq. 41). Therefore, the point representing state 2 will be above and to the right of the point chosen to represent state 1. If the specific heat c_v is a constant for the gas, the process itself will be represented by a curve which is concave upwards, as shown in Fig. 48*b*. The curve has this form since, for any infinitesimal part of the process,

$$ds = \frac{dQ}{T} = \frac{c_v\,dT}{T};$$

and therefore the slope of the curve is

$$\frac{dT}{ds} = \frac{T}{c_v}.$$

Hence, if c_v is a constant, the slope of the constant volume line on a *T*-*s* diagram increases as the temperature T increases, and the curve is concave upwards as shown.

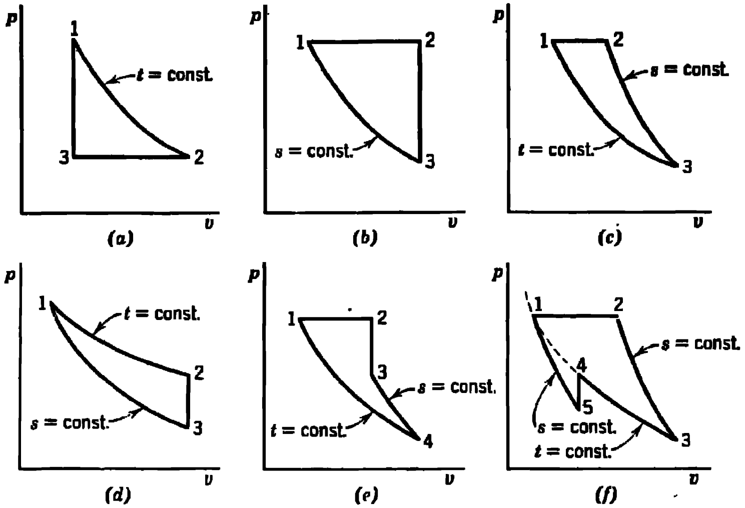
Suppose next that the gas is heated from state 1 to state 3 by a *constant pressure* reversible or internally reversible process, as shown in Fig. 48*a*. Since heat is assumed to be added to the gas, the temperature of the gas will rise and its entropy will increase. Therefore, on a *T*-*s* diagram the point representing state 3 will be above and to the right of the point representing state 1. If c_p for the gas remains constant, the curve will be concave upwards, like the constant volume curve, but will be less steep, as shown in Fig. 48*b*. This is true because the slope of the constant pressure curve is evidently equal to T/c_p, and c_p for a perfect gas is always larger than c_v (since $c_p - c_v = R/J$). Hence, at any given value of T the slope of the constant pressure line is less than the slope of the constant volume line; so curve 1–3 in Fig. 48*b* must fall below curve 1–2.

Now suppose that the gas is heated from state 1 to state 4 by an *isothermal* reversible or internally reversible process, as shown in Fig. 48*a*. Since heat is assumed to be added to the gas, the entropy of the gas will increase, and the point representing state 4 on the *T*-*s* diagram will be to the right of the point representing state 1. Therefore, the process will appear as the horizontal line shown in Fig. 48*b*.

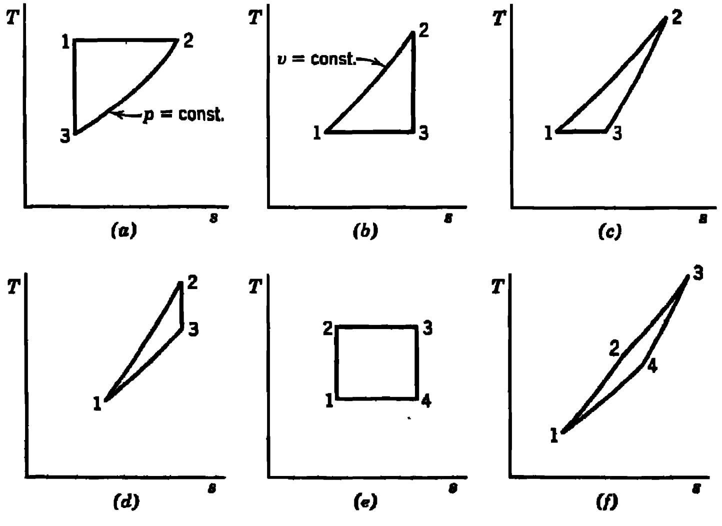
Finally, suppose that the gas is expanded from state 1 to state 5 by an *adiabatic* reversible or internally reversible process, as shown in Fig. 48*a*. Since the gas expands, its temperature will drop (see Eq. 28*a*); but, since the process is adiabatic, its entropy will remain constant. Therefore, the point representing state 5 on the *T*-*s* diagram will be directly below the point representing state 1, and the process will appear as the vertical line shown in Fig. 48*b*.

PROBLEMS

272. A perfect gas undergoes the cycles shown on the p-v diagrams in the accompanying figure. Sketch each cycle on a T-s diagram, and number the states corresponding to the ones marked on the p-v diagrams. Assume that c_v and c_p for the gas are constants.



273. A perfect gas undergoes the cycles shown on the T-s diagrams in the accompanying figure. During those processes which appear as curved lines, either the



volume or the pressure of the gas remains constant. Sketch each cycle on a p-v diagram, and number the states which correspond to the ones marked on the T-s diagram. Assume that c_v and c_p for the gas are constants.

274. An imaginary substance undergoes a constant volume process during which its temperature rises from 70° to 250°F and its entropy increases by 0.050 Btu per lb deg F. If the process appears as a straight line on a T-s diagram, what is the instantaneous specific heat at constant volume for the substance at 150°F?

Ans. 0.1695 Btu per lb deg F.

275. The curve on a T-s diagram representing a certain reversible process undergone by 1 lb of a perfect gas has a slope equal to $5.1T$°F per Btu per lb deg F. If the gas is initially at 70°F, and if 40 Btu of heat are added to it during the process, by how much does its entropy increase? *Ans.* 0.0635 Btu per lb deg F.

276. Suppose that a perfect gas having a constant value of c_v undergoes a reversible process which appears as a straight line on a T-s diagram. Show that the pressures and volumes of the gas at any two of its states are related by the equation

$$\frac{p_2v_2 - p_1v_1}{\alpha R c_v} = \ln\left[\left(\frac{p_2}{p_1}\right)\left(\frac{v_2}{v_1}\right)^k\right],$$

where α denotes the slope of the straight line (that is, $\alpha = dT/ds$).

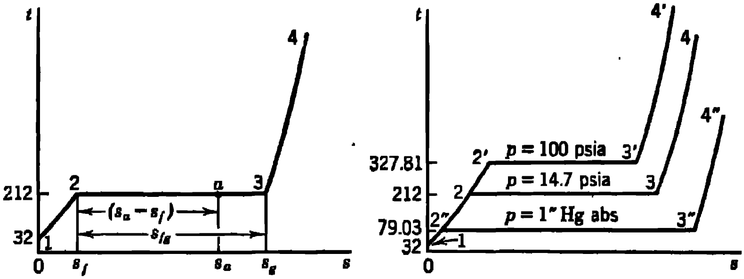
78. The Temperature-Entropy Diagram for Steam.

The temperature-entropy diagram for a system which may be in the form of a liquid, a vapor, or a liquid-vapor mixture will now be considered. Although the temperature-entropy diagrams for some systems of this type differ in certain respects from the diagram for steam, the diagram for steam is typical of most of them.

Let us first show on a t-s diagram the points representing the states through which a pound of water passes in changing at a constant pressure of 14.7 psia from water at 32°F into superheated steam at some temperature above 212°F. The initial state of the water may be represented by a point lying anywhere on the horizontal 32°F temperature line. The position of the point on this line may be chosen arbitrarily; that is, any value may be assigned to the entropy of the water in its initial state. Since the entropy of *saturated* water at 32°F has been arbitrarily set equal to zero in the *Steam Tables*, however, let us follow the same convention. As explained in Art. 73, at moderate pressures the entropy of subcooled water is very nearly the same as the entropy of saturated water at the same temperature. Therefore, the entropy of water at 32°F and 14.7 psia is approximately zero, and the initial state of the water may be represented by point 1 in Fig. 49*a*.

Now let us consider the state of the water after enough heat has been added to raise its temperature to 212°F (state 2). Since heat is added, the entropy of the water increases; so the point representing state 2 is above and to the right of the point representing state 1. Note that the entropy at state 2 is equal to s_f at 14.7 psia and that the states through which the water passes during this part of the process are represented by points falling on a curve which is concave upwards, as shown.

Next, consider the part of the process during which the fluid changes from saturated water to saturated steam (state 3). Since heat is added to the water, the entropy increases; but, since the temperature remains constant during this vaporization process, state 3 is represented by a point directly to the right of point 2, as shown in Fig. 49*a*. Note that the entropy of the saturated steam at state 3 is equal to s_g at 14.7 psia. Note also that any point along the horizontal line 2–3 represents a mixture of saturated water and saturated steam. Moreover, point 2 corresponds to a quality of 0%, point 3 to a quality of 100%, and any other point along this line corresponds to a quality directly proportional to the position of the point along the line. For example, point *a* is three-quarters of the way from point 2 and corresponds to a quality of 75%. That this is true follows from Eq. 44*a*. Thus, the entropy s_a of a mixture having the quality x_a is



(*a*) Atmospheric pressure (*b*) Several pressures

FIG. 49. Temperature-entropy diagrams showing the states through which water passes in changing *at constant pressure* from subcooled water to superheated steam.

$$s_a = s_f + x_a s_{fg}, \tag{44a}$$

and, therefore,

$$\frac{s_a - s_f}{s_{fg}} = x_a.$$

Finally, consider the part of the process during which the fluid changes from saturated to superheated steam at some temperature t_4. Since heat is added, the entropy again increases; so the point representing state 4 is above and to the right of point 3, as shown in Fig. 49*a*. The points representing the states through which the steam passes again fall on a curve that is concave upwards, as shown.

Now let us add to the diagram curves showing the states through which the system passes in changing from water at 32°F into superheated steam if the pressure is kept constant at 100 psia. The initial state of the water will again be represented approximately by point 1 in Figs. 49*a* and 49*b* since at moderate pressures the entropy of subcooled water is very nearly equal to the entropy of saturated water at the same temperature. As the water is heated, the points representing the states through which it passes will fall on the same curve as for a pressure of 14.7 psia. However, since at 100 psia the system will remain a liquid until it reaches a temperature of 327.81°F (the saturation temperature corresponding to 100 psia), the curve will continue up to point 2′ shown in Fig. 49*b*. As the heating is continued, the fluid first vaporizes at constant temperature and then becomes superheated. The nature of the entropy changes during these parts of the process is the same as when the pressure is 14.7 psia. If the system is changed from water at 32°F to superheated steam at some constant pressure below 14.7 psia, such as 1 in. Hg abs, the curve for the process will appear as curve 1-2″-3″-4″ in Fig. 49*b*.

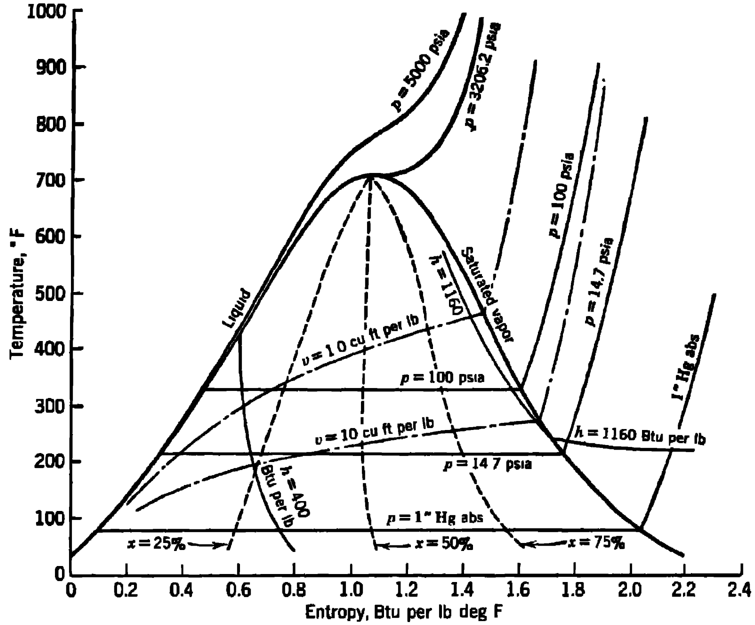


FIG. 50. Temperature-entropy diagram for steam.

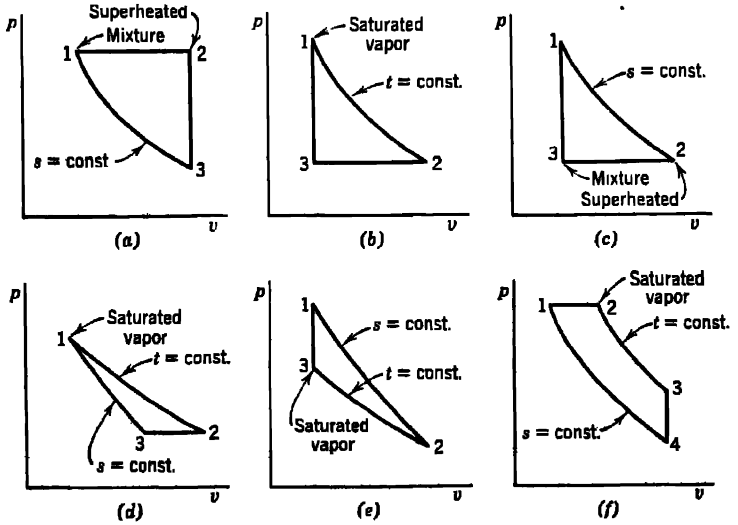
Note that points 3, 3′, and 3″ in Fig. 49*b* all represent saturated vapor. If additional constant pressure lines are added to the figure, and if a curve is drawn through the points representing saturated vapor, the figure obtained will appear as in Fig. 50. Note that the *liquid line* and the *saturated vapor line* meet at a temperature of 705.40°F. This is the "critical point" discussed in Art. 13. At any pressure above the critical pressure of 3206.2 psia, a constant temperature vaporization no longer takes place. Note, for example, that the constant pressure line for 5000 psia has no horizontal portion. Note also that, at pressures as high as this, there is a small difference between the entropy of subcooled and of saturated water at the same temperature.

Several lines of constant quality, constant volume, and constant enthalpy are also shown in Fig. 50. Note that, in the mixture region, the constant volume lines are concave downward, whereas in the superheat region they are concave upward and are steeper than the constant pressure lines. Note also that the constant enthalpy lines in the superheat region become nearly horizontal at low pressure; that is, for superheated steam at low pressure the enthalpy is approximately a function of the temperature only. This is to be expected since at low pressures superheated steam approaches perfect gas behavior, and the enthalpy of a perfect gas is a function of the temperature only (see Art. 38).

The attention of the student is called to the temperature-entropy diagram that appears as Fig. 9 in the *Steam Tables*.

PROBLEMS

277. A pound of H_2O undergoes the cycles shown on the accompanying p-v diagrams (not to scale). Whether the H_2O is in the form of saturated vapor, super-

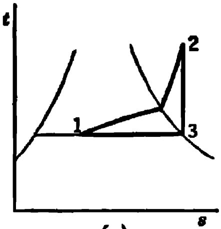


(a) (b) (c)

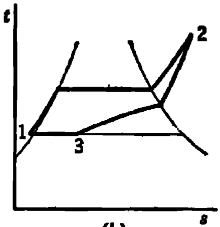
(d) (e) (f)

heated vapor, or a mixture of saturated water and saturated steam is indicated for certain states. Sketch each cycle on a T-s diagram, and number the states corresponding to the ones marked on the p-v diagram.

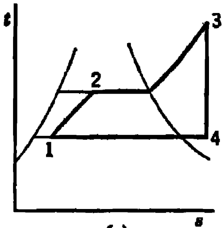
278. A pound of H_2O undergoes the cycles shown on the accompanying t-s diagrams. During each process either the pressure, volume, temperature, or entropy



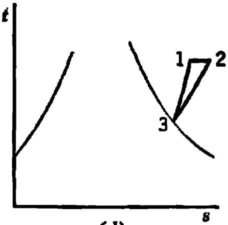
(a)



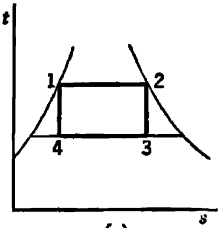
(b)



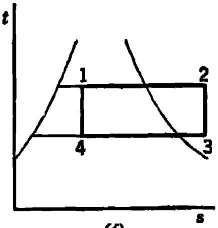
(c)



(d)



(e)



(f)

of the H_2O remains constant. Sketch each cycle on a p-v diagram, and number the states which correspond to the ones marked on the T-s diagram.

279. Show each of the following processes on a t-s diagram. For the throttling processes show only the initial and final states but draw a dotted line to show the constant enthalpy line on which these states lie.

(*a*) A pound of steam initially at 20 psia and having a quality of 10% is compressed adiabatically and reversibly to 200 psia.

(*b*) A pound of H_2O initially at 200 psia and 80°F is compressed adiabatically and reversibly to 1000 psia, then heated at constant pressure to 700°F, and finally throttled back to 200 psia.

(*c*) Steam at 100 psia and having a quality of 98% flows from a steam main into a throttling calorimeter. The pressure in the calorimeter is 14.7 psia.

(*d*) Water at 250 psia and 350°F flows through an insulated valve into a region where the pressure is 50 psia.

79. The Mollier Diagram for Steam. In the operation of many kinds of apparatus of interest to engineers, a fluid is compressed or expanded. For example, fluids are compressed or expanded in compressors, turbines, internal combustion engines, pumps, and nozzles. Almost invariably the compression or expansion takes place at relatively high speed, and there is little time for heat to be transferred to or from the fluid during the process; that is, these processes are approximately adiabatic. Ideally, they would also be reversible and would therefore be *isentropic*. This suggests that, for engineering purposes, graphical presentations of the properties of any fluid should be on diagrams having the *entropy* of the fluid as one coordinate. Also, most kinds of engineering apparatus operate in a steady-flow manner; and, since the property enthalpy appears in the steady-flow energy equation, this suggests that the other coordinate be *enthalpy*. Such diagrams are available for many fluids of importance in engineering and are usually called **Mollier diagrams** in honor of Richard Mollier, who first suggested them.

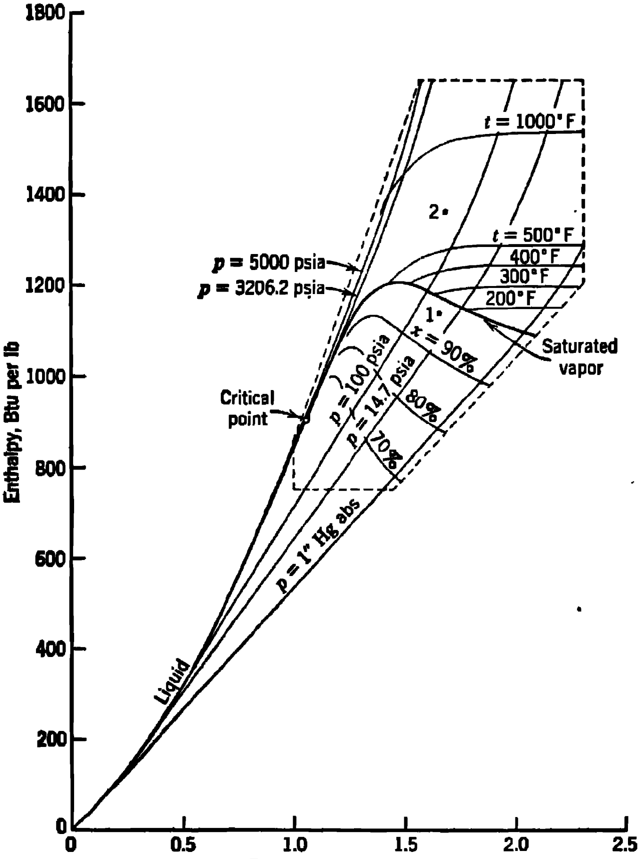


Fig. 51. Mollier diagram for steam.

A skeleton Mollier diagram for steam is shown in Fig. 51. Note that a point such as 1 represents a mixture of saturated water and saturated steam, and a point such as 2 represents superheated steam. Note also that constant pressure lines in the mixture region are straight lines. The fact that these lines must be straight can be shown by deriving an expression for their slope (= dh/ds). Since the enthalpy of a mixture is

$$h = h_f + xh_{fg}, \tag{20a}$$

and since for any given pressure h_f and h_{fg} are constants, it follows that *at constant pressure*

$$dh = h_{fg}\,dx.$$

Similarly, at constant pressure

$$ds = s_{fg}\,dx.$$

Therefore, the slope of any constant pressure line in the mixture region is

$$\frac{dh}{ds} = \frac{h_{fg}\,dx}{s_{fg}\,dx} = \frac{h_{fg}}{s_{fg}} = T = \text{a constant,}$$

and the line is a straight line. The foregoing expression for the slope also shows that at higher pressures (and therefore at higher saturation temperatures) the constant pressure lines in the mixture region are steeper than at lower pressures.

A large scale Mollier diagram for steam, suitable for engineering purposes, is included in a pocket in the back cover of the Keenan and Keyes *Steam Tables*. This diagram shows only the portion of Fig. 51 enclosed within the dotted lines. Examination of this diagram will reveal that lines of "constant superheat" are also shown. The **degrees of superheat** of any superheated vapor is defined as the difference between the temperature of the superheated vapor and the saturation temperature corresponding to the pressure of the vapor. For example, steam at 14.7 psia and 312°F is said to have 100° of superheat. This term is gradually dropping into disuse, however, and will not be used in this text.

PROBLEMS

The following problems are to be solved by means of the Mollier chart in the back pocket of the *Steam Tables*.

280. Steam enters a turbine at 220 psia and 540°F and leaves at 2 in. Hg abs. If the steam flowed through the turbine adiabatically and reversibly, and if it entered and left the turbine at low velocities, how much work would each pound do?

Ans. 377 Btu per lb.

281. If the steam supplied to the turbine in the preceding problem were throttled adiabatically to a pressure of 160 psia before it entered the turbine, and if it were then expanded in the turbine to a pressure of 2 in. Hg abs, how much work would each pound do? Again assume that the steam would flow through the turbine adiabatically and reversibly and that it would enter and leave at low velocities.

282. Steam enters a converging-diverging nozzle (Fig. 116) at 120 psia and 440°F and leaves at 40 psia. If it enters at a low velocity, and if it expands adiabatically and reversibly within the nozzle, at what velocity does it leave?

Ans. 2160 ft per sec.

283. The pressure and temperature in a throttling calorimeter are 14.7 psia and 240°F. If steam flows into the calorimeter from a main in which the pressure is 120 psia, what is the quality of the steam in the main?

284. Steam at 450 psia and 600°F flows first through a valve and then through a section of bare pipe. If it leaves the pipe at 150 psia with a quality of 98%, how much heat does each pound of it lose while flowing through the apparatus? Assume that its initial and final velocities are low. *Ans.* 126 Btu.

285. If steam at pressures above 500 psia is only slightly superheated, it may become saturated on being throttled to a lower pressure. To what pressure must steam at 1500 psia and 600°F be throttled in order for it to become saturated? Assume that the throttling process takes place adiabatically and that the initial and final velocities of the steam are low.

80. The Entropy Principle. As explained in the preceding article, many processes involved in the operation of engineering equipment are approximately adiabatic because of the speed at which the equipment operates. If the equipment is assumed to operate in an ideal manner, these processes are also reversible and therefore isentropic. Probably the most important use engineers make of the property entropy is in the analysis of such isentropic processes. It is useful also in the analysis of isothermal non-flow processes, but, since such processes are difficult to realize in actual practice (with the exception of the isothermal vaporization of a saturated liquid, during which process the pressure also remains constant), this use is not of great importance. Finally, entropy is used for purposes of exposition, in that, as explained in Art. 76, the states through which a system passes during any reversible or internally reversible process may be represented by a curve on a T-s diagram, and the area under such a curve represents the heat added. The foregoing are the only uses of entropy usually made in engineering thermodynamics. In the broader science of thermodynamics, it has other uses, one of the most interesting of which is its use as a criterion of equilibrium. This application is based on the fact that, *whenever any system undergoes an adiabatic process, its entropy always either increases or remains constant.* This statement is known as the **Entropy Principle.** It follows from this principle that, whenever a system undergoing an adiabatic process reaches a state where its entropy is a maximum (that is, where any further changes would have to be to states for which the entropy is lower), the system can undergo no further changes and must therefore be in a state of equilibrium.

The Entropy Principle applies to any system undergoing an adiabatic process, regardless of whether the system is in equilibrium at the beginning and at the end of the process, and regardless of whether it undergoes a change in chemical composition. The principle will be proved, however, only for the special case of a system which is at a uniform pressure and temperature in both its initial and final states and which undergoes no change in chemical composition. Suppose that such a system is initially in the state represented by point 1 in Fig. 52, and that at the end of an adiabatic process it is in the state represented by point 2. If the process is not reversible, the system will not in general pass through an infinite series of equilibrium states; so the process cannot be represented by a curve on the diagram. For this reason, the process is indicated in Fig. 52 only by an arrow. It is evident that the system could be returned from state 2 to state 1, first by compressing (or expanding) it adiabatically and reversibly to any arbitrarily chosen temperature T, then compressing (or expanding) it isothermally and reversibly to a state such that another adiabatic and reversible expansion (or compression) will return it to its initial state. One such series of processes is shown in Fig. 52. Note that, after the system has been returned to its initial state, it has undergone a cycle during which it has received or rejected heat *only during the isothermal process from a to b*. Let us assume that the system *receives* heat during this process; that is, let us assume that Q_{a-b} is positive. In accordance with the First Law (see Eq. 10), the net heat received equals the net work done by the system; and, since the net heat ($= Q_{a-b}$) is assumed to be positive, the net work done by the system is also positive. Thus, the system constitutes a machine, operating in a cycle, that converts all the heat received by it into work. Since this is a direct violation of the Second Law (see the Kelvin-Planck statement given in Illustrative Problem

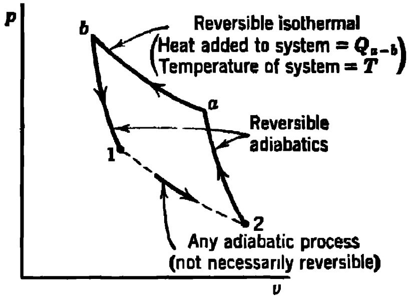

p
b
Reversible isothermal
(Heat added to system = Q_{a-b})
(Temperature of system = T)
a
Reversible
adiabatics
1
2
Any adiabatic process
(not necessarily reversible)
v

FIG. 52. A cycle which violates the Second Law if Q_{a-b} is positive.

222), we conclude that heat cannot be supplied to the system during the isothermal process from a to b: that is, we conclude that

$$Q_{a-b} \leq 0.$$

Since the change $s_b - s_a$ in the entropy of the system during the isothermal reversible process from a to b is, by Eq. 41, equal to Q_{a-b}/T, it follows that

$$s_b - s_a \leq 0,$$

or

$$s_a - s_b \geq 0.$$

But $s_a = s_2$ and $s_b = s_1$ since the entropy change during an adiabatic reversible process is zero. Therefore, finally, it follows that

$$s_2 - s_1 \geq 0, \tag{47}$$

and the Entropy Principle is proved for the special case of a system which in its initial and final states is at a uniform temperature and pressure. This proof may be readily extended to the general case where the various parts of the system are not in equilibrium with each other in the initial and final states or where the system undergoes chemical changes.

It may be noted that, since the equality

$$s_2 - s_1 = 0$$

holds if any system undergoes an adiabatic reversible process, the inequality

$$s_2 - s_1 > 0$$

must hold if the system undergoes an adiabatic process that is not reversible. It may also be noted that, although the Entropy Principle applies only to systems undergoing adiabatic processes, this condition does not restrict its usefulness as much as it might seem to at first. This is true because, by including within the system everything affected during any process, all processes may be made adiabatic.

As an illustration of the application of the Entropy Principle, consider the following problem: Suppose that a 2-lb aluminum sphere initially at 500°F is dropped into 3 lb of water initially at 60°F, and suppose that no heat is lost to or gained from the surroundings during the process. We know from the First Law that the heat lost by the aluminum will equal the heat gained by the water. Also, we know from experience that in the final state the aluminum and the water will be at the same temperature. Hence, denoting this final temperature by t_2, we may write

$$2 \times 0.224(500 - t_2) = 3 \times 1.00(t_2 - 60),$$

where the constants 0.224 and 1.00 are the specific heats at constant pressure of aluminum and water respectively. Solving for t_2, we get

$$t_2 = 117°\text{F}.$$

Now let us see if we can arrive at this result, not by making use of the fact that in the final state the aluminum and the water will be at the same temperature, but by applying the Entropy Principle. Since this principle applies only to a system undergoing an adiabatic process, we must consider the system to be *both* the aluminum and the water. This system does undergo an adiabatic process. Now the First Law tells us only that the energy lost by the aluminum equals the energy gained by the water; that is,

$$2 \times 0.224(500 - t_{\text{Al}}) = 3 \times 1.00(t_{\text{H}_2\text{O}} - 60),$$

where t_{Al} and $t_{\text{H}_2\text{O}}$ denote the temperatures of the aluminum and the water at any instant during the process. By means of this equation we can determine, for example, that, if the temperature $t_{\text{H}_2\text{O}}$ of the water is 100°F, the temperature t_{Al} of the aluminum must be 232°F. Note that any pair of values of $t_{\text{H}_2\text{O}}$ and t_{Al} (including pairs for which $t_{\text{H}_2\text{O}}$ is higher than t_{Al}) that satisfy the foregoing equation represents possible states of the system, so far as the First Law is concerned. We can determine the pair corresponding to the final state by finding the pair for which the entropy is a maximum. To calculate the change in the entropy of the system, we note first that the change in the entropy per pound of any substance for which c_p remains constant is, by Eq. 41,

$$\begin{aligned}\Delta s &= \int \frac{dQ}{T} \qquad (41)\\ &= \int \frac{c_p\,dT}{T}\\ &= c_p \ln\left(\frac{T_2}{T_1}\right).\end{aligned}$$

Hence, the total change in the entropy of the 3 lb of water in changing from 60°F (or 520°F abs) to any other temperature $T_{\text{H}_2\text{O}}$ is

$$\Delta s_{\text{H}_2\text{O}} = 3 \times 1.00 \ln\left(\frac{T_{\text{H}_2\text{O}}}{520}\right),$$

and the total change in the entropy of the 2 lb of aluminum in changing from 500°F (or 960°F abs) to any other temperature T_{Al} is

$$\Delta s_{Al} = 2 \times 0.224 \ln \left(\frac{T_{Al}}{960}\right).$$

The change Δs in the entropy of the entire system is equal to the sum of the changes in the entropy of its component parts; that is,

$$\Delta s = \Delta s_{H_2O} + \Delta s_{Al}.$$

For example, the change in the entropy of the system when it passes from its initial state to the state where $t_{H_2O} = 100$°F and $t_{Al} = 232$°F is

$$\begin{aligned} \Delta s &= 3 \times 1.00 \ln \left(\tfrac{560}{520}\right) + 2 \times 0.224 \ln \left(\tfrac{692}{960}\right) \\ &= +0.0758 \text{ Btu per deg F.} \end{aligned}$$

The values of Δs shown in Fig. 53 are calculated in this way for several pairs of values of t_{H_2O} and t_{Al}. According to the Entropy Principle, the final state of the system will be the state at which its entropy (or the change Δs in its entropy) is a maximum. As expected, the entropy turns out to be a maximum at the state where t_{H_2O} and t_{Al} are equal.

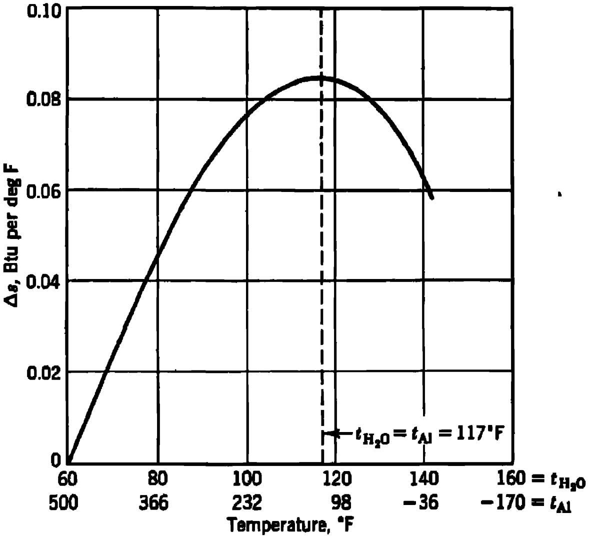


Fig. 53.

Now let us consider the application of the Entropy Principle to another process, the result of which is not so obvious as was the result in the preceding example. Suppose that air is flowing in steady flow through a long tube having a constant cross-sectional area; and suppose that, at some section 1, the pressure p_1 is 100 psia, the temperature t_1 is 70°F, and the velocity V_1 is 300 ft per sec. Suppose further that the tube is perfectly insulated, so that the flow is adiabatic (but not frictionless), and that the air discharges from the end of the tube into a region at atmospheric pressure, as shown in Fig. 54. At first thought it seems logical to expect that the pressure of the air, as it flows from section 1 to the end of the tube, will decrease steadily and reach atmospheric pressure just before emerging from the end. To determine whether the air actually behaves in this manner, let us calculate what its entropy would have to be if it could expand *while still inside the tube* to all pressures in the range from 100 to 14.7 psia. Then, if it should be found that its entropy reaches a maximum value at some pressure above 14.7 psia, we know from the Entropy Principle that this must be the lowest pressure to which the air can expand while still inside the tube.

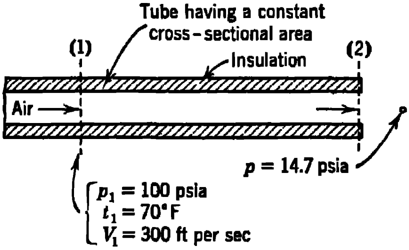


FIG. 54.

To determine the entropy of the air (or the change in its entropy) after it has expanded to any pressure p, we note first that, since the flow is adiabatic and no shaft-work is supplied to the air, the steady-flow energy equation (Eq. 13a) reduces to

$$h_1 + \frac{V_1^2}{2gJ} = h + \frac{V^2}{2gJ}.$$

Also, since the cross-sectional area of the tube is constant, the continuity equation (Eq. 15) reduces to

$$\frac{V_1}{v_1} = \frac{V}{v}.$$

Eliminating the velocity V from these two equations, we obtain

$$h_1 + \frac{V_1^2}{2gJ} = h + \left(\frac{v}{v_1}\right)^2 \times \frac{V_1^2}{2gJ}.$$

Assuming that air behaves as a perfect gas and that c_p for air is constant, we may express the difference in enthalpies $h - h_1$ and the ratio of the specific volumes v/v_1 in terms of the pressures and temperatures of the air by means of the relations

$$h - h_1 = c_p(T - T_1) \tag{18a}$$

and

$$\frac{pv}{T} = \frac{p_1 v_1}{T_1}. \tag{3}$$

Introducing these expressions into the foregoing equation and solving for p,* we obtain

$$p = p_1 \times \frac{T}{T_1} \times \frac{V_1}{\sqrt{V_1^2 + 2gJc_p(T_1 - T)}}.$$

This equation contains only the two unknowns p and T and expresses the relation that must exist between them in order to satisfy the First Law. For example, if the temperature of the air drops to 60°F ($T = 520$°F abs), it is found from this equation that the pressure p must be 64.1 psia. The difference between the entropy of each pound of air at this condition and at the condition existing at section 1 can be calculated by Eq. 42*a*:

$$s - s_1 = c_p \ln\left(\frac{T}{T_1}\right) - \frac{R}{J}\ln\left(\frac{p}{p_1}\right) \tag{42a}$$

$$= 0.241 \ln\left(\frac{520}{530}\right) - \frac{53.3}{778}\ln\left(\frac{64.1}{100}\right)$$

$$= +0.0258 \text{ Btu per lb deg F.}$$

Thus, for any value of T, it is possible to calculate what the values of p and $s - s_1$ must be. The curve shown in Fig. 55 is obtained by plotting several such sets of values. This curve shows that, as the pressure of the air decreases, its entropy increases to a maximum value at a pressure of 24.3 psia and then begins to decrease. Since, according to the Entropy Principle, the entropy of any system undergoing an adiabatic process can never decrease, it follows that, as long as the air remains inside the tube, it cannot expand to any pressure below 24.3 psia. Moreover, it must reach this pressure just before emerging from the end of the tube since otherwise the flow would have to be isentropic (and therefore frictionless) in the last part of the tube after this pressure was reached.

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* It is also possible to solve for T, but the resulting equation is more cumbersome.

It is of interest to determine the velocity of the air just before it emerges from the end of the tube. The temperature T_2 corresponding to $p_2 = 24.3$ psia is 441°F abs. With this value, the velocity V_2 can be readily calculated by the steady-flow energy equation. It is found to be 1030 ft per sec. The velocity of sound in air at 441°F abs is also 1030 ft per sec. Thus, it appears that the maximum velocity the air can attain under the given conditions is the acoustic velocity.

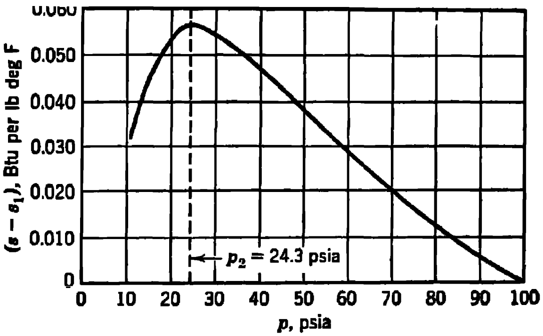


FIG. 55.

Although little use is made of the Entropy Principle in the chapters that follow, it is well to remember that the property entropy is useful for reasons other than those ordinarily considered in engineering thermodynamics.

PROBLEMS

286. A 3-lb aluminum sphere initially at 500°F is dropped into saturated water at 14.7 psia. Show by applying the Entropy Principle that 0.200 lb of the water will evaporate if the pressure is kept constant at 14.7 psia. Assume that no heat is lost to or gained from the surroundings. The specific heat of aluminum is 0.224 Btu per lb deg F.

287. Suppose that air is flowing through a long tube whose cross-sectional area is constant. Suppose further that at some section 1 the air is at a pressure of 20 psia, a temperature of 70°F, and is flowing with a velocity of 2000 ft per sec. Determine what the pressure of the air is just inside the end of the tube if (*a*) the tube is perfectly insulated, and (*b*) the air discharges into a region where the pressure is 14.7 psia.

288. Suppose that a cylinder contains air initially at 20 psia and 70°F, that the cylinder is provided with a frictionless piston free to move in a vertical direction, that the area of the piston face is 10 sq in., and that the cylinder and piston are made of a material which is perfectly non-conducting to heat. If a 150-lb weight were placed on the piston, the piston would move downward, would oscillate for a while, but would eventually come to rest. The final pressure of the air would, of course, be (20 + 150/10 =) 35 psia.

The final pressure would not have to be 35 psia to satisfy the First Law of Thermodynamics, however. Instead, the First Law would be satisfied if the final pressure and temperature were any pair of values which satisfied the equation

$$\begin{aligned} 0 &= mc_v(T_2 - T_1)J + F(x_2 - x_1) \\ &= c_v(T_2 - T_1)J + \frac{F}{a}(v_2 - v_1) \\ &= 0.172(T_2 - 530) \times 778 + \frac{350 \times 144}{10}\left[\frac{53.3T_2}{p_2 \times 144} - \frac{53.3 \times 530}{20 \times 144}\right], \end{aligned}$$

where F denotes the force exerted on the air by the piston and weight, $x_2 - x_1$ denotes the distance the piston moves, and a denotes the area of the piston face. By applying the Entropy Principle, show that the final pressure of the air must be 35 psia.

GENERAL PROBLEMS

289. An insulated tank contains 1 lb of air initially at 50 psia and 70°F. If the air is violently agitated by paddles, and if 15,000 ft-lb of work are done on it in this manner, by how much does its entropy change? Assume that in its initial and final states the air is at rest. *Ans.* 0.0327 Btu per lb deg F.

290. Two pounds of air initially at 300°F are cooled at constant pressure. How much heat must be absorbed from the air in order to cause its entropy to decrease by 0.150 Btu per deg F? *Ans.* 98.4 Btu.

291. A rigid tank contains 3 lb of air. If adding 90 Btu of heat to the air causes its entropy to increase by 0.135 Btu per deg F, what is the final temperature of the air?

292. (*a*) A pound of air initially at 20 psia and 70°F is heated at constant volume until its pressure reaches 35 psia. The air is then cooled at constant pressure until its entropy is the same as it was when the air was at 20 psia and 70°F. Determine the final temperature of the air.

(*b*) If a pound of air initially at 20 psia and 70°F is compressed adiabatically and reversibly to 35 psia, what will be its final temperature?

293. An 8.5-cu-ft tank contains air initially at 70°F. How much heat must be added to the air to raise its pressure to 50 psia and to cause its entropy to increase by 0.093 Btu per deg F?

294. Basing your calculations on the entropies listed in the *Steam Tables* at 200 psia and 400°F and at 200 psia and 600°F, determine an average c_p for superheated steam at 200 psia. *Ans.* 0.565 Btu per lb deg F.

295. Assuming that the entropy of liquid ammonia at 50 psia and −40°F is very nearly the same as that of saturated liquid ammonia at −40°F, calculate (*a*) the value of s_f for ammonia at 50 psia, (*b*) the value of s_g for ammonia at 50 psia, and (*c*) the value of s for superheated ammonia at 50 psia and 300°F. Compare your results with the values listed in the ammonia tables. (*Suggestion*: Determine average values of c_p for liquid and for superheated ammonia by finding $\Delta h/\Delta t$.)

296. If a pound of water initially saturated at 100°F were compressed isothermally and reversibly to 5000 psia by a non-flow process, how much heat would have to be absorbed from it, and how much work would have to be done on it?

Ans. 1.72 Btu; 0.09 Btu.

297. A pound of water initially at 200 psia and 100°F is compressed isentropically to 6000 psia. Determine how much work is required if the water is compressed in a pump operating in a steady-flow manner and the velocities of the water entering and leaving the pump are negligible.

298. Steam enters a turbine at 650 psia and 740°F and leaves at 1½ in. Hg abs. It enters with a velocity of 150 ft per sec and leaves with a velocity of 300 ft per sec. If the steam flowed through the turbine adiabatically and without friction, how many pounds per hour would be needed in order for the turbine to deliver 25,000 hp? *Ans.* 129,000 lb per hr.

299. A system consisting initially of 3 lb of water at 220 psia and 280°F undergoes the following non-flow reversible processes: (*a*) The system expands isentropically until its pressure drops to 12 psia, and then (*b*) it is heated at constant pressure until its temperature is again 280°F. Determine the total work done by the system, and show the two processes on a *T*-*s* diagram.

300. A vapor compression refrigerating machine (Fig. 38) uses ammonia as its refrigerant. Calculate the coefficient of performance of the machine if (*a*) the pressure in the condenser is 200 psia, (*b*) the ammonia leaves the condenser as a subcooled liquid at 80°F, (*c*) it leaves the evaporator at a pressure of 20 psia with a quality of 90%, and (*d*) it is compressed isentropically in the compressor. Assume that the velocity of the ammonia is low throughout the plant. *Ans.* 3.43.

301. Steam initially at 270°F and having a specific volume of 9 cu ft per lb undergoes the following adiabatic steady-flow processes: (*a*) It is compressed reversibly until its temperature reaches 540°F, and (*b*) it is then throttled back to its initial pressure. If its velocity at the start of the throttling process is negligible but at the end is 300 ft per sec, what is its final temperature? Show the two processes on a *T*-*s* diagram.

302. If the fluid in the preceding problem were air instead of steam, what would its final temperature be? *Ans.* 532.5°F.

303. Suppose that steam at a pressure of 20 in. Hg abs and a quality of 90% enters a compressor and is compressed isentropically. To what pressure will it be raised if 200 Btu of work are done on each pound? Assume that the steam enters and leaves the compressor at low velocities.

304. A system consisting initially of 1.6 lb of saturated water and 0.4 lb of saturated steam at 90 psia undergoes a reversible non-flow process during which its temperature remains constant. If the system does 275 Btu of work, how much heat does it absorb?

305. A pound of steam undergoes a non-flow power cycle consisting of the following reversible processes: (*a*) The steam, initially at 40 psia and having a quality of 50%, is heated at constant volume until its pressure rises to 210 psia; (*b*) it is then expanded isothermally to 40 psia; and, finally, (*c*) it is cooled at constant pressure back to its initial state. Sketch the *T*-*s* diagram for the cycle, and determine what the thermal efficiency of a heat engine would be if it operated on the cycle.

306. One pound of a certain gas undergoes the following reversible non-flow power cycle: (*a*) The gas is heated at constant pressure from 100° to 400°F; (*b*) it is then expanded isentropically until its temperature drops to 100°F; and, finally, (*c*) it is cooled at constant temperature back to its initial state. If c_p for the gas is a constant, what would be the thermal efficiency of a heat engine operating on the cycle? *Ans.* 20.0%.

307. Suppose that a power plant uses steam as its working fluid and operates on the following steady-flow cycle: (*a*) The steam, initially at 180 psia and 600°F, is throttled to 40 psia; (*b*) it is then expanded isentropically until it becomes saturated; (*c*) it is next cooled at constant pressure until 90% of it condenses; (*d*) it is then compressed isentropically to 180 psia; and, finally, (*e*) it is heated at constant pressure back to its initial state. Sketch the *T*-*s* diagram for the cycle, and determine the thermal efficiency of the plant. Assume that the steam flows at low velocities throughout the plant.

308. A pound of H_2O undergoes the following non-flow power cycle: (*a*) The H_2O, initially in the form of saturated water at 500°F, is heated isothermally and reversibly until its pressure becomes 60 psia; (*b*) it is then cooled at constant volume until its pressure drops to 30 psia; (*c*) it is next cooled at constant pressure to a state such that (*d*) an isentropic compression returns it to its initial state. Determine what the thermal efficiency of a heat engine operating on the cycle would be, and sketch the *T*-*s* diagram for the cycle. *Ans.* 24.3%.

SYMBOLS

c_p	specific heat at constant pressure
c_v	specific heat at constant volume
g	acceleration that a force of 1 lb gives to a mass of 1 lb
h	enthalpy per unit mass; h_f = enthalpy per unit mass of saturated liquid; h_g = enthalpy per unit mass of saturated vapor; $h_{fg} = h_g - h_f$
J	mechanical equivalent of heat
k	ratio of c_p to c_v
p	pressure
Q	heat added to system
R	gas constant
s	entropy per unit mass; s_f = entropy per unit mass of saturated liquid; s_g = entropy per unit mass of saturated vapor; $s_{fg} = s_g - s_f$
t	temperature
T	absolute temperature
u	internal energy per unit mass
v	specific volume; v_f = specific volume of saturated liquid; v_g = specific volume of saturated vapor
V	velocity
W	work done by system
x	quality; weight of saturated vapor per unit weight of mixture

PART III

APPLICATIONS

In Part I consideration is given first to the properties *pressure, specific volume,* and *temperature.* These properties are used to describe the state of a system, and the relation among them is known as the *equation of state* for the system. Next, the quantities *work* and *heat,* which can cause a system to change from one state to another, are defined; and the fundamental postulate relating to them, known as the *First Law of Thermodynamics,* is presented. It is then shown that as a consequence of the First Law another property, called *internal energy,* must exist. Finally, the relations among work, heat, and the properties of a system when the system undergoes either a *non-flow* or a *steady-flow process* are developed. Although these equations are applicable to any system, particular attention is paid to perfect gases and to fluids for which tables of thermodynamic properties are available.

In Part II *power cycles* and *refrigeration cycles* are defined. The *Second Law of Thermodynamics* is then used to determine the highest efficiency attainable by any heat engine and to determine the highest coefficient of performance attainable by any refrigerating machine. These concepts make possible the definition of a scale of temperature known as the *thermodynamic scale.* This scale is of interest, partly because it is an absolute scale and is independent of any thermometric fluid, but principally because it allows us to discover still another property. This property is called *entropy* and is of great importance in the general science of thermodynamics. It is of particular value to the engineer in solving problems involving isothermal or adiabatic reversible processes.

Having developed in Parts I and II methods for calculating the work done by and the heat added to any system undergoing either a non-flow or a steady-flow process, we are now able to analyze the performance of heat-driven equipment, such as steam power plants and internal-combustion engines, and of work-driven equipment, such as refrigerating plants and air compressors. These engineering applications of thermodynamics, together with the application to nozzles, turbines, certain flow-measuring devices, and air-conditioning processes, are considered in Part III.

Chapter 10

STEAM POWER PLANTS

A *heat engine* is defined in Art. 52 as any engine whose working fluid executes a power cycle. An example of a practical heat engine is the steam power plant. It consists, not of the steam engine or turbine alone, but of all the pieces of equipment through which the steam flows. If the plant operates on the simple Rankine cycle, for instance, the heat engine consists of the boiler, the engine or turbine, the condenser, and the pump that returns the condensate from the condenser back to the boiler.

The *thermal efficiency* of a heat engine is defined as the net work delivered by the engine divided by the heat supplied to it from some high temperature source. The thermal efficiency of any actual heat engine depends upon the kind of power cycle its working fluid undergoes and upon the extent to which friction and the irreversible flow of heat are avoided.

In the present chapter the possibility of constructing a practical power plant to operate on a cycle approximating the Carnot power cycle is considered first. The cycles actually used in steam power plants are discussed next, and the thermal efficiencies which can in theory be obtained with these cycles are calculated. Finally, some of the losses that occur in an actual power plant are discussed.

81. The Limiting Temperatures. In Art. 60 it was proved that, *of all heat engines receiving heat from the same constant temperature source and rejecting heat to the same constant temperature receiver, none can be more efficient than a reversible engine;* and in Art. 61 it was shown that the efficiency of a reversible engine operating under the aforenamed conditions would be

$$\eta_{\text{rev}} = \frac{T - T_0}{T}, \tag{36a}$$

where T denotes the temperature of the source, and T_0, the temperature of the receiver. In view of the italicized statement, we may logically ask, why not build actual heat engines in such a way that their method of operation approaches that of a reversible engine? We shall answer this question in the next article. In the present article we shall consider the limitations which would exist on the temperatures at which the working fluid of the engine could receive and reject heat.

The usual source of heat for any heat engine is the products of combustion of some fuel, such as coal, oil, or natural gas. Since the temperature of these products is likely to be in the order of 3000° to 4000°F, it is possible in theory to have the working fluid of the heat engine approach these temperatures during the heat reception part of the cycle. In actual practice, however, this is not possible because the working fluid must be separated from the products of combustion by some sort of retaining wall which must be at a temperature between that of the products and that of the working fluid. Since the wall must be made of a material strong enough to withstand whatever pressure may be exerted by the working fluid, and since this requirement is satisfied only by a metal such as steel, the highest temperature to which the working fluid may be raised is fixed by the highest temperature to which the metal may be safely heated. This upper limit is about 1000°F at the present time. Thus, even though a source of heat well above this temperature is available, it is impossible to take full advantage of it.

Now let us consider the limitations on the temperature at which the working fluid rejects heat. It is evident from Eq. 36*a* that the receiver should be at as low a temperature as possible in order that the efficiency of the heat engine will be as high as possible. It is also clear that the receiver must be capable of absorbing large quantities of heat without undergoing any appreciable rise in temperature. These requirements are satisfied either by the atmosphere or by the water from a river or lake. The only way that any receiver at a lower temperature could continuously absorb heat without eventually rising in temperature would be to maintain its temperature below that of the atmosphere by a refrigerating machine. But, since the work required to drive the refrigerating machine would have to be supplied by the heat engine, there can be no increase in the *net* work output from the engine. If there were, the heat engine and refrigerating machine together would constitute a heat engine violating the Carnot Principle.

Thus, we conclude that the highest thermal efficiency attainable at present by any heat engine would be that of a reversible engine receiving heat at approximately 1000°F and rejecting heat at atmospheric temperature. Taking 60°F as an average temperature for the atmosphere, we find by Eq. 36*a* that this upper limiting efficiency is

$$\eta_{rev} = \frac{1460 - 520}{1460}$$

$$= 64.4\%.$$

82. The Impracticability of the Carnot Power Cycle. The only reversible power cycle operating between a constant temperature source of heat and a constant temperature receiver discussed up to this point is the Carnot power cycle. This cycle, as explained in Art. 54, consists of (*a*) an isothermal reversible expansion during which the working fluid absorbs heat from the source, (*b*) an adiabatic reversible expansion during which the temperature of the fluid drops from that of the source to that of the receiver, (*c*) an isothermal reversible compression during which the fluid rejects heat to the receiver, and, finally, (*d*) an adiabatic reversible compression during which the temperature of the fluid rises back to that of the source.

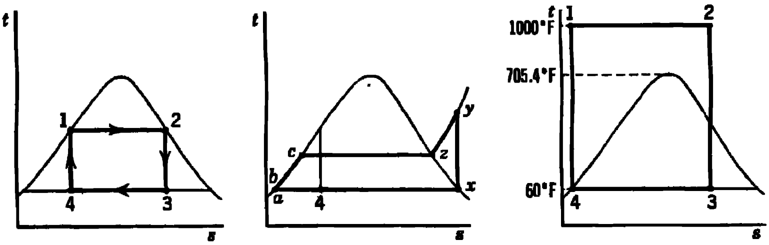
Let us consider the possibility of building a practical heat engine to operate on the Carnot power cycle. Any fluid could be used as its working fluid without affecting the efficiency of the engine since, as proved in Art. 61, *all reversible heat engines receiving heat from the same constant temperature source and rejecting heat to the same constant temperature receiver have the same thermal efficiency.* Let us suppose, first, that air is to be used and, second, that steam is to be used.

If air is used as the working fluid, the two adiabatic reversible processes would be rather easy to approximate. This is true since, to make a process approximately adiabatic, it is necessary only to carry it out rapidly and thereby allow little time for any transfer of heat to take place; and to make an adiabatic process approximately reversible, it is necessary only to eliminate all friction as nearly as possible.

The two isothermal processes would be more difficult to approximate. To explain, let us suppose that some air is compressed inside a cylinder and that the cylinder walls are kept cool by cooling water. The air directly in contact with the walls will remain approximately at the temperature of the walls, while the air at the center will rise in temperature. In order for the air at the center to return to the temperature of the walls, heat must flow *through the air* to the cylinder walls. Because of the low thermal conductivity of air, this can be accomplished only by allowing plenty of time to elapse. Therefore, for a compressor or engine to operate isothermally and to have any useful capacity, it would have to be very large and slow moving. It is for this reason that a power plant using air as its working fluid and operating on the Carnot power cycle is impracticable.

Let us consider next the possibility of building a power plant to operate on the Carnot power cycle with steam as the working fluid. Let us suppose that at the start of the isothermal heat reception process the fluid is saturated water (state 1), and that at the end it is saturated steam (state 2), as shown in Fig. 56*a*. When states 1 and 2 are chosen in this way, the heat reception process takes place, not only at constant temperature, but also at constant pressure. Since constant pressure processes are relatively easy to approximate in actual practice, little difficulty is encountered in carrying out this part of the cycle. Similarly, the heat rejection part of the cycle (3–4) is at constant pressure as well as at constant temperature and can therefore be easily approximated.

The adiabatic reversible expansion (2–3) can be approximated rather easily by expanding the steam rapidly and reducing friction as much as possible by proper designing. The adiabatic reversible compression (4–1), however, is difficult to approximate because at 4 the fluid is a *mixture* of saturated water and saturated steam. Suppose, for example, that a pound of this mixture is confined inside a cylinder equipped with a piston. Some of the water might be dispersed throughout the vapor in the form of mist, but most of it collects at the bottom of the cylinder. Now, if the compression is carried out rapidly, there will be little transfer of heat between the vapor and the liquid, and each phase will be compressed adiabatically. The vapor part of the mixture will become superheated (process x to y in Fig. 56b), while the saturated liquid will be compressed with virtually no change in temperature (process a–b), and the two phases will no longer be in equilibrium. If they are allowed to return to equilibrium while the pressure remains constant, heat will flow from the superheated vapor to the compressed water, the vapor will change from y to z, and the water will change from b to c. Thus, after equilibrium is restored, the cylinder will again contain a mixture of saturated water and saturated steam. It is evident that, to cause the mixture to stay approximately in equilibrium at all times (that is, to cause its state to follow approximately the isentropic line through point 4), the compression would have to be carried out slowly. Since this would again require bulky equipment, it is not practicable to attempt to build power plants to operate on the cycle shown in Fig. 56a.



(a) (b) (c)

FIG. 56.

It should also be noted that, aside from the difficulty explained in the preceding paragraph, it is impossible to operate a power plant on the Carnot cycle with steam as the working fluid if the temperature of the steam during the isothermal heat absorption process is to be near 1000°F. This is true because the critical temperature of steam (705.4°F) is below this value. Let us suppose, for example, that, at the start of the adiabatic reversible compression, the fluid is saturated water (point 4 in Fig. 56*c*). It can be raised to 1000°F (process 4–1) only by going to pressures far beyond the practicable range. Even if this could be done, the isothermal heat absorption process (1–2) would no longer be at constant pressure and could be carried out only with great difficulty.

Thus, we are led to the conclusion that to build a power plant to operate approximately on the Carnot power cycle with either air or steam as the working fluid is impracticable.

83. The Ideal Fluid. It would be possible to operate a power plant approximately on the Carnot power cycle if a fluid could be found whose saturated liquid and saturated vapor lines on a temperature-entropy diagram were as shown in Fig. 57. The critical temperature of the fluid would have to be above 1000°F so that the isothermal heat reception process from 1 to 2 could be made to take place at constant pressure and therefore could be easily approximated. Its triple point would have to be below atmospheric temperature so that the fluid would not solidify during the isothermal heat rejection process from 3 to 4. Finally, its saturated liquid line on the temperature-entropy diagram would have to be vertical in the range from atmospheric temperature to 1000°F so that during the adiabatic compression from 4 to 1 the fluid would remain in the liquid phase. The difficulties encountered when a mixture of saturated liquid and saturated vapor is compressed would thus be avoided.

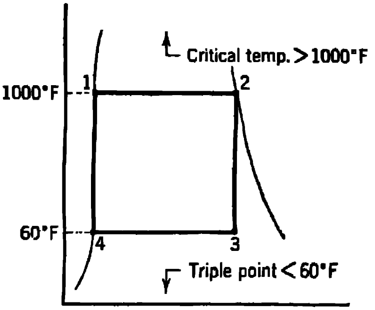


FIG. 57. Temperature-entropy diagram for an ideal fluid.

In addition to having the foregoing characteristics, an ideal fluid would have to satisfy certain other requirements. Its saturation pressure at 1000°F could not be excessively high. If it were, it would be difficult and expensive to build a boiler in which the vaporization process from 1 to 2 could be carried out. Likewise, its saturation pressure at atmospheric temperature could not be exceedingly low, for then the volume of the vapor at the end of the adiabatic reversible expansion (state 3) would be very large, and this would require too bulky equipment. Also, if the pressure were below atmospheric, it would be necessary either to guard against air leaking into the equipment or to provide some means for removing air that did enter. It would be necessary also that the fluid be chemically stable and non-corrosive and desirable that it be inexpensive and nontoxic.

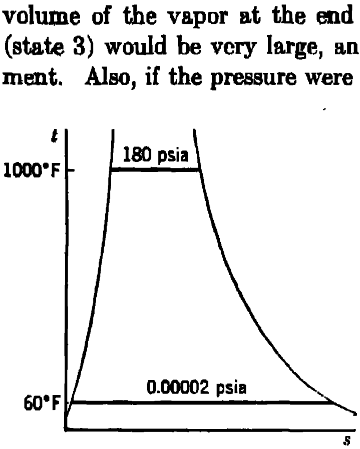


Fig. 58. Temperature-entropy diagram for mercury.

There are a number of fluids that satisfy some of the foregoing requirements, but none of them satisfies all. Mercury, for example, has a triple point temperature of −38°F, a critical temperature in excess of 2800°F, and a saturated liquid line on the temperature-entropy diagram that is quite steep, as shown in Fig. 58. Moreover, its saturation pressure at 1000°F is only 180 psia; and, being an element, it is of course chemically stable. However, its saturation pressure at 60°F is about 0.00002 psia—a pressure entirely out of the practicable range in engineering.

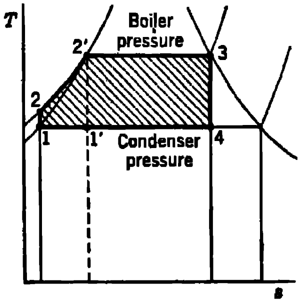


Fig. 59. Temperature-entropy diagram for the Rankine cycle.

84. The Rankine Cycle. The practicable power cycle most like the Carnot cycle is the **Rankine cycle.** As already explained in Art. 51, a power plant built to operate on this cycle must include a pump, a boiler, a turbine or reciprocating engine, and a condenser; and these pieces of equipment must be arranged as shown in Fig. 35. Let us again consider the series of processes that the fluid would undergo if the plant could operate in an ideal manner, and let us show these processes on a temperature-entropy diagram. We shall suppose that the working fluid is water. The condensate leaves the condenser as saturated water and is compressed in the pump adiabatically and reversibly to the boiler pressure. This process is represented by the vertical line from 1 to 2 in Fig. 59. For moderate boiler pressures point 2 almost coincides with point 1, but this portion of the figure is distorted slightly for the sake of clarity. In the boiler the water leaving the pump is first heated at constant pressure to the saturation temperature corresponding to the boiler pressure (point 2′) and is then vaporized. The steam leaving the boiler might be saturated, or it might be superheated. If it is saturated, its state is represented by point 3. In the turbine the steam is expanded adiabatically and reversibly to the condenser pressure, the process being represented by the vertical line from 3 to 4. Finally, in the condenser the mixture leaving the turbine is condensed at constant pressure, the process being represented by the horizontal line from 4 to 1.

Since we are assuming that the plant is operating in an ideal manner (that is, that all the processes are reversible), the heat supplied to each pound of fluid flowing through the boiler is represented by the area under the constant pressure line from 2 to 2′ to 3 (see Art. 76). Likewise, the heat rejected by each pound of fluid flowing through the condenser is represented by the area under the constant pressure line from 4 to 1. In accordance with Eq. 10 in Chapter 3, the difference between the heat supplied to and the heat rejected by each pound of fluid passing completely around the cycle is equal to the net work done by each pound. Therefore, the net work done is represented by the shaded area in Fig. 59. Furthermore, since the thermal efficiency of a heat engine is defined as the ratio of the net work done to the heat supplied, the ideal thermal efficiency of a power plant operating on this cycle is represented by the ratio of the shaded area to the total area under the constant pressure line from 2 to 2′ to 3.

The ideal thermal efficiency of a power plant operating on the Rankine cycle can be calculated by Eq. 35*a*:

$$\eta = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}}. \tag{35a}$$

To determine the heat supplied (Q_{in}) to each pound of fluid flowing through the boiler, we note that the shaft-work supplied to the boiler is zero. Hence, if the velocities entering and leaving the boiler are assumed to be negligible, the steady-flow energy equation applied to the boiler reduces to

$$Q_{\text{in}} = h_3 - h_2,$$

where the subscripts refer to the section numbers marked on Fig. 35. Similarly, the shaft-work supplied to the condenser is zero; and therefore, if the entering and leaving velocities are again assumed to be negligible the steady-flow energy equation applied to the condenser reduces to

$$Q_{4\text{-}1} = h_1 - h_4,$$

where Q_{4-1} denotes the heat *added* to each pound flowing through the condenser. Since Q_{out} denotes the heat *removed* from each pound,

$$Q_{\text{out}} = h_4 - h_1.$$

The enthalpies at states 1, 2, 3, and 4 can be determined by the methods explained in Arts. 40 and 75.

An actual power plant cannot, of course, operate in the ideal manner just described because reversible processes cannot be realized in practice. These processes can be approximated, however, and actual plants can at least approach the ideal Rankine cycle. Some of the deviations from the ideal are discussed in Arts. 85 and 90.

The Rankine cycle shown in Fig. 59 could be transformed into a Carnot cycle if the adiabatic reversible compression from 1 to 2 were replaced by an adiabatic reversible compression from 1′ to 2′. The cycle would then consist of the adiabatic reversible process from 1′ to 2′, an *isothermal* heat reception process from 2′ to 3, an adiabatic reversible expansion from 3 to 4, and an isothermal heat rejection process from 4 to 1′. The ideal thermal efficiency for this cycle is represented graphically by the ratio of the rectangular portion of the shaded area to the total area under the line from 2′ to 3. It is evident that the efficiency of the Rankine cycle is lower than that of the Carnot cycle because part of the heat supplied is used to raise the temperature of the water from 2 to 2′ (that is, the heat is not all received at the upper temperature). However, this heat, represented by the area under 2–2′, is usually less than that supplied from 2′ to 3, represented by the area under 2′–3. Therefore, the Rankine cycle is a reasonably close approach to the Carnot cycle. Its lower efficiency is compensated for by the fact that the process from 1 to 2 is one that can be carried out in practice easily and with simple equipment.

Although in many respects water is a desirable fluid to use in a power plant operating on the Rankine cycle, it does have certain undesirable characteristics. One of these, already mentioned, is the fact that its critical temperature is below 1000°F. Another is that, if the temperature during the isothermal heat reception part of the cycle (2′–3 in Fig. 59) is raised to near the critical temperature, the boiler pressure becomes quite high. For example, at 400°F the saturation pressure of water is less than 250 psia, whereas at 650°F (55° below the critical) it is greater than 2200 psia.

The fact that the saturation pressure of water during the heat rejection part of the cycle (4 to 1) is below atmospheric pressure is also undesirable since this results in air leaking into the plant. This air collects in the condenser, and, in order for the condenser to operate properly, it must be removed. This requires that a suitable compressor (called a *vacuum pump*) be provided. Also, on account of the low pressure, the volume of the vapor leaving the turbine is quite large. Note, for example, that the specific volume of saturated steam at 400°F is less than 2 cu ft per lb, whereas at 60°F it is greater than 1200. This means that both the turbine and the condenser must be capable of handling large volumes of steam.

Another objection to water as a working fluid is that condensation takes place during the adiabatic reversible expansion part of the cycle (process 3–4). Since most turbines rotate either at 1800 or at 3600 rpm, the condensate present in the turbine causes erosion of the turbine blades. As is evident from Fig. 59, the amount of condensate increases as the boiler pressure is raised because raising the pressure causes point 3 (and therefore also point 4) to move to the left. The amount of condensation can be reduced somewhat by superheating the steam before it is admitted to the turbine.

It may be noted that, if an ideal fluid having the characteristics listed in the preceding article existed, and if it were used as the working fluid in a power plant built to operate on the Rankine cycle, the plant would operate on the Carnot cycle as well because the two cycles would become identical.

PROBLEMS

309. A steam power plant operates on the Rankine power cycle. If the steam entering the turbine is saturated at 400 psia, and if the steam leaving is at 2½ in. Hg abs, what is the ideal thermal efficiency of the plant? *Ans.* 32.7%.

310. Suppose that the turbine of a power plant operating on the Rankine cycle is supplied with steam at 200 psia and 500°F. How much will the work ideally obtained from each pound of steam be increased if the condenser pressure is lowered from 14.7 psia to 1 in. Hg abs? Lowering the condenser pressure by this amount will increase the ideal thermal efficiency by how much?

311. The condensation which ordinarily occurs in the low pressure end of a steam turbine is undesirable because it causes blade erosion and reduces the mechanical efficiency of the turbine. It could be avoided, at least in theory, by superheating the steam to a sufficiently high temperature before it enters the turbine. What temperature would be necessary if the steam entered at 250 psia, if it left at 1½ in. Hg abs, and if it expanded isentropically within the turbine?

312. Suppose that the steam leaving the boiler of a power plant operating on the Rankine cycle is at 3200 psia and 860°F, that it is throttled to 600 psia before it enters the turbine, and that it leaves the turbine at 2 in. Hg abs. (*a*) Determine the ideal thermal efficiency of the plant. (*b*) What would the thermal efficiency be if the condition of the steam entering and leaving the turbine were unchanged but the boiler were operated at 600 instead of 3200 psia? *Ans.* (*a*) 35.4%.

313. Suppose that an imaginary fluid having the following properties were used as the working fluid in a power plant operating on the Rankine cycle: (*a*) Its liquid phase is incompressible, (*b*) the value of c_p for the liquid phase is 0.50 Btu per lb deg F, (c) the heat of vaporization at 400°F is 350 Btu per lb, and (d) the saturated vapor line on a T-s diagram has a negative slope. Determine what the ideal thermal efficiency of the plant would be if saturated vapor at 400°F were supplied to the turbine and the temperature of the vapor in the condenser were 100°F.

Ans. 30.4%.

85. Turbine Efficiency. In the preceding article adiabatic reversible expansion of the steam through the turbine was assumed. The actual expansion is likely to be nearly adiabatic because of the short time required for the steam to flow through the turbine, but it is not reversible. The principal source of irreversibility is the fluid friction occurring when the high speed jets from the turbine nozzles enter and flow through the moving turbine blades. The effect of this friction is to increase the entropy of the steam. That this is true follows from the *Entropy Principle*, discussed in Art. 80. This principle states that, *whenever any system undergoes an adiabatic process, its entropy always either increases or remains constant.* Since the steam does undergo a nearly adiabatic process, and since its entropy could stay constant only if the process were reversible, the actual expansion must be accompanied by an increase in entropy.

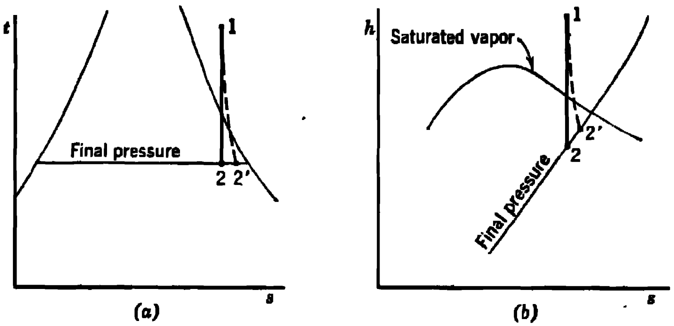


FIG. 60. t-s and h-s diagrams for the expansion of steam through a turbine.

The effect of this increase in entropy on the work done by the steam expanding from a given initial state to a given final pressure can be shown as follows: Suppose that the initial state of the steam is represented by point 1 in Fig. 60*a*, and that the final pressure is represented by the constant pressure line shown. The final state of the steam after an adiabatic *reversible* expansion is represented by point 2, whereas its final state at the end of the *actual* expansion is represented by point 2′. These states appear on an h-s diagram as shown in Fig. 60*b*. The ideal work done by each pound of steam flowing through the turbine is

$$W' = h_1 - h_2,$$

whereas the actual work done is

$$W' = h_1 - h_{2'}.$$

These equations follow from the steady-flow energy equation if the expansion is assumed to be adiabatic and the velocities entering and leaving the turbine are assumed to be negligible. It is evident from Fig. 60*b* that $h_{2'}$ will always be larger than h_2 since constant pressure lines on an *h-s* diagram all have positive slopes. Therefore, the effect of friction is always to decrease the work output from the turbine.

This decrease in work done is, of course, undesirable. However, the effect of the friction is not completely without benefit since, if state 2 is in the mixture region, state 2′ will be at a higher quality or perhaps will still be in the superheat region. This is beneficial in that it reduces the amount of water present in the low pressure end of the turbine.

The extent to which a turbine approaches isentropic performance is indicated by the **turbine** (or "engine") **efficiency.** This term is defined as the ratio of the work W'_t actually delivered by the turbine to the work it would deliver if the expansion were adiabatic and if no friction occurred (also see Art. 127); that is,

$$\eta_t = \frac{W'_t}{J(h_1 - h_2)}, \tag{48}$$

where $h_1 - h_2$ denotes the difference between the enthalpy of the steam entering the turbine and its enthalpy after an *isentropic* expansion to the exhaust pressure. If a turbine operates approximately adiabatically, and if the entering and leaving velocities are negligible, $W'_t = J(h_1 - h_{2'})$, where $h_{2'}$ denotes the enthalpy of the steam at the end of the actual expansion. Under these conditions the efficiency of the turbine may be written

$$\eta_t = \frac{h_1 - h_{2'}}{h_1 - h_2}. \tag{48a}$$

Turbine efficiencies of large central station turbines are likely to be 80% or higher, whereas the efficiencies of small turbines may be less than 60%.

A term frequently used in comparing the performance of various turbines is the so-called "steam rate." The **steam rate of a turbine** is defined as the pounds of steam required per horsepower-hour or per kilowatthour delivered by the turbine. If the rate at which steam flows through the turbine and the power output from the turbine are known, the steam rate can be found by dividing the first of these rates by the second. Or, if the work W'_t actually delivered by the turbine per pound of steam flowing through it is known, the steam rate can be found by dividing the constants 2545 or 3413 by the work W'_t.

PROBLEMS

314. Steam at 300 psia and 600°F enters the turbine of a power plant operating on the Rankine cycle. It leaves the turbine at 1½ in. Hg abs. (*a*) Determine the ideal thermal efficiency of the plant. (*b*) What would the thermal efficiency of the plant be if its boiler, condenser, and pump operated in an ideal manner but its turbine operated with an efficiency of 80%? *Ans.* (*a*) 33.5%; (*b*) 26.8%.

315. Steam flows through a large turbine at the rate of 240,000 lb per hr when the generator which it drives is delivering 30,000 kw. The steam enters the turbine at 850 psia and 900°F and leaves at 1½ in. Hg abs. If the efficiency of the generator is 98%, what is the efficiency of the turbine?

316. If steam were supplied to a turbine at 400 psia and were expanded to 1 in. Hg abs, and if the turbine had an efficiency of 82%, at what temperature would the steam have to be supplied to avoid the occurrence of condensation within the turbine?

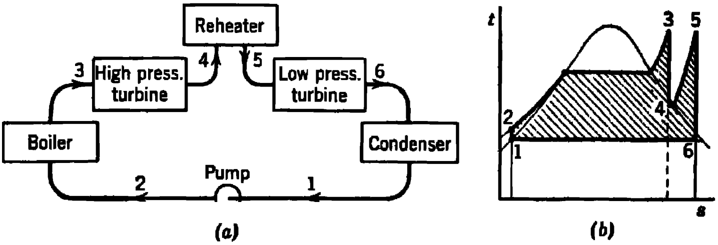
317. The following data were obtained from a test on a small steam turbine:

Pressure at turbine inlet = 110.5 psi gage
Pressure at turbine exhaust = 1.5 psi gage
Rate of steam flow = 1600 lb per hr
Power delivered at shaft = 50 hp
Barometric pressure = 29.5 in. Hg abs.

A throttling calorimeter was connected to the steam main at the turbine inlet. The pressure in the calorimeter was atmospheric, and the temperature was 260°F. Determine the efficiency of the turbine. *Ans.* 53.5%.

✓ **86. The Reheating Cycle.** The condensation that normally occurs within a turbine could, in theory, be avoided entirely by superheating the steam to a sufficiently high temperature before admitting it to the turbine. This cannot be done in practice, however, because a temperature above the metallurgical limit would be required. For the conditions stated in Problem 316, for example, a temperature of almost 1200°F would be needed. A practicable method for reducing the amount of condensation is to allow the steam, initially superheated, to expand first in a high pressure turbine to a pressure at which condensation is just starting, then to resuperheat the steam at approximately constant pressure in a second superheater (called a *reheater*), and finally to expand it to condenser pressure in a low pressure turbine. The resulting cycle is called the **reheating cycle.** A flow-diagram for a plant operating on this cycle is shown in Fig. 61*a*, and a temperature-entropy diagram for the ideal condition in which the expansion through each of the turbines is isentropic is shown in Fig. 61*b*.

The ideal thermal efficiency of the reheating cycle can be calculated by Eq. 35*a* after the heat supplied (Q_{in}) and the heat rejected (Q_{out}) have been determined. The heat Q_{in} is equal to the heat supplied to each pound of fluid flowing through the boiler plus the heat supplied to each pound flowing through the reheater. Assuming that the velocities entering and leaving the boiler and reheater are negligible, and noting that the shaft-work supplied to each is zero, we obtain from the steady-flow energy equation



(a) *(b)*

FIG. 61. The reheating cycle.

$$Q_{in} = (h_3 - h_2) + (h_5 - h_4).$$

The heat rejected, as in the Rankine cycle, is

$$Q_{out} = h_6 - h_1.$$

The heat Q_{in} is represented in Fig. 61*b* by the area under the constant pressure line from 2 to 3 plus the area under the constant pressure line from 4 to 5. The heat Q_{out} is represented by the area under the constant pressure line from 6 to 1. Hence, the net work done ($= Q_{in} - Q_{out}$) is represented by the shaded area, and the ideal thermal efficiency is represented by the ratio of the shaded area to the total area under 2345. It is evident from the figure that the ideal thermal efficiency for the reheating cycle is not very different from the ideal efficiency for the Rankine cycle. It should be remembered in this connection that the primary purpose of the reheating cycle is to reduce condensation within the turbine (and thereby reduce blade erosion) and not to improve the thermal efficiency. The *actual* thermal efficiency of the plant is improved, however, because the friction within the turbine is reduced.

It may be remarked that, although condensation within the turbine can be reduced by reheating, the reheater and the additional piping and controls required increase the initial cost of the plant.

PROBLEMS

318. Steam at 600 psia and 700°F enters the high pressure turbine of a power plant operating on the reheating cycle. It leaves at 100 psia and is reheated to 700°F. It then enters the low pressure turbine and expands to 1 in. Hg abs. (*a*) Determine the ideal thermal efficiency of the plant. (*b*) Determine what the ideal thermal efficiency would be if the steam were not reheated (that is, if the plant operated on the Rankine cycle). *Ans.* (*a*) 38.7%.

319. Suppose that the steam supplied to the high pressure turbine of a power plant operating on the reheating cycle is at 1200 psia and 700°F and that it expands in the high pressure turbine until it becomes saturated. Suppose that it is next reheated at constant pressure to 700°F and is then supplied to the low pressure turbine. If it leaves the low pressure turbine at 1 in. Hg abs, what is the ideal thermal efficiency of the plant?

320. Steam at 1200 psia and 700°F enters the high pressure turbine of a power plant operating on the reheating cycle. The steam leaves the turbine saturated and flows into the reheater where its temperature is raised to 700°F again. It then enters the low pressure turbine and expands to 1 in. Hg abs. If the boiler, reheater, condenser, and pump all operated in an ideal manner, but if the two turbines operated with an efficiency of 80%, what would be the thermal efficiency of the plant?

Ans. 33.8%.

87. The Ideal Regenerative Cycle. The only reversible power cycle operating between a constant temperature source of heat and a constant temperature receiver discussed so far is the Carnot power cycle. Another reversible cycle, also operating between a constant temperature source and constant temperature receiver, is the **regenerative cycle.**

An example of the regenerative cycle is shown in Fig. 62. During the process from 1 to 2 the working fluid receives heat (represented by the area under the line from 1 to 2), and its temperature rises from T_0 to T; from 2 to 3 the fluid receives more heat while its temperature remains constant at T; from 3 to 4 it rejects heat, and its temperature drops from T to T_0; and, finally, from 4 to 1 it rejects heat while its temperature remains constant at T_0. The nature of the processes from 1 to 2 and from 3 to 4 is immaterial *as long as the curves representing these processes on a temperature-entropy diagram are identical.* For simplicity, they are shown as straight lines in Fig. 62.

It is clear that, if the heat supplied from 1 to 2 were taken from the source at temperature T, and if the heat rejected from 3 to 4 were rejected to the receiver at temperature T_0, the cycle would not be a *reversible* cycle because heat flow through a finite difference in temperature would take place. However, if the heat rejected from 3 to 4 is rejected *reversibly* to an infinite number of reservoirs ranging in temperature from T to T_0, these reservoirs could be used to supply the heat which is required to heat the fluid *reversibly* from 1 to 2. Since the heat rejected by the fluid during the process from 3 to 4 is exactly equal to the heat received by the fluid during the process from 1 to 2 (the areas in Fig. 62 representing these quantities are identical), the net effect on the reservoirs at the completion of the cycle is zero.

The only heat taken from the source at T is that received by the fluid during the isothermal reversible process from 2 to 3. Likewise, the only heat rejected to the receiver at T_0 is that rejected by the fluid during the isothermal reversible process from 4 to 1. Therefore, the thermal efficiency of this cycle, like that of the Carnot cycle or any other reversible cycle operating between a constant temperature source and a constant temperature receiver, is

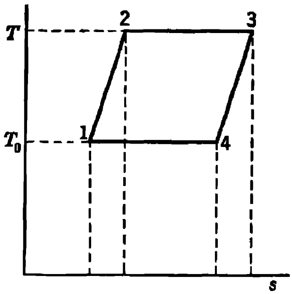


FIG. 62. Temperature-entropy diagram for an ideal regenerative cycle.

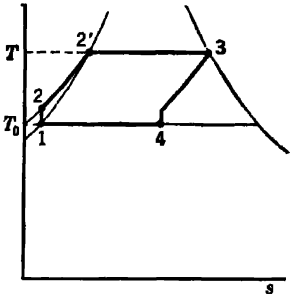


FIG. 63. Temperature-entropy diagram for an ideal regenerative cycle with water as the working fluid.

$$\eta_{\text{rev}} = \frac{T - T_0}{T}. \tag{36a}$$

Note that the working fluid may be any fluid.

Now let us consider how the regenerative cycle could be carried out, at least in theory, if the working fluid were water. The cycle might consist of the processes shown in Fig. 63. The water initially saturated at temperature T_0 (point 1) is compressed isentropically to the saturation pressure corresponding to temperature T (process 1–2). Next, it is kept at constant pressure while heat is added to it, first to raise its temperature to T (process 2–2′) and then to change it isothermally into saturated vapor (process 2′–3). The steam is then allowed to expand, not adiabatically, but in such a way that its state follows the curve shown from 3 to 4. If the heat that must be removed from the steam during this process is allowed to flow reversibly into an infinite number of reservoirs at temperatures ranging from T to T_0, it can later be used to heat the water reversibly from 2 to 2′. Consequently, the only heat taken from the source at temperature T is that absorbed by the fluid during the isothermal reversible process from 2′ to 3, and the only heat rejected to the receiver at T_0 is that rejected during the isothermal reversible process from 4 to 1.

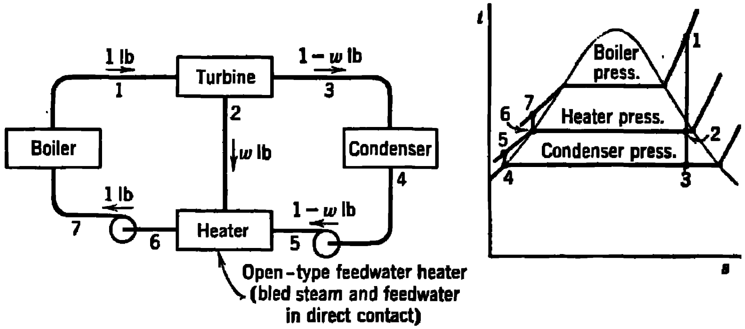
88. The Practical Approach to the Regenerative Cycle. It would be difficult in actual practice to build a power plant to operate on the regenerative cycle shown in Fig. 63. The isentropic compression between states 1 and 2 and the constant pressure processes between states 2 and 3 and between states 4 and 1 would all be easy to approximate. But the process between states 3 and 4 would be quite difficult to approximate if an attempt were made to absorb heat from the steam as it expanded through a turbine. This is true because the time required for steam to flow through a turbine is very short, and there would be insufficient time available for the heat transfer to take place. Besides, the condensation resulting within the turbine would be undesirable.

The regenerative cycle can be approximated in practice if the process from 3 to 4 in Fig. 63 is replaced by the following: The steam is allowed to expand through the turbine as nearly adiabatically and reversibly as possible, but some of it is extracted, or "bled," from the turbine at one or more sections between the inlet and the exhaust. The bled steam is condensed outside the turbine at approximately constant pressure, and the heat given up is used to raise the temperature of the feedwater (the condensate from the condenser) before admitting it to the boiler. This heating of the feedwater by means of steam bled from the turbine is known as **regenerative feedwater heating.**

Regenerative feedwater heating is done in what are called **feedwater heaters.** Feedwater heaters are of two types, known as **open-type** and **closed-type** heaters. In the open type the steam bled from the turbine is brought into direct contact with the colder feedwater. In the closed type the steam condenses on the outside of metal tubes while the feedwater flows through the inside of the tubes. It is apparent that in the open-type heater the bled steam and the feedwater must be at the same pressure, whereas in the closed type they may be at different pressures.

The flow-diagram for a power plant using a single open-type feedwater heater is shown in Fig. 64*a*. For the ideal condition where the expansion through the turbine is isentropic and the water leaving the condenser and leaving the feedwater heater is saturated, the states of the steam at the various sections numbered on the flow-diagram are shown in Fig. 64*b*. Note that the temperature at which the bled steam condenses (the temperature at 6) is higher than the temperature at which the feedwater leaves the pump (the temperature at 5). Consequently, the heat given up by the steam can be used to raise the temperature of the feedwater from T_5 to T_6.

The ideal thermal efficiency of the foregoing plant can be calculated by Eq. 35a after the heat supplied to the fluid flowing through the boiler and the heat rejected by the steam flowing through the condenser have been calculated. Note that heat is neither received from the source nor rejected to the receiver in the feedwater heater. In fact, feedwater heaters are always covered with insulation. If we make the usual assumption that the velocities entering and leaving the boiler are negligible, the steady-flow energy equation applied to the boiler reduces to



(a) Flow-diagram

(b) t-s diagram

FIG. 64. A steam power plant with one open-type feedwater heater.

$$Q_{\text{in}} = h_1 - h_7.$$

Similarly, if we assume that the velocities entering and leaving the condenser are negligible, the heat rejected by each pound of steam flowing through the condenser is $h_3 - h_4$. But, since for each pound of fluid flowing through the boiler only $1 - w$ pounds of steam flow through the condenser,

$$Q_{\text{out}} = (1 - w)(h_3 - h_4).$$

To determine the weight w of steam bled from the turbine per pound of steam entering, we must apply the steady-flow energy equation (Eq. 14) to the feedwater heater. Assuming that the velocities of the streams entering and leaving the heater are all negligible, and noting that $\bar{Q}$ and $\bar{W}'$ are both zero, we find that Eq. 14 reduces to

$$\Sigma(wh)_{\text{in}} = \Sigma(wh)_{\text{out}}.$$

Hence, we may write

$$wh_2 + (1 - w)h_5 = h_6.$$

Since we can determine all the enthalpies appearing in the foregoing equations by the usual methods, we can calculate the ideal thermal efficiency of the plant.

A closer approach to the ideal regenerative cycle can be realized by using more feedwater heaters. Let us consider, as an example, a power plant provided with two closed-type heaters, and let us suppose that the flow-diagram for the plant is as shown in Fig. 65. The condensate from the second feedwater heater flows through a **trap** (a float-controlled valve that allows liquid, but not vapor, to pass) into the vapor space of the first, while the condensate from the first flows through a trap into the condenser. Other arrangements are possible, but the one shown is frequently used.

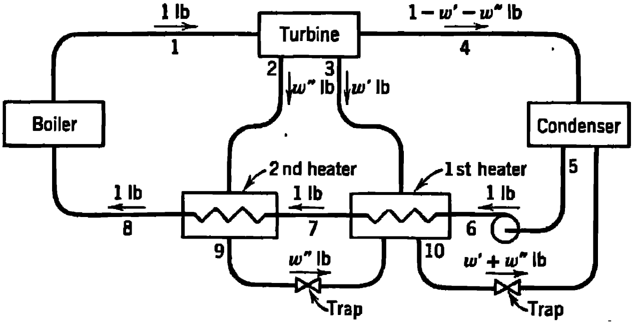


Fig. 65. Flow-diagram for a steam power plant with two closed-type feedwater heaters.

The ideal thermal efficiency of the plant can again be found by Eq. 35*a*. Assuming that the velocities entering and leaving each piece of equipment are negligible, we obtain the following equations from the steady-flow energy equation: for the boiler,

$$Q_{\text{in}} = h_1 - h_8;$$

for the condenser,

$$Q_{\text{out}} = (1 - w' - w'')h_4 + (w' + w'')h_{10} - h_5;$$

for the first feedwater heater,

$$w'h_3 + w''h_9 + h_6 = (w' + w'')h_{10} + h_7;$$

and for the second feedwater heater,

$$w''h_2 + h_7 = w''h_9 + h_8.$$

Hence, if we assume that the expansion through the turbine is isentropic, and if we assume that the condensate leaving the condenser and each of the heaters is saturated, we can calculate the ideal thermal efficiency of the plant. Note that the flow through the traps is a throttling process, and therefore that the enthalpy of the mixture leaving each is equal to the enthalpy of the condensate entering.

The foregoing examples show how regenerative feedwater heating can be used in steam power plants and show how the ideal thermal efficiencies of such plants can be calculated. It is clear that, to make a plant approximate the ideal regenerative cycle even more closely, a larger number of heaters is required. If too many are used, however, the gain in efficiency does not compensate for the increased complexity and increased initial cost of the plant. The maximum number usually is four.

The reheating cycle and the regenerative cycle have been discussed separately in this and in the two preceding articles. It is evident, however, that the two cycles can be combined to form a **reheating-regenerative cycle.** Such cycles are economically justifiable only in large central station plants.

PROBLEMS

321. Suppose that the steam entering the turbine of the power plant shown in Fig. 64*a* is at 420 psia and 600°F, that the pressure in the open-type feedwater heater is 100 psia, and that the pressure in the condenser is 2 in. Hg abs. (*a*) Determine the ideal thermal efficiency of the plant. (*b*) What would the ideal thermal efficiency be if the feedwater heater were not used (that is, if the plant operated on the Rankine cycle)? *Ans.* (*a*) 36.0%.

322. What would be the ideal thermal efficiency of the power plant shown in Fig. 65 if (*a*) the steam supplied to the turbine were at 420 psia and 600°F, (*b*) the pressure in the first heater were 15 psia, (*c*) the pressure in the second heater were 100 psia, and (*d*) the pressure in the condenser were 2 in. Hg abs? Assume that the condensate leaving each heater would be saturated.

323. Suppose that the closed-type feedwater heaters shown in Fig. 65 are replaced by open-type heaters and that two pumps are added to the plant, one to pump the feedwater from the first heater into the second and the other to pump the feedwater from the second heater into the boiler. What would be the ideal thermal efficiency of the plant if the condition of the steam entering the turbine and the pressures in the feedwater heaters and in the condenser were the same as in the preceding problem? *Ans.* 36.9%.

324. A steam power plant has a high pressure and a low pressure turbine, a reheater, and one closed-type feedwater heater. Steam at 450 psia and 700°F enters the high pressure turbine and leaves at 80 psia. Part of the steam leaving this turbine flows into the reheater, is reheated to 600°F, and then flows into the low pressure turbine. The rest flows into the feedwater heater and condenses. The condensate from the heater flows through a trap into the condenser. If the pressure in the con-

denser is 1½ in. Hg abs, what is the ideal thermal efficiency of the plant? Assume that the condensate leaving the heater is saturated.

89. The Binary-Vapor Cycle. It has been shown that the highest efficiency attainable by any power plant operating between a constant temperature source of heat and a constant temperature receiver is that which would be obtained by a plant operating on a *reversible cycle.* It has also been shown how, by making use of regenerative feedwater heating, a steam power plant can be made to operate on a cycle that is approximately reversible. The only objection to such a plant is that, since water is used as the working fluid, the highest temperature at which heat can be received isothermally is below the critical temperature of steam. As explained in Art. 83, this objection could be overcome by using some other working fluid having a critical temperature above 1000°F, a triple point temperature below atmospheric temperature, and saturation pressures that are neither too high at 1000°F nor too low at atmospheric temperature. Unfortunately, no such fluid is known. One way of getting around this difficulty is to make use of what is known as the **binary-vapor cycle.** A power plant operating on this cycle uses two fluids, one having desirable characteristics at high temperatures and the other having desirable characteristics at low temperatures.

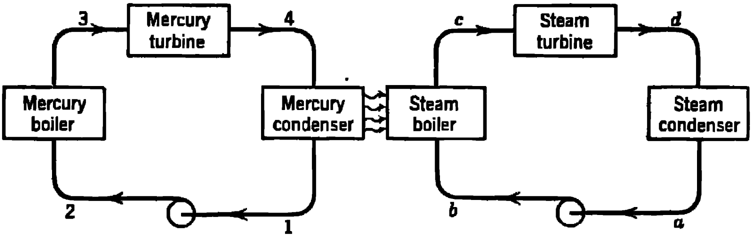


FIG. 66. Flow-diagram for a binary-vapor power plant.

The simplest practicable binary-vapor cycle is one in which each fluid undergoes a Rankine cycle. A flow-diagram for such a plant using mercury and water as the working fluids is shown in Fig. 66. The fluids need not be mercury and water, but we shall use them for purposes of illustration since they are the ones actually used in the few existing plants of this type. Note that the only heat taken from the source is that supplied to the mercury boiler, the only heat rejected to the receiver is that rejected in the steam condenser, and the heat rejected by the mercury in the mercury condenser is used to generate steam in the steam boiler.

A temperature-entropy diagram for the plant, based on the idealizing assumptions that the expansion in the two turbines is isentropic and that the temperature difference between the condensing mercury and the boiling water is infinitesimal, is shown in Fig. 67. Since the heat rejected in the mercury condenser is absorbed in the steam boiler, the diagram is constructed so that the area under the line from 4 to 1 is equal to the area under the constant pressure line from *b* to *c*. Note that, since the heat rejected by 1 lb of mercury in the mercury condenser is not equal to the heat absorbed by 1 lb of water flowing through the steam boiler, the rates at which the two fluids undergo their respective cycles are not equal. It happens that, for each pound of water undergoing the steam cycle, approximately 12 or 13 lb of mercury must undergo the mercury cycle. For this reason the abscissa of Fig. 67 is not the entropy per pound of each fluid but the total entropy of each.

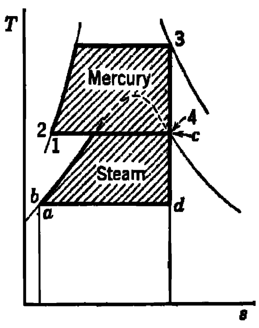


FIG. 67. Temperature-entropy diagram for the binary-vapor power plant shown in FIG. 66.

The ideal thermal efficiency of a plant operating on the binary-vapor cycle can be found by Eq. 35*a*. Let us denote the heat supplied to and rejected *by each pound* of the high temperature fluid by Q_{2-3} and Q_{4-1} (see Fig. 66), and let us denote the heat supplied to and rejected *by each pound* of the low temperature fluid by Q_{b-c} and Q_{d-a}. These quantities can be determined by the methods discussed in connection with the Rankine cycle. The number of pounds *n* of the high temperature fluid undergoing the high temperature cycle while 1 lb of the low temperature fluid undergoes its cycle is

$$n = \frac{Q_{b-c}}{Q_{4-1}}.$$

After this number is determined, the quantities Q_{in} and Q_{out} in Eq. 35*a* can be found since

$$Q_{\text{in}} = nQ_{2-3},$$

and

$$Q_{\text{out}} = Q_{d-a}.$$

Although binary-vapor power plants have higher thermal efficiencies than steam power plants, it is unlikely that many of them will be built. Their initial cost is so high that it usually more than offsets the decrease in operating cost.

PROBLEMS

325. Suppose that mercury is used as the working fluid in a power plant operating on the Rankine cycle. If the mercury supplied to the turbine is saturated at 1000°F (p = 180 psia), and if the vapor in the condenser condenses at 500°F (p = 1.9 psia), what is the ideal thermal efficiency of the plant? The value of c_p for liquid mercury may be taken as 0.0314 Btu per lb deg F, and the heat of vaporization of mercury at 1000°F as 124.2 Btu per lb. Neglect the work required to operate the pump.

326. The power plant described in the preceding problem could be made part of a binary-vapor plant by letting the heat rejected by the mercury be used to generate steam in a steam power plant operating on the Rankine cycle. If saturated steam at 500°F is supplied to the turbine of the steam plant, and if the steam in the condenser condenses at 100°F, what is the ideal thermal efficiency of the binary-vapor plant? *Ans.* 56.6%.

90. Losses Occurring in an Actual Steam Power Plant. Up to this point it has been assumed (*a*) that no pressure drop due to friction takes place in the piping, boiler, or condenser of a steam power plant and (*b*) that the turbines and pumps operate isentropically. It is of interest now to consider briefly some of the losses that occur in an actual plant. The more important of these losses are shown in Fig. 68.

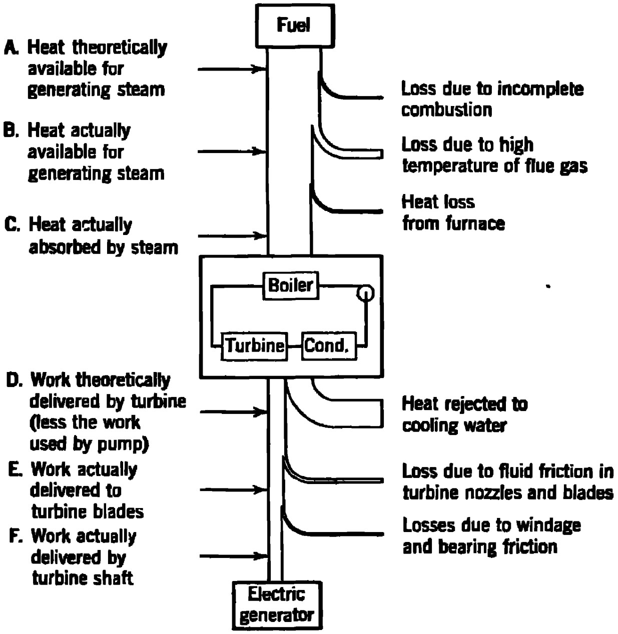

Fuel
A. Heat theoretically available for generating steam
B. Heat actually available for generating steam
C. Heat actually absorbed by steam
Loss due to incomplete combustion
Loss due to high temperature of flue gas
Heat loss from furnace
Boiler
Turbine
Cond.
D. Work theoretically delivered by turbine (less the work used by pump)
E. Work actually delivered to turbine blades
F. Work actually delivered by turbine shaft
Heat rejected to cooling water
Loss due to fluid friction in turbine nozzles and blades
Losses due to windage and bearing friction
Electric generator

FIG. 68. Diagram showing some of the more important losses occurring in a steam power plant.

Let us consider first the heat *theoretically* available when the fuel is burned (item A on the diagram). Since the combustion process in a steam power plant is a steady-flow process in which fuel and air enter the furnace and the products of combustion leave, we may apply the steady-flow energy equation. Assuming that the velocities of the streams entering and leaving the furnace are negligible and that any shaft-work supplied is also negligible, we find that the steady-flow energy equation reduces to

$$Q = H_2 - H_1,$$

or, if we denote the heat *leaving* the furnace by Q', to

$$Q' = H_1 - H_2,$$

where H_1 = the total enthalpy of the air and fuel entering the furnace, and

H_2 = the total enthalpy of the products of combustion leaving.

It is desired, of course, that Q' be as large as possible, and this in turn requires that the enthalpy H_2 of the products of combustion be as low as possible. The value of H_2 will be a minimum if (*a*) the combustion is very nearly complete, and (*b*) the products leave the furnace at atmospheric temperature and pressure. Hence, the maximum value of Q' (the heat theoretically available for generating steam) can be determined experimentally by burning the fuel completely in a steady-flow calorimeter and cooling the products to atmospheric temperature.

It is easy to carry out this experiment if the fuel is a liquid or a gas, but it is difficult if the fuel is a solid. It is much easier to burn a solid fuel *at constant volume* in a "bomb calorimeter." But, since this process is a non-flow process, the quantity determined is a change in internal energy rather than a change in enthalpy. It is not the true heating value of the fuel under the conditions existing in a steam power plant, but it is used regularly in evaluating the performance of such plants because it is easily determined. It may be remarked that heating values determined in a bomb calorimeter are somewhat higher than values determined in a steady-flow calorimeter because more of the water vapor resulting from combustion of the hydrogen in the fuel is condensed. Heating values determined in a bomb calorimeter are called **higher heating values.**

Now let us consider some of the losses that cause the heat *actually* available when fuel is burned (item B on the diagram) to be less than the heat theoretically available. One of these losses is the result of incomplete combustion. Because of an insufficient supply of air or because of poor mixing of the air and fuel while still in the combustion space, some combustible matter may be carried out with the gaseous products (usually in the form of soot, hydrogen, and carbon monoxide) and some with the solid refuse (in the form of unburned carbon). If the fuel is a solid, this loss can be reduced by pulverizing the fuel, by using more air than the amount theoretically required for complete combustion (called **excess air**), and by creating turbulence within the combustion space.

Another loss which causes the heat actually available to be less than the theoretical is due to the products of combustion leaving the boiler at temperatures well above atmospheric temperature. This loss can be reduced by decreasing the amount of excess air supplied (thereby decreasing the weight of the products of combustion). It can also be reduced by means of "economizers" and "air-preheaters." An **economizer** consists of a set of tubes over which the hot gases from the boiler flow and through which the feedwater flows before entering the boiler. An **air-preheater** is a heat exchanger that allows heat to flow from the hot gases leaving the economizer to the air used for combustion. Both these devices increase the heat available for generating steam by reducing the temperature at which the flue gases leave the plant.

Most of the heat actually available for generating steam is absorbed by the water in the boiler and in the economizer. A small fraction of it is lost to the atmosphere, however. This loss is kept small by covering the furnace with insulation.

Since the working fluid of the heat engine (which consists of the boiler, turbine, condenser, pump, and perhaps reheater and regenerative feedwater heaters) undergoes a cycle, all the heat absorbed by it leaves in the form of either work or heat. Most of the energy rejected in the form of heat is absorbed by the cooling water, although some of it is lost to the atmosphere through the insulation on the boiler, the piping, and the turbine. The amount of heat rejected to the cooling water can be decreased (and the theoretical work output from the plant thereby increased) only by changing to a more efficient cycle. However, even if the best conceivable cycle (any reversible cycle) were used, the heat rejected to the cooling water could never be reduced to zero. Note that, by comparison, all the other losses shown in Fig. 68 can, at least in theory, be made negligibly small.

The difference between the heat actually absorbed by the steam (item C) and the heat rejected to the cooling water is equal to the *net* work theoretically delivered at the turbine shaft (the turbine work minus the pump work). The work actually delivered is less than this because of fluid friction within the turbine nozzles and blades, windage within the turbine, and mechanical friction in the bearings. These losses have already been mentioned in Art. 85 in connection with turbine efficiencies.

The ratio of item C to item A in Fig. 68 may be called the **combined boiler and furnace efficiency.** This efficiency indicates how serious are the losses shown between items A and C. For a large power plant, it is likely to be between 80% and 90%. Note that in theory it can be made to approach 100%. The ratio of item D to item C is the **ideal thermal efficiency** of the heat engine, discussed in the foregoing articles. Its highest possible value is not 100% but is the efficiency of a reversible engine. The ratio of item F to item D is the **turbine efficiency,** discussed in Art. 85. It can in theory be made to approach 100%. The ratio of item F to item A may be called the **actual over-all thermal efficiency** of the plant. For modern plants at best load, it is likely to be between 25% and 35%.

SYMBOLS

h	enthalpy per unit mass
H	total enthalpy
J	mechanical equivalent of heat
n	weight of high temperature fluid circulated per unit weight of low temperature fluid
Q	a quantity of heat; $\bar{Q}$ = rate at which heat is added to a system in steady flow
s	entropy per unit mass
t	temperature
T	absolute temperature
w	weight
W'	shaft-work done by a system in steady flow; $\bar{W}'$ = rate at which shaft-work is done by a system in steady flow
η	thermal efficiency of a heat engine; η_t = turbine efficiency

Chapter 11

INTERNAL-COMBUSTION ENGINES

Steam power plants and internal-combustion engines are alike in that both are devices for converting into work part of the heat liberated when a fuel is burned. They are different, however, in that the working fluid of a steam power plant undergoes a *cycle* (in the thermodynamic sense) whereas that of an internal-combustion engine does not. Thus, the working fluid in a steam power plant returns periodically to the same state and repeats the same series of processes over and over. On the other hand, the working fluid of an internal-combustion engine is the fuel itself together with the air required to burn it. It flows through the engine only once and does not return to its initial state.

An exact analysis of the processes involved in the operation of an internal-combustion engine requires that the changes taking place in the chemical composition of the working fluid be taken into account. This could be done without too great difficulty if it were not for the fact that dissociation (the breaking up of a chemical compound into its constituents) takes place. This dissociation occurs during the combustion process and is caused by the high temperatures attained. Since an exact analysis is beyond the scope of a first course in engineering thermodynamics, only an approximate analysis is presented in this chapter. The approximation made is to replace the actual internal-combustion engine by an engine operating on what is known as an "air-standard cycle."

91. The Otto Engine. The commonest type of internal-combustion engine is the **Otto** engine. It consists of one or more cylinders, each provided with a piston, an intake valve for admitting the fresh charge of fuel and air, a spark plug for igniting this charge, and an exhaust valve for discharging the products of combustion. The operation of the engine can be understood by referring to the "indicator diagram" for the engine. Let us digress for a moment to consider the nature of such diagrams.

An **indicator diagram** shows the relation between the pressure inside the cylinder of a reciprocating engine and the position of the piston.

It consists of the pressure plotted as ordinate against the *cylinder volume* as abscissa. Since the weight of the fluid inside the cylinder of most reciprocating engines does not remain constant, the abscissa of an indicator diagram is not specific volume. In this respect the diagram differs from the pressure-volume diagrams drawn heretofore.

A typical indicator diagram for an Otto engine is shown in Fig. 69. At point 1 the cylinder contains a mixture of air and fuel (in the form of a vapor) plus a small amount of the products of combustion from the previous cycle of operations. Unless the engine is equipped with a supercharger, the pressure at this point is below atmospheric. With both the intake and exhaust valves closed, the piston moves inward and compresses the charge approximately adiabatically. As a result, both the pressure and temperature of the charge rise. Near the end of the compression stroke the charge is ignited by a spark. Since the piston is moving slowly at this time, and since the charge burns rapidly, the combustion process from 2 to 3 takes place nearly at constant volume. The piston next moves outward, and the gases within the cylinder expand approximately adiabatically. At point 4 the exhaust valve opens, and some of the products of combustion flow out to the atmosphere as the piston completes its outward travel. At point 5 the piston has returned to its initial position, but the cylinder now contains only the products of combustion. In order to discharge these and to draw in a fresh charge of fuel and air, the piston again travels inward, forcing most of the spent gases out through the exhaust valve. On account of the resistance to flow imposed by the exhaust valve and exhaust manifold, the pressure during this process is slightly above atmospheric. At point 6 the exhaust valve closes, and the intake valve opens. As the piston again travels outward, a fresh charge of fuel and air is drawn into the cylinder through the intake valve. On account of the resistance to flow in the carburetor, intake manifold, and intake valve, the pressure during this process is below atmospheric. The cycle of operations is completed when the piston reaches the end of its travel.

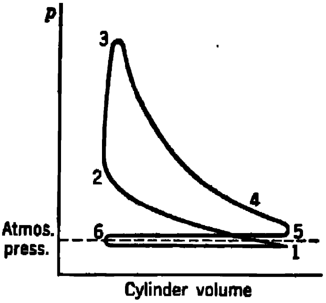


Fig. 69. Typical indicator diagram for a four-cycle Otto engine.

In order to complete the cycle of operations just described, it is necessary that the piston make four strokes. For this reason an engine operating on this cycle is called a **four-cycle** engine. It is also possible to complete the cycle of operations in two strokes of the piston by providing an auxiliary compressor to blow the fresh charge into the cylinder at the end of the expansion stroke. The fresh charge is directed into the cylinder in such a way that it forces most of the spent gases out through the exhaust opening. Since some of the fresh charge inevitably flows out with the spent gases, the fuel consumption of these engines is somewhat higher than for a four-cycle engine. They are known as **two-cycle** engines.

Returning to the indicator diagram for a four-cycle engine shown in Fig. 69, we note that the piston does work *on the gases* within the cylinder during the processes from 1 to 2 and from 5 to 6, and that the gases do work *on the piston* during the processes from 3 to 5 and from 6 to 1. Since these various amounts of work are represented by the areas under each of these curves, it is evident that the *net* work done on the piston during one complete cycle of operations is represented by the area of the upper loop on the diagram minus the area of the lower loop.

It is of interest to consider what the effect is on the indicator diagram shown in Fig. 69 if the load on the engine is decreased. The power delivered by the engine is controlled by a "throttle valve" located between the carburetor and the intake valve of the engine. Partially closing this valve increases the resistance to flow. As a result the pressure at point 1 is lowered, and the area of the lower loop is increased. Since the cylinder volume at point 1 remains unchanged, and since the temperature of the charge at this point is also approximately the same as before, lowering the pressure decreases the weight of the charge drawn into the cylinder. With less fuel inside the cylinder the pressure at 3 does not rise to as high a value, and the area of the upper loop on the diagram decreases. The result, therefore, is that the net work done decreases.

92. The Analysis of the Ideal Otto Engine. Now let us consider some of the factors that we should have to take into account if we wished to make an accurate analysis of the processes involved in the operation of an *ideal* four-cycle Otto engine. For an ideal engine the pressure drop through the inlet and exhaust valves would be zero, and the curves from 5 to 6 and from 6 to 1 in Fig. 69 would coincide with the atmospheric pressure line, as shown in Fig. 70. Since the area of the lower loop would become zero, this part of the cycle of operations could be disregarded in calculating the net work done. The compression from 1 to 2 would be adiabatic and reversible. Although the assumption that the specific heats c_v and c_p of the gas mixture remain constant during this process would not be permissible, the work done and the pressure and temperature of the mixture at 2 could all be determined without too great difficulty (see Problem 190). The combustion process from 2 to 3 would take place at constant volume, as shown in Fig. 70. If the combustion could be assumed to proceed to completion at 3, no serious difficulty would be encountered in determining the pressure and temperature at this point. Even in an ideal engine, however, the combustion would not be complete. Instead, on account of the high temperatures reached (in the order of 5000°F), *dissociation* would take place. Ideally, the expansion process from 3 to 4 would take place adiabatically and reversibly; but it would be necessary to take into account the fact that, as the temperature dropped during this expansion, the combustion could proceed more nearly to completion. Instead of the exhaust valve opening at point 4 shown in Fig. 69, in the ideal engine it would remain closed until the end of the expansion stroke, as shown in Fig. 70. Finally, in the ideal engine, when the exhaust valve opened, part of the spent gases would flow out of the cylinder so rapidly that the pressure would drop to atmospheric pressure instantaneously, as shown by the vertical line from 4 to 5 in Fig. 70.

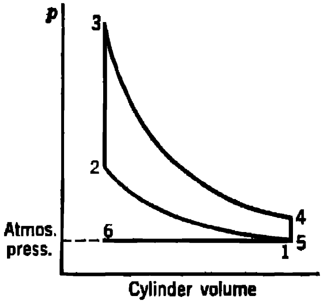


FIG. 70. Ideal indicator diagram for a four-cycle Otto engine.

It is possible to calculate the ideal thermal efficiency of the Otto engine, taking into account the various factors mentioned in the preceding paragraph.* Such an analysis is beyond the scope of this text, however. In place of it the analysis of a cycle known as the "air-standard Otto cycle" is presented. This analysis is of particular value in showing how the compression ratio affects the thermal efficiency of an Otto engine.

93. The Air-Standard Otto Cycle. The **air-standard cycle** for any internal-combustion engine is the cycle on which a heat engine using *air* as its working fluid must operate to make the pressure-volume diagram for the heat engine *look like* the indicator diagram for the internal-combustion engine. The manner in which the heat engine operates, however, is quite different from that in which the internal-combustion engine operates. It differs in the following respects:

(*a*) The working fluid of the heat engine is air alone, and the weight of air inside the engine remains constant. The cylinder of an internal-combustion engine, on the other hand, contains gases that vary both in chemical composition and in amount.

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* H. C. Hottel, G. C. Williams, and C. N. Satterfield, *Thermodynamic Charts for Combustion Processes*, John Wiley and Sons, Inc., 1949.

(*b*) The working fluid of the heat engine returns periodically to the same state, whereas the working fluid of an internal-combustion engine does not.

(*c*) During one part of its cycle the working fluid of the heat engine receives heat from a source external to the engine. Ideally, the combustion process in an internal-combustion engine takes place adiabatically.

(*d*) During another part of its cycle the working fluid of the heat engine rejects heat to a receiver external to the engine. The cycle of operations in an internal-combustion engine is completed by letting the products of combustion flow out of the engine and a fresh charge of fuel and air flow in.

Because of the foregoing differences, the results obtained from the analysis of an air-standard cycle are of limited utility only. However, conclusions of some value can frequently be drawn.

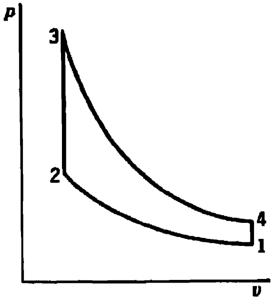


FIG. 71. Pressure-volume diagram for the air-standard Otto cycle.

The pressure-volume diagram for the **air-standard Otto cycle** is shown in Fig. 71. The cycle is made up of the following processes: Air initially at atmospheric pressure and temperature (state 1) is first compressed isentropically to state 2; it is then heated at constant volume to state 3 by heat taken from a source external to the engine; the air is next expanded isentropically to state 4; and, finally, it is cooled at constant volume back to state 1. The heat rejected during the last process is rejected to a receiver external to the engine. Note the similarity between Figs. 70 and 71.

We can calculate the ideal thermal efficiency of a heat engine operating on the air-standard Otto cycle by Eq. 35*a*:

$$\eta = \frac{Q_{\text{in}} - Q_{\text{out}}}{Q_{\text{in}}}. \tag{35a}$$

Let us suppose, for example, that the following data are given: (*a*) the temperature T_1 of the air at state 1 (Fig. 71), (*b*) the ratio v_1/v_2 of the volume of the air when the piston is at the extremes of its travel (called the **compression ratio** and denoted by the symbol r_c), and (*c*) the heat Q_{in} supplied to the air during the constant volume process from 2 to 3. We must determine the heat Q_{out} rejected during the constant volume process from 4 to 1. Since we can easily determine this quantity after we have found the temperature at state 4, the problem reduces to one of finding T_4.

Assuming that the specific heats c_v *and* c_p *for air are constant,* we can calculate the temperature T_2 at the end of the isentropic compression from 1 to 2 by Eq. 28*a*. Thus,

$$T_2 = T_1\left(\frac{v_1}{v_2}\right)^{k-1} = T_1 r_c^{\,k-1}.$$

Then, knowing the temperature T_2 and the heat Q_{in} supplied to the air during the constant volume process from 2 to 3, we can calculate the temperature T_3 by the equation

$$Q_{in} = c_v(T_3 - T_2).$$

Finally, since $v_4/v_3 = v_1/v_2 = r_c$, we can calculate the temperature T_4 at the end of the isentropic expansion from 3 to 4 by again using Eq. 28*a*:

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = T_3\left(\frac{1}{r_c}\right)^{k-1}.$$

The foregoing method of analysis may be applied, not only to the air-standard Otto cycle, but to any other air-standard cycle. It happens that a simple expression for the ideal thermal efficiency of the air-standard Otto cycle can be derived in terms of the compression ratio alone. Thus, since $Q_{in} = c_v(T_3 - T_2)$ and $Q_{out} = c_v(T_4 - T_1)$, Eq. 35*a* may be written

$$\begin{aligned}\eta &= \frac{Q_{in} - Q_{out}}{Q_{in}}\\ &= 1 - \frac{Q_{out}}{Q_{in}}\\ &= 1 - \frac{T_4 - T_1}{T_3 - T_2}.\end{aligned}$$

Introducing the expressions for T_2 and T_4 stated in the preceding paragraph, we get

$$\begin{aligned}\eta &= 1 - \frac{T_3(1/r_c)^{k-1} - T_1}{T_3 - T_1 r_c^{\,k-1}}\\ &= 1 - \left(\frac{1}{r_c}\right)^{k-1}\left(\frac{T_3 - T_1 r_c^{\,k-1}}{T_3 - T_1 r_c^{\,k-1}}\right)\\ &= 1 - \left(\frac{1}{r_c}\right)^{k-1} \qquad (49)\end{aligned}$$

Hence, the ideal thermal efficiency of the air-standard Otto cycle depends only upon the compression ratio r_c and increases as r_c increases.

It is of interest to compare the ideal thermal efficiency of the air-standard Otto cycle with the actual thermal efficiency of an Otto engine. Figure 72 shows the air-standard Otto cycle efficiency, calculated by Eq. 49, and the actual thermal efficiency of an Otto engine plotted against the compression ratio r_c. Note that, although the first curve is higher than the second, they have the same general shape.

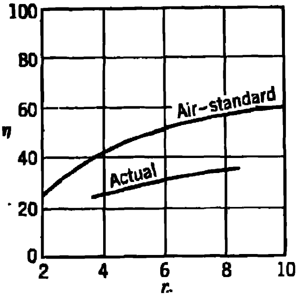


Fig. 72. Comparison of air-standard and actual thermal efficiencies for the Otto engine.

As shown in Fig. 72, the thermal efficiency of an actual Otto engine can be increased by increasing the compression ratio of the engine. There is an upper limit above which the compression ratio may not be raised, however. If the ratio is too high, the temperature during the compression stroke becomes so high that the charge of fuel and air ignites prematurely. Also, **detonation** (the "knock" resulting from high localized pressures within the combustion space) is likely to occur and injure the engine. At the present time compression ratios between 6 and 8 are the highest commonly used.

The reason why the ideal thermal efficiency of the air-standard Otto cycle increases as the compression ratio increases can be shown graphically by means of the temperature-entropy diagram for the cycle. The temperature-entropy diagrams for two cycles, both having the same heat input, but one having a compression ratio of 4 and the other of 8, are shown in Fig. 73. The states numbered on these diagrams correspond to the states numbered on the pressure-volume diagram shown in Fig. 71. The temperature T_1 at the start of the compression process is assumed to be the same for both cycles. For this reason the temperature T_2 at the end of the process is higher for the cycle having the higher compression ratio, as shown in Fig. 73. Since the heat supplied is assumed to be the same for both cycles, the diagrams are drawn so that the areas under the constant volume curves from 2 to 3 are equal. Since the heat rejected is represented by the areas under the constant volume curves from 4 to 1, the shaded areas represent the net work done. It is evident from the diagrams why the ideal thermal efficiency (represented by the ratio of the shaded area to the total area under curve 2–3) is greater for the cycle having the higher compression ratio.

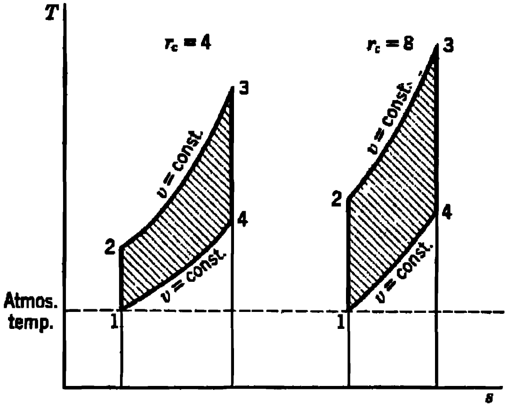


FIG. 73. Temperature-entropy diagrams for two air-standard Otto cycles.

PROBLEMS

327. What would be the thermal efficiency of a heat engine operating on the air-standard Otto cycle if (*a*) the temperature of the air at the start of the compression process were 70°F, (*b*) the compression ratio r_c were 6, and (*c*) 500 Btu of heat were supplied per pound of air during the constant volume heat reception process? Determine the efficiency by Eq. 35*a* after finding the heat rejected per pound of air, and then check your answer by means of Eq. 49. Assume that c_v for air remains constant and is equal to 0.172 Btu per lb deg F.

328. Suppose a heat engine operates on a cycle similar to the air-standard Otto cycle except that, referring to Fig. 71, (*a*) the expansion process 3–4 is continued to the point where $p_4 = p_1$, and (*b*) the constant volume process 4–1 is replaced by a constant pressure process. What would be the thermal efficiency of the engine if the temperature of the air at state 1 were 70°F, the ratio of v_1 to v_2 were 6, and the heat supplied per pound of air during the process 2–3 were 500 Btu? Assume that c_v and c_p for air are constants and equal to 0.172 and 0.241 Btu per lb deg F respectively. *Ans.* 61.0%.

329. Suppose that a heat engine operates on the air-standard Otto cycle and that, referring to Fig. 71, the temperature of the air at state 1 is 70°F and at state 3 is 3500°F. What would be the thermal efficiency of the engine if the heat supplied during the process 2–3 were such that the maximum possible work per cycle were delivered by the engine? Assume that c_v for air is a constant.

94. The Diesel Engine and the Dual-Combustion Engine.

A high compression ratio is desirable, not only in an Otto engine, but in any internal-combustion engine. This is true because increasing the compression ratio raises the temperature at the end of the compression stroke, and as a result the combustion process takes place at higher temperatures. This, in turn, causes the thermal efficiency of the engine to be higher, as was shown in Fig. 73 for the air-standard Otto cycle.

It was explained in the preceding article that the highest compression ratio permissible in an Otto engine is fixed by the compression ratio at which detonation begins. A higher compression ratio could be used if, instead of compressing a combustible mixture, air alone were compressed and the fuel were not introduced until after the compression was completed. This is done in the Diesel engine.

A **Diesel** engine, like an Otto engine, may have one or more cylinders, each of which is provided with a piston, an intake valve, and an exhaust valve. Unlike an Otto engine, however, a Diesel engine has a nozzle in each cylinder for injecting the fuel but does not have spark plugs. The operation of a Diesel engine can be explained with the help of an indicator diagram.

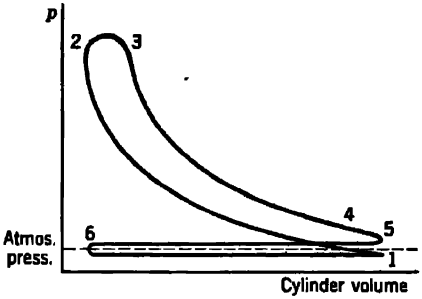


Fig. 74. Typical indicator diagram for a four-cycle Diesel engine.

A typical indicator diagram for a Diesel engine is shown in Fig. 74. At point 1 the cylinder contains air plus a small amount of the products of combustion from the previous cycle of operations. With both the intake and exhaust valves closed, the piston moves inward, compressing the charge of air approximately adiabatically. As the piston starts its outward travel, liquid fuel is injected into the cylinder in a finely atomized spray. The fuel ignites because of the high temperature of the air at point 2, and the heat released causes the pressure to remain at a high value. The fuel injection stops at point 3, and the products of combustion expand approximately adiabatically from 3 to 4. The exhaust valve opens at 4, and, as the piston completes its outward travel, some of the spent gases flow out the exhaust valve. To purge most of the remaining spent gases from the cylinder, the piston again travels inward, forcing more of these gases out the exhaust valve. At point 6 the exhaust valve closes, and the intake valve opens; and, as the piston travels outward, a fresh charge of air is drawn into the cylinder.

The cycle of operations just described is that of a **four-cycle** Diesel engine. Like an Otto engine, a Diesel engine can also operate as a

two-cycle engine if an auxiliary compressor is provided to blow the fresh charge of air into the cylinder at the end of the expansion stroke.

The speed and power output from a Diesel engine are controlled, not by a throttle valve in the air intake manifold, but by varying the amount of fuel injected. If less fuel is injected, point 3 in Fig. 74 moves to the left, the area of the upper loop decreases, and the net work done decreases.

In the *ideal* Diesel engine the fuel is injected at a rate such that the combustion process from 2 to 3 takes place at *constant pressure.* If the fuel is injected in such a way that the combustion process takes place, not at constant pressure only, but first at constant volume and then at constant pressure, the engine is called an ideal **dual-combustion** engine. The operation of an actual Diesel engine is between that of the ideal Diesel and the ideal dual-combustion engine.

95. The Air-Standard Diesel and Dual-Combustion Cycles.

The pressure-volume diagram for the **air-standard Diesel cycle** is shown in Fig. 75. It consists of an isentropic compression from 1 to 2, a constant pressure heating from 2 to 3, an isentropic expansion from 3 to 4, and a constant volume cooling from 4 back to 1.

The ideal thermal efficiency of the air-standard Diesel cycle can be determined by Eq. 35*a*. Suppose, for example, that we know (*a*) the temperature at state 1, (*b*) the compression ratio r_c $(= v_1/v_2)$, and (*c*) the amount of heat Q_{in} supplied to the air during the constant pressure process from 2 to 3. We must determine the heat Q_{out} rejected during the constant volume process from 4 to 1, which we can do easily after we have found the temperature at state 4. To find T_4 we may proceed as follows:

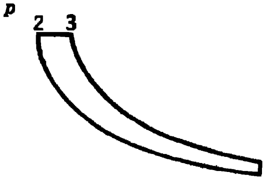


FIG. 75. Pressure-volume diagram for the air-standard Diesel cycle.

Assuming that c_v and c_p for air are constant, we can find T_2 by Eq. 28*a*:

$$T_2 = T_1 \left(\frac{v_1}{v_2}\right)^{k-1} = T_1 r_c^{k-1}.$$

We can now calculate T_3 by the relation

$$Q_{in} = c_p(T_3 - T_2).$$

Next, we can find the ratio v_4/v_3 (called the **expansion ratio** and denoted by the symbol r_e) by making use of the fact that

$$\frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3},$$

whence

$$\frac{v_2}{v_3} = \frac{T_2}{T_3},$$

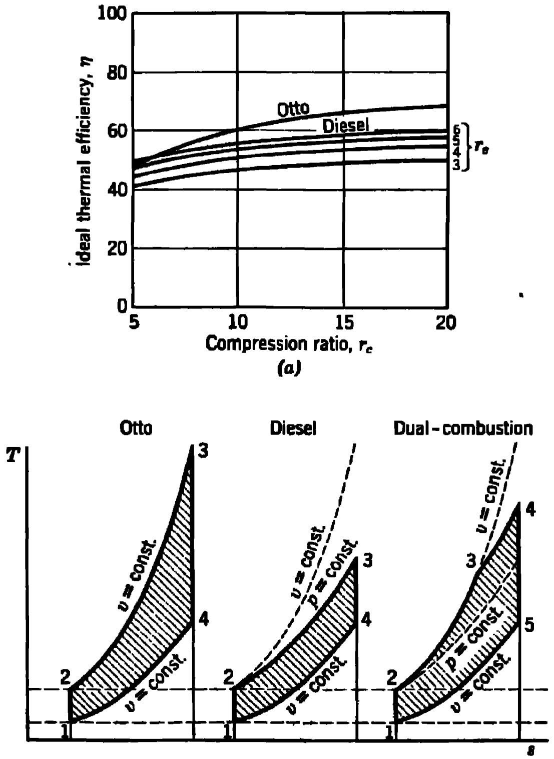
and therefore

$$r_e = \frac{v_4}{v_3} = \frac{v_1}{v_3} = \frac{v_1}{v_2} \times \frac{v_2}{v_3} = r_c \times \left(\frac{T_2}{T_3}\right).$$

Finally, after we have found the expansion ratio r_e by the above equation, we can calculate the temperature T_4 by Eq. 28*a*:

$$T_4 = T_3\left(\frac{v_3}{v_4}\right)^{k-1} = T_3\left(\frac{1}{r_e}\right)^{k-1}$$

Just as the ideal thermal efficiency of the air-standard Otto cycle depends only upon the compression ratio r_c (Eq. 49), the ideal thermal



(a)

(b)

FIG. 76. Comparison of the ideal thermal efficiencies of the air-standard Otto and Diesel cycles.

efficiency of the air-standard Diesel cycle depends only upon the compression ratio r_c and the expansion ratio r_e. Thus, it can be shown that the ideal thermal efficiency of the Diesel cycle is

$$\eta = 1 - \frac{1}{k}\left[\frac{(1/r_e)^k - (1/r_c)^k}{(1/r_e) - (1/r_c)}\right]. \tag{50}$$

It is interesting to compare the ideal thermal efficiencies of the air-standard Otto and Diesel cycles. The values shown in Fig. 76*a* have been calculated by means of Eqs. 49 and 50. Note that for the same compression ratio r_c the efficiency of the Otto cycle is higher than that of the Diesel cycle. This can be explained by means of the temperature-entropy diagrams for the two cycles, shown in Fig. 76*b*. It is assumed that the temperature T_1, the compression ratio r_c, and (for simplicity) the heat Q_{out} are the same for both cycles. With the same T_1 and the same r_c, T_2 is the same for both cycles. Also, with Q_{out} the same for both, the curves from 4 to 1 are identical. Therefore, since a constant volume curve starting at point 2 is steeper than a constant pressure curve starting at the same point (see Art. 77), the ideal thermal efficiency (represented by the ratio of the shaded area to the total area under the curve from 2 to 3) is greater for the air-standard Otto cycle than for the air-standard Diesel cycle. The *actual* thermal efficiency of a Diesel engine is usually higher than that of an Otto engine, however, since a higher compression ratio may be used in a Diesel engine. It should be noted also that a Diesel engine has an advantage over an Otto engine in that it can operate on a less highly refined and consequently cheaper fuel.

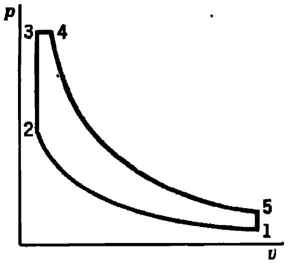


FIG. 77. Pressure-volume diagram for the air-standard dual-combustion cycle.

The pressure-volume diagram for the **air-standard dual-combustion cycle** is shown in Fig. 77. The ideal thermal efficiency of this cycle can be calculated by Eq. 35*a* if the temperature T_1, the compression ratio r_c $(=v_1/v_2)$, and the amounts of heat supplied to the air during the processes from 2 to 3 and from 3 to 4 are given. The temperature T_5 must be determined, and this can be done by first calculating the temperature of the air at states 2, 3, and 4. Note that for this cycle

$$r_e = \frac{v_5}{v_4} = \frac{v_1}{v_4} = \frac{v_1}{v_3} \times \frac{v_3}{v_4} = r_c \times \frac{T_3}{T_4}.$$

If the temperature T_1 and the compression ratio r_c are the same for the air-standard Otto, Diesel, and dual-combustion cycles, the ideal thermal efficiency of the dual-combustion cycle falls between that of the Otto and that of the Diesel cycles. That this is true is evident from the temperature-entropy diagram for the air-standard dual-combustion cycle, shown in Fig. 76*b*.

PROBLEMS

330. Derive Eq. 50.

331. What would be the thermal efficiency of a heat engine operating on the air-standard Diesel cycle if (*a*) the temperature of the air at the start of the compression process were 70°F, (*b*) the compression ratio r_c were 15, and (*c*) 500 Btu of heat were supplied per pound of air during the constant pressure heat reception process? Determine the efficiency by Eq. 35*a* after finding the heat rejected per pound of air. Assume that c_v and c_p for air remain constant. *Ans.* 58.8%.

332. Figure 76*a* shows that, if the compression ratio of a heat engine operating on the air-standard Diesel cycle is kept constant, the thermal efficiency of the engine is increased by increasing its expansion ratio. If such an engine has a compression ratio of 15, what is the maximum thermal efficiency it can conceivably have?

333. Figure 76*a* shows that, if the compression ratio of a heat engine operating on the air-standard Diesel cycle is kept constant, the thermal efficiency of the engine is lowered by decreasing its expansion ratio. If such an engine has a compression ratio of 5, what is the lowest thermal efficiency it can have? *Ans.* 20.2%.

334. What would be the thermal efficiency of a heat engine operating on the air-standard dual-combustion cycle if (*a*) the temperature of the air at the start of the compression process were 70°F, (*b*) the compression ratio r_c were 15, (*c*) 250 Btu of heat were supplied per pound of air during the constant volume heat reception process, and (*d*) 250 Btu of heat were supplied per pound of air during the constant pressure heat reception process? Assume that c_v and c_p for air remain constant.

96. The Gas Turbine Power Plant. Since the volume of working fluid which can flow through a reciprocating engine is small compared to that which can be handled by a turbine type of engine, it is impossible to build either an Otto engine or a Diesel engine in sizes comparable to those of large steam power plants. This inherent disadvantage of reciprocating internal-combustion engines has stimulated interest in the gas turbine power plant.

The simplest **gas turbine power plant** consists of an air compressor, a combustion chamber, and a gas turbine, arranged as shown in Fig. 78. Its method of operation is as follows: Air enters the compressor at atmospheric pressure and temperature and is compressed to a pressure usually in the order of 60 or 70 psia. This compression process is carried out in a multistage axial-type compressor because large volumes of air must be compressed. The air leaving the compressor flows into the combustion chamber, where a stream of liquid fuel is sprayed into it. The fuel is burned at constant pressure, and the rise in temperature resulting from this combustion process causes the volume of the gases leaving the combustion chamber to be greater than the volume of the air entering. It may be mentioned that the *rate* at which fuel may be burned is governed by the temperature at which the products of combustion may enter the gas turbine. To avoid injury to the turbine, the gases entering it must not be at a temperature higher than about 1200° or 1300°F. The products of combustion expand to atmospheric pressure in the turbine and finally discharge from the turbine to the atmosphere. Part of the work from the turbine drives the compressor, and the remainder is the net work output from the plant. It is evident that, unless both the compressor and the turbine are reasonably efficient, the net work output might be zero or might even be negative. Although it has been possible to build fairly efficient turbines for many years, only recently have the efficiencies of axial compressors been improved to the point where power plants of this type can be considered practicable.

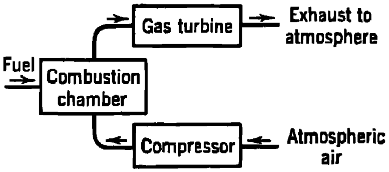


FIG. 78. Flow-diagram for a simple gas turbine power plant.

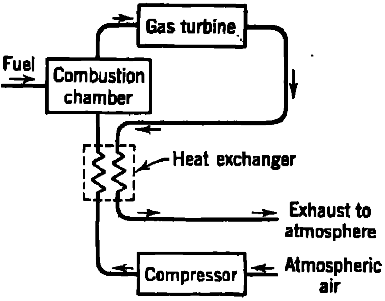
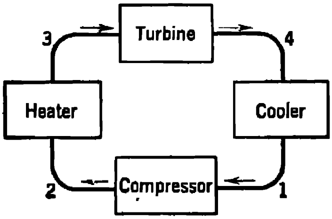


FIG. 79. Flow-diagram for a gas turbine power plant with a regenerative heat exchanger.

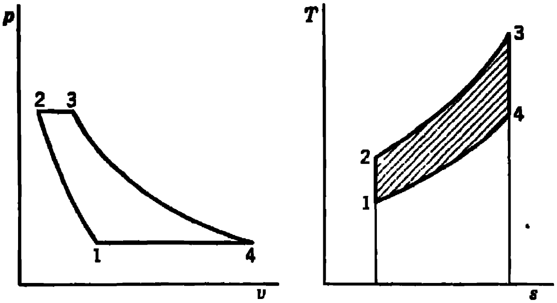
The temperature of the gases leaving the turbine is well above the temperature of the air entering the combustion chamber. Therefore, the thermal efficiency of the plant can be increased by adding a heat exchanger, as shown in Fig. 79. With such a regenerative heat exchanger added, less fuel must be burned to raise the gases flowing through the combustion chamber to the temperature desired at the turbine inlet.

Other more complex gas turbine power plants have been proposed, but the two described illustrate the basic principles on which all such plants operate.

97. The Air-Standard Gas Turbine Cycle. If a heat engine were built to operate on the simple **air-standard gas turbine cycle,** its flow-diagram would be as shown in Fig. 80*a*. Its manner of operation would be as follows: At section 1 the air would be at atmospheric pressure and temperature. It would first be compressed isentropically in the compressor and then heated at constant pressure in the heater. The heat supplied would be taken from a source external to the engine. The air would be expanded isentropically back to atmospheric pressure in the turbine, and finally it would be cooled at constant pressure back to atmospheric temperature in the cooler. Each of the processes making up the cycle would be a steady-flow process. The states through which the air would pass are shown on a pressure-volume diagram and on a temperature-entropy diagram in Figs. 80*b* and 80*c*.



(*a*) Flow-diagram



(*b*) Pressure-volume diagram (*c*) Temperature-entropy diagram

FIG. 80. The simple air-standard gas turbine cycle.

It is interesting to note the similarity between the flow-diagram for an engine operating on the simple air-standard gas turbine cycle and that for a steam power plant operating on the Rankine cycle (Fig. 35). If each plant operated in an ideal manner, the processes from 1 to 2 and 3 to 4 would be isentropic and the processes from 2 to 3 and 4 to 1 would be at constant pressure. The plants would differ in that the working fluid of the steam plant would undergo a change of phase whereas that of the air plant would not. They would also differ in that the work required to drive the pump in the steam plant would be only a small fraction of the work output from the turbine, whereas the work required to drive the compressor in the air plant would be a relatively large fraction of the turbine output. For this reason a low pump efficiency has little effect on the actual thermal efficiency of a steam power plant, whereas a low compressor efficiency greatly reduces the actual thermal efficiency of a gas turbine power plant.

The ideal thermal efficiency of the simple air-standard gas turbine cycle can be calculated by Eq. 35*a*. Suppose, for example, that the pressure p_1 in the cooler, the pressure p_2 in the heater, the temperature T_1 entering the compressor, and the temperature T_3 entering the turbine are all given. Suppose, also, that *c_v and c_p for air are assumed to be constant.* The temperature T_2 leaving the compressor and the temperature T_4 leaving the turbine can be calculated by Eq. 28*b* since both the compressor and the turbine are assumed to operate isentropically. Thus, denoting the ratio of the heater pressure to the cooler pressure by the symbol r_p $(= p_2/p_1 = p_3/p_4)$, we may write

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k} = T_1 r_p^{(k-1)/k},$$

and

$$T_4 = T_3\left(\frac{p_4}{p_3}\right)^{(k-1)/k} = T_3\left(\frac{1}{r_p}\right)^{(k-1)/k}$$

The heat Q_{in} supplied to the air as it flows through the heater and the heat Q_{out} rejected by the air as it flows through the cooler can next be calculated by the steady-flow energy equation (Eq. 13*a*). Thus, assuming that the velocities of the air at sections 1, 2, 3, and 4 are all negligible, and making use of the fact that for a perfect gas $\Delta h = c_p\,\Delta t$ (Eq. 18*a*), we can apply the steady-flow energy equation to the heater and to the cooler to obtain the equations

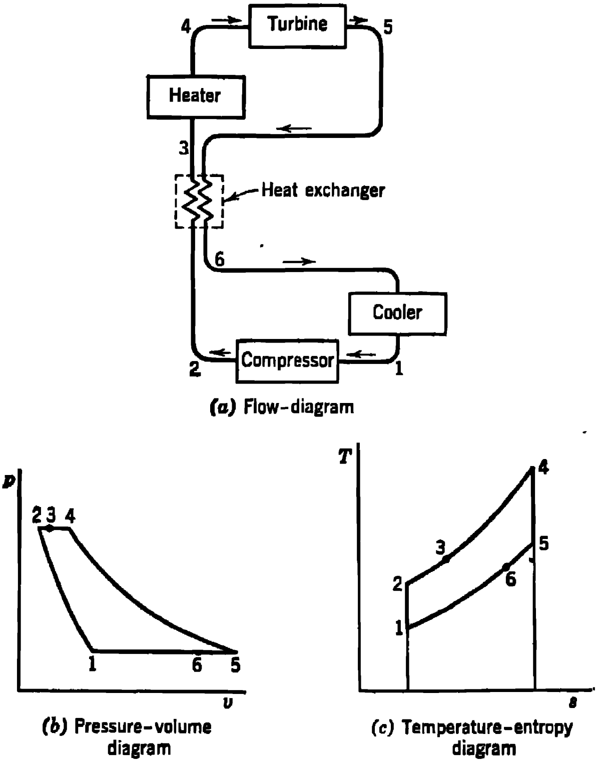
$$Q_{\text{in}} = h_3 - h_2 = c_p(T_3 - T_2)$$

and

$$Q_{\text{out}} = h_4 - h_1 = c_p(T_4 - T_1).$$

Having determined Q_{in} and Q_{out}, we can calculate the ideal thermal efficiency by Eq. 35*a* (see also Problem 336).

The flow-diagram, pressure-volume diagram, and temperature-entropy diagram for the air-standard gas turbine cycle with a regenerative heat exchanger added are shown in Fig. 81. The ideal thermal efficiency of this cycle can be calculated by Eq. 35*a* after the temperatures entering and leaving the heater and the cooler are known. Suppose, for example, that the pressure in the cooler, the pressure in the heater, and the temperatures at sections 1, 4, and 6 are all given. The temperatures at sections 2 and 5 can be calculated by Eq. 28b, as explained in the preceding paragraph. The temperature at section 3 can be determined by applying the steady-flow energy equation (Eq. 14) to the heat exchanger. Since the rates of flow of the two streams passing through the exchanger are equal, since the velocities at which each stream enters and leaves may be assumed to be negligible, and since the exchanger may be assumed to be perfectly insulated, Eq. 14 reduces to



(a) Flow-diagram

(b) Pressure-volume diagram

(c) Temperature-entropy diagram

FIG. 81. The air-standard gas turbine cycle with a regenerative heat exchanger.

$$h_2 + h_5 = h_3 + h_6.$$

But, by Eq. 18a, $h_3 - h_2 = c_p(T_3 - T_2)$ and $h_5 - h_6 = c_p(T_5 - T_6)$. Hence, the foregoing equation reduces to

$$T_3 = T_2 + T_5 - T_6.$$

Finally, after the temperatures entering and leaving the heater and the cooler have been determined, Q_{in} and Q_{out} can be calculated, and the ideal thermal efficiency can be found.

PROBLEMS

335. What would be the thermal efficiency of a heat engine operating on the simple air-standard gas turbine cycle if (*a*) air entered the compressor at 14.5 psia and 70°F, (*b*) it left the compressor at 65 psia, and (*c*) it entered the turbine at 1250°F? Assume that c_v and c_p for air remain constant.

336. Show that the thermal efficiency of a heat engine operating on the simple air-standard gas turbine cycle is given by the expression

$$\eta = 1 - \left(\frac{1}{r_p}\right)^{(k-1)/k}$$

where r_p denotes the ratio of the heater pressure to the cooler pressure. Check your answer to the preceding problem by this expression.

337. Suppose that a heat engine operated on the air-standard gas turbine cycle shown in Fig. 81. What would be the thermal efficiency of the engine if (*a*) the pressure and temperature of the air at section 1 were 14.5 psia and 70°F, (*b*) the pressure and temperature at section 4 were 65 psia and 1250°F, and (*c*) the temperature at section 6 were the same as at section 2? Assume that c_v and c_p for air remain constant. *Ans.* 52.4%.

338. Suppose that a heat engine operates on the simple air-standard gas turbine cycle and that, referring to Fig. 80, the pressure and temperature of the air at section 1 are 14.5 psia and 70°F and the temperature at section 3 is 1250°F. (*a*) What would the pressure in the heater have to be in order for the engine to deliver the maximum possible amount of work per pound of air circulated? (*b*) What would be the thermal efficiency of the engine with this heater pressure?

98. Actual Thermal Efficiency and Specific Fuel Consumption. In several of the foregoing articles the method of calculating the *ideal* thermal efficiency of various *air-standard* internal-combustion engine cycles has been discussed. Let us now consider how the *actual* thermal efficiency of the internal-combustion engines themselves can be found.

The **actual thermal efficiency** of an internal-combustion engine is defined as the work actually delivered at the engine shaft per pound of fuel consumed divided by the heating value of the fuel. If the horsepower delivered at the engine shaft and the rate of fuel consumption are known, the actual thermal efficiency of the engine can be calculated by the equation

$$\eta_{\text{act}} = \frac{\text{bhp} \times 2545}{w \times HV}, \qquad (51)$$

where bhp = the horsepower delivered at the engine shaft (called the **shaft or brake horsepower**),

w = the rate of fuel consumption, in pounds per hour,

HV = the heating value of the fuel, in British thermal units per pound, and

2545 = the number of British thermal units in a horsepower-hour.

The heating value of the fuel is usually taken arbitrarily as its *higher heating value* (see Art. 90).

It is sometimes simpler to compare the performance of internal-combustion engines on the basis of their "specific fuel consumptions" rather than on the basis of their actual thermal efficiencies. The **specific fuel consumption** of an internal-combustion engine is defined as the pounds of fuel consumed per horsepower-hour output at the engine shaft. If the pounds of fuel consumed per hour and the horsepower delivered by the engine are known, the specific fuel consumption can be calculated by dividing the first of these two rates by the second; that is,

$$\text{specific fuel consumption} = \frac{w}{\text{bhp}}. \tag{52}$$

Note that the specific fuel consumption is more easily determined than the actual thermal efficiency, but it does not take into account the nature of the fuel burned. Consequently, it is not an accurate basis for comparing the performance of various engines unless all of them use the same fuel.

If the ideal thermal efficiency η of an internal-combustion engine and the heating value HV of the fuel are known, the *ideal* specific fuel consumption of the engine can be calculated by the equation

$$\text{ideal specific fuel consumption} = \frac{2545}{\eta \times HV}. \tag{53}$$

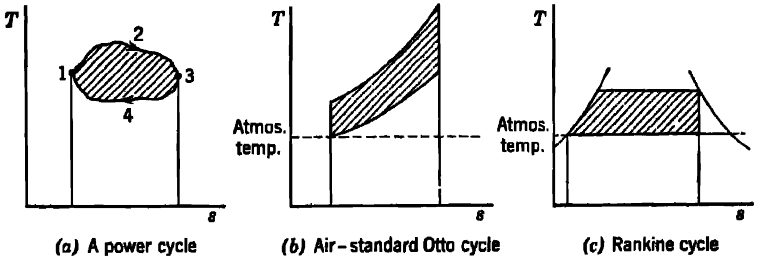
PROBLEMS

339. An Otto engine consumes 16 lb of gasoline during a 20-min test. The engine runs at a speed of 1800 rpm and delivers a torque of 260 lb-ft. If the gasoline has a higher heating value of 20,500 Btu per lb, what is the actual thermal efficiency of the engine? What is its specific fuel consumption?

340. If the engine in the preceding problem has a compression ratio of 5.5, what is its air-standard ideal thermal efficiency? What is its ideal specific fuel consumption on the basis of this efficiency?

99. Comparison of Internal-Combustion Engines and Steam Power Plants. The thermal efficiency of any heat engine is increased by (*a*) raising the temperature at which its working fluid receives heat or (*b*) lowering the temperature at which its working fluid rejects heat. Suppose, for example, that the working fluid undergoes the cycle shown in Fig. 82*a*. Since the ideal thermal efficiency of the engine is represented by the ratio of the enclosed area (the shaded area) to the total area under the curve 1–2–3, it is evident that the thermal efficiency can be increased either by raising the temperature during the heat reception process 1–2–3 or by lowering the temperature during the heat rejection process 3–4–1.

Now let us consider how a heat engine operating on the air-standard Otto cycle and a steam power plant operating on the Rankine cycle compare in regard to the temperatures at which their working fluids receive and reject heat. The temperature-entropy diagrams for the two cycles are shown in Figs. 82*b* and 82*c*. It is evident from these diagrams that the air-standard Otto cycle is superior to the Rankine cycle in that the average temperature during the heat reception process is higher for the Otto cycle. However, it is also apparent that the Rankine cycle is superior to the Otto cycle in that the average temperature during the heat rejection process is lower for the Rankine cycle. The same comparison holds for any air-standard internal-combustion engine cycle and any steam power plant cycle. Let us consider why this is true.



(*a*) A power cycle (*b*) Air-standard Otto cycle (*c*) Rankine cycle

FIG. 82. Temperature-entropy diagrams for the air-standard Otto cycle and for the Rankine cycle.

It was explained in Art. 81 that the highest temperature to which the working fluid in a steam power plant may be raised is fixed by the temperature to which the retaining wall separating the working fluid from the products of combustion may be heated. Although the temperature of the working fluid in a heat engine operating on an *air-standard* cycle would be subject to the same restriction, the working fluid in an actual internal-combustion engine is not. This is true because heat is not transferred to the working fluid of an internal-combustion engine through the cylinder walls of the engine. Instead, the temperature of its working fluid is raised by the combustion process *occurring within the fluid itself.* The cylinder walls of the engine must be kept at a relatively low temperature to avoid their being injured and to make lubrication of the piston possible, but this does not restrict the temperature to which the gases within the cylinder may rise. As mentioned in Art. 92, temperatures in the order of 5000°F actually occur during the combustion process. These temperatures are far above the metallurgical limit, but they do not damage the engine for two reasons: (*a*) the time during which they exist is short, and (*b*) the layer of gas in direct contact with the cylinder walls remains at a much lower temperature than does the gas at the center of the cylinder. The fact that very high temperatures are permissible in internal-combustion engines is the principal reason why the thermal efficiencies of such engines are higher than those of steam power plants.

The lower average temperature at which a steam power plant receives heat is partially compensated for by the fact that the temperature during its heat rejection process is lower than the corresponding temperature for an internal-combustion engine. As shown in Fig. 82, heat is rejected in a steam power plant at virtually atmospheric temperature, whereas in an internal-combustion engine the products of combustion leave at a much higher temperature. The temperature of the gases leaving an internal combustion engine could be lowered somewhat if the expansion process were continued down to atmospheric pressure (see Problem 328); but, even if this were practicable, the gases would still leave at a temperature well above atmospheric temperature.

Aside from their higher thermal efficiencies, internal-combustion engines have an advantage over steam power plants in that they are more compact and are lighter in weight. This accounts for their use where the plant must be mobile, as in automobiles and airplanes. For large stationary plants, however, the initial cost of a steam power plant is usually less than that of an internal-combustion engine plant of the same capacity. Moreover, the steam power plant can burn cheaper fuels, and therefore its operating cost is likely to be lower than that of an internal-combustion engine plant.

SYMBOLS

c_p	specific heat at constant pressure
c_v	specific heat at constant volume
h	enthalpy per unit mass
HV	heating value

k	ratio of c_p to c_v
p	pressure
Q	a quantity of heat
r_c	compression ratio
r_e	expansion ratio
r_p	ratio of heater pressure to cooler pressure
s	entropy per unit mass
T	absolute temperature
v	specific volume
w	rate of fuel consumption
η	thermal efficiency

Chapter 12

REFRIGERATION

A *refrigerating machine* was defined in Art. 56 as a machine whose working fluid (the "refrigerant") undergoes a refrigeration cycle. Since the purpose of such a machine is to absorb heat from a region at low temperature (that is, to produce "refrigeration"), and since the machine is able to do this only at the expense of some work (or its equivalent) supplied to it, it is desirable that the ratio of the refrigeration produced to the work supplied be as high as possible. This ratio is called the *coefficient of performance* (c.p.) of the machine; that is,

$$\text{c.p.} = \frac{Q_{\text{in}}}{W'_{\text{cycle}}/J}, \qquad (37)$$

where Q_{in} represents the heat absorbed from the low temperature region, and W'_{cycle}/J the net work supplied to the refrigerating machine.

It was proved in Art. 62 that, if the temperature of the low temperature region and the temperature of the receiver to which heat is rejected remain constant, the highest possible coefficient of performance is that of a refrigerating machine operating on a *reversible* cycle. In this chapter we shall consider first why it is impractical to build a refrigerating machine to operate on a reversible cycle, and then we shall consider some of the refrigeration cycles actually used in practice.

100. The Ton of Refrigeration. The rate at which work is delivered by a power plant is regularly measured, not in British thermal units per unit time, but in horsepower or in kilowatts. Similarly, the rate at which refrigeration is produced by a refrigerating machine is usually measured, not in British thermal units per unit time, but in "tons of refrigeration." A **ton of refrigeration** is defined as the absorption of heat from a low temperature region at the rate of 12,000 Btu per hr (or 200 Btu per min). This unit is used as a matter of convenience since it allows rates of refrigeration to be expressed in smaller numbers. For example, it is more convenient to describe a refrigerating machine as a "5-ton" machine than as one having a capacity of 60,000 Btu per hr of refrigeration.

Another term sometimes used in refrigeration work is the "ton-day." This term is a quantity of refrigeration rather than a rate. Thus, a **ton-day** of refrigeration is the amount of heat absorbed from a low temperature region in 24 hr by a refrigerating machine having a capacity of 1 ton. It is equal to (12,000 × 24 =) 288,000 Btu of refrigeration.

It is customary in practice to compare refrigerating machines on the basis of the horsepower that must be supplied to them per ton of refrigeration produced rather than on the basis of their coefficients of performance. These quantities are related by the equation

$$\frac{\text{hp}}{\text{ton}} = \frac{12{,}000 \text{ Btu of refrig.}}{\text{ton-hr}} \left| \frac{\text{Btu of work}}{(\text{c.p.}) \text{ Btu of refrig.}} \right| \frac{\text{hp-hr}}{2545 \text{ Btu of work}}$$

$$= \frac{4.72}{\text{c.p.}} \tag{54}$$

The student may wonder why the "ton of refrigeration" is called by that name. This unit was originally defined as a rate of heat absorption equal to that which would be realized by the melting of 1 ton of ice in 24 hr. This definition would be identical with the present definition if the latent heat of fusion of ice were 144 Btu per lb. The latent heat of fusion is actually slightly less than this (143.35 Btu per lb).

101. The Impracticability of the Carnot Refrigeration Cycle. It was proved in Art. 62 that, if the temperatures of the source and receiver of heat remain constant, no refrigerating machine can have a coefficient of performance higher than that of a machine operating on a *reversible* cycle. One reversible cycle is the Carnot refrigeration cycle. This cycle, previously described in Art. 57, consists of (*a*) an isothermal reversible process during which the refrigerant absorbs heat from the low temperature source, (*b*) an adiabatic reversible compression during which the temperature of the refrigerant rises from that of the source to that of the receiver, (*c*) an isothermal reversible process during which the refrigerant rejects heat to the receiver, and, finally, (*d*) an adiabatic reversible expansion during which the temperature of the refrigerant drops from that of the receiver back to that of the source. Let us consider the practicability of building a refrigerating machine to operate approximately on this cycle if it uses as its refrigerant (*a*) air, or any other gas, and (*b*) ammonia, or any other condensable vapor.

It is impracticable to build a refrigerating machine to operate on the Carnot refrigeration cycle using air as the refrigerant for the same reason that it is impracticable to build a power plant to operate on the Carnot power cycle using air as the working fluid. Since this reason was discussed in detail in Art. 82, it will be reviewed only briefly. To compress or to expand air approximately adiabatically, it is necessary only to compress or expand it rapidly; and, to make the process approximately reversible, it is necessary only to eliminate friction as nearly as possible. Since both can be done, the two adiabatic reversible processes can be approximated without too great difficulty. The two isothermal processes, on the other hand, cannot be approximated easily on account of the low thermal conductivity of air. Thus, if these processes were carried out in a cylinder equipped with a piston, the temperature of the air in contact with the cylinder walls could be kept approximately constant, but the temperature of the air at the center of the cylinder could not unless the process were carried out very slowly. Since large, slow-moving equipment would be required, to attempt to approximate the two isothermal processes would be impracticable.

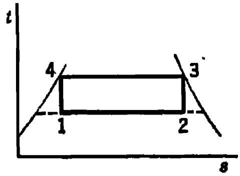
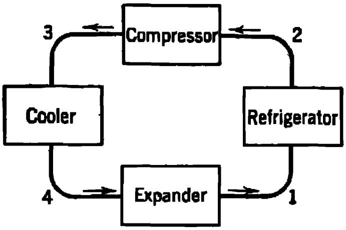


FIG. 83. Carnot refrigeration cycle with ammonia as the refrigerant.

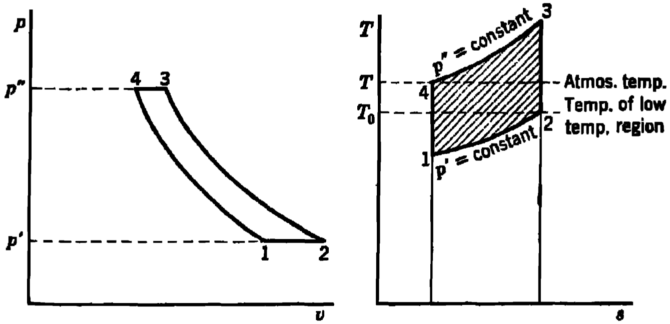
Let us consider next the possibility of building a refrigerating machine to operate approximately on the Carnot refrigeration cycle with ammonia as the refrigerant. It happens that this can be done without too great difficulty if the ammonia is allowed to pass through the states shown in Fig. 83. For this cycle both the isothermal heat reception process from 1 to 2 and the isothermal heat rejection process from 3 to 4 are constant pressure processes, and therefore they can be approximated easily in practice. The isentropic process from 2 to 3 can also be easily approximated. In fact, this process, known as **wet compression,** is actually used in some refrigerating plants. It has the disadvantage of requiring that liquid refrigerant be introduced into the compressor. This liquid, if not completely vaporized during the compression stroke, may cause damage to the compressor. The isentropic expansion from 4 to 1 also can be approximated in practice. However, the work obtained from the expander engine would be so small that it would not be economical to try to obtain it. Thus, although the Carnot refrigeration cycle could be approximated in practice without too great difficulty if ammonia were used as the refrigerant, it is not economically practicable to do so.

The practicable refrigeration cycle that most closely approaches the Carnot refrigeration cycle is the *vapor compression cycle,* described in Art. 56. Before discussing this cycle further, let us consider again the *air refrigeration cycle,* also described in Art. 56.

102. The Air Refrigeration Cycle. Refrigerating machines operating on the *air refrigeration cycle* were at one time used rather extensively, especially on board ships, because the refrigerant (air) is noninflammable and non-poisonous. They are no longer common because of their high power consumption per ton of refrigeration produced, but they are used to some extent for aircraft air-conditioning systems because of their safety and light weight. We shall consider this cycle in some detail because it is the only familiar refrigeration cycle in which the refrigerant remains always in the gaseous phase.



(a) Flow-diagram



(b) Pressure-volume diagram (c) Temperature-entropy diagram

FIG. 84. The air refrigeration cycle.

A flow-diagram for a refrigerating machine operating on the air refrigeration cycle is shown in Fig. 84*a*, and the states through which the air would pass if the machine operated in an ideal manner are shown on a pressure-volume diagram and on a temperature-entropy diagram in Figs. 84*b* and 84*c*. As is evident from Fig. 84*c*, the cycle is like the Carnot refrigeration cycle except that the two difficult-to-approximate isothermal processes are replaced by two easily approximated constant pressure processes.

Ideally, the plant operates in the following manner: The air leaving the expander engine (state 1) is at a low pressure p' and at a low temperature T_1. It flows through the refrigerator at constant pressure and absorbs heat from the region to be refrigerated. Its temperature during this process must be below the temperature T_0 of the region from which heat is to be absorbed, as shown in Fig. 84*c*. The air leaves the refrigerator at temperature T_2 (ideally equal to T_0) and is compressed isentropically in the compressor to some higher pressure p''. During this process its temperature rises to T_3. This temperature must be above the temperature T of the atmosphere (or of the cooling water) so that the air can reject heat to the atmosphere as it flows at constant pressure through the cooler. Finally, the air enters the expander at temperature T_4 (ideally equal to T) and expands isentropically back to pressure p' and temperature T_1. The expander supplies part of the work required to drive the compressor. The remainder must be taken from some external source.

The ideal coefficient of performance of a refrigerating machine operating on the air refrigeration cycle can be calculated by Eq. 37*a*:

$$\text{c.p.} = \frac{Q_{\text{in}}}{Q_{\text{out}} - Q_{\text{in}}}, \tag{37a}$$

where Q_{in} = the heat absorbed from the region to be refrigerated, and
Q_{out} = the heat rejected to the atmosphere.

Suppose, for example, that the temperature T_0 of the region to be refrigerated, the temperature T of the atmosphere, the air pressure p' in the refrigerator, and the air pressure p'' in the cooler are all given. Since, ideally, $T_2 = T_0$ and $T_4 = T$, it is necessary to determine only the temperatures T_1 and T_3 to be able to calculate Q_{in} and Q_{out}. These two temperatures can be found by Eq. 28*b* (if it is assumed that c_v and c_p for air are constant) since the expansion from 4 to 1 and the compression from 2 to 3 are assumed to be isentropic. Thus,

$$T_1 = T_4\left(\frac{p'}{p''}\right)^{(k-1)/k} \qquad \text{and} \qquad T_3 = T_2\left(\frac{p''}{p'}\right)^{(}$$

The heat Q_{in} absorbed by each pound of air as it flows through the refrigerator can now be calculated by the steady-flow energy equation (Eq. 13*a*). Assuming that the velocities of the air entering and leaving the refrigerator are negligible, and making use of the fact that for a perfect gas, $\Delta h = c_p \, \Delta t$ (Eq. 18*a*), we may apply the steady-flow energy equation to the refrigerator to obtain

$$Q_{\text{in}} = h_2 - h_1 = c_p(T_2 - T_1).$$

Similarly, assuming that the velocities of the air entering and leaving the cooler are negligible, we may apply the steady-flow energy equation to the cooler to obtain

$$Q_{3-4} = h_4 - h_3 = c_p(T_4 - T_3),$$

or

$$Q_{\text{out}} = c_p(T_3 - T_4).$$

Having determined the ideal values of Q_{in} and Q_{out}, we can calculate the ideal coefficient of performance by Eq. 37*a*.

It is of interest to inquire into the reason why the ideal coefficient of performance for the air refrigeration cycle is usually low. The reason is apparent from the temperature-entropy diagram shown in Fig. 84*c*. The heat Q_{in} is represented by the area under the curve from 1 to 2, and the heat Q_{out} by the area under the curve from 3 to 4. Therefore, in accordance with Eq. 37*a*, the coefficient of performance is represented by the ratio of the area under 1–2 to the shaded area. It is clear that the ideal coefficient of performance for this (or any other) refrigeration cycle is increased by keeping the temperature of the refrigerant as near T_0 as possible during the heat reception process. Likewise, it is increased by keeping the temperature of the refrigerant as near T as possible during the heat rejection process. In fact, if the temperatures of the refrigerant during these two processes were kept exactly at T_0 and at T, the cycle would become the Carnot refrigeration cycle. It is evident that the ideal coefficient of performance for the cycle shown in Fig. 84*c* is low because the temperature of the air is too far below T_0 during the process from 1 to 2 and too far above T during the process from 3 to 4.

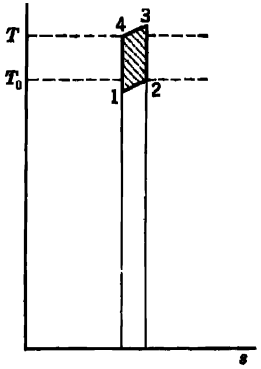


Fig. 85.

The ideal coefficient of performance for the air refrigeration cycle can be increased by raising the temperature T_1 and lowering the temperature T_3, as shown in Fig. 85. In fact, it can be made to approach that of the Carnot refrigeration cycle simply by making T_1 only slightly lower than T_0 and making T_3 only slightly higher than T. This can be accomplished by making the pressure ratio p''/p' only slightly higher than the minimum permissible value calculated by the equation

$$\left(\frac{p''}{p'}\right)_{\text{min}} = \left(\frac{T}{T_0}\right)^{k/(k-1)}$$

It is of interest to note at this point that the ideal coefficient of performance for the air refrigeration cycle is in reality a function of the pressure ratio p''/p' only. Thus, it can be shown that for this cycle

$$(\text{c.p.})_{\text{ideal}} = \frac{1}{(p''/p')^{(k-1)/k} - 1}. \tag{55}$$

This equation has been used to calculate the values shown in Fig. 86. Note that, as p''/p' approaches its minimum permissible value, the ideal coefficient of performance approaches that of the Carnot cycle.

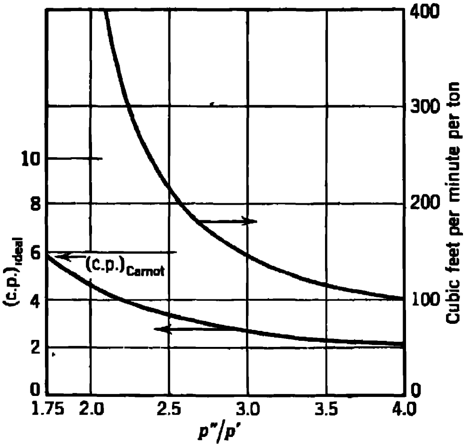


FIG. 86. Relation between $(\text{c.p.})_{\text{ideal}}$, the volume of air entering the compressor, and the pressure ratio p''/p' for the air refrigeration cycle (on the basis that, referring to FIG. 84, $p' = 14.7$ psia, $t_2 = 5°\text{F}$, and $t_4 = 86°\text{F}$).

Since the ideal coefficient of performance of the air refrigeration cycle can be made to approach that of the Carnot refrigeration cycle simply by reducing the pressure ratio p''/p', the question arises, why not do this in actual practice? It is impracticable because the volume of air that must be circulated becomes exceedingly large when p''/p' is reduced to near its minimum permissible value. Let us assume, for example, that the air temperature at 2 in Fig. 84 is 5°F and at 4 is 86°F and that the plant operates in an ideal manner. The cubic feet of air which must enter the compressor per minute per ton of refrigeration is plotted against the pressure ratio p''/p' in Fig. 86. These values are calculated as follows: The weight of air which must be circulated per minute per ton of refrigeration is first calculated by the equation

$$\text{pounds of air circulated per minute per ton} = \frac{200}{Q_{\text{in}}}.$$

This weight rate is then converted into the desired volume rate by multiplying it by the specific volume of the air at state 2. The specific volume is calculated by the equation $pv = RT$. Figure 86 shows that, as the pressure ratio p''/p' is reduced, the ideal coefficient of performance approaches that of the Carnot refrigeration cycle while the volume of air to be handled approaches infinity.

The actual coefficient of performance of an air refrigerating plant is, of course, lower than the ideal. It is affected by the inefficiency of both the compressor and the expander engine. The processes taking place in these parts of the plant are likely to be approximately adiabatic, but they are not isentropic because of friction. In accordance with the *Entropy Principle* discussed in Art. 80, the entropy of the air must *increase* as it flows through each. The effect on the temperature-entropy diagram for the cycle is shown in Fig. 87. The refrigeration produced by each pound of air, represented by the area under the curve from 1 to 2, is decreased; and the heat rejected by each pound of air, represented by the area under the curve from 3 to 4, is increased. Therefore, the work which must be supplied to the machine, represented by the difference between these areas, is increased. Thus, the inefficiency of the compressor and of the expander seriously affects the actual coefficient of performance of the plant (represented by the ratio of the area under curve 1–2 to the shaded area).

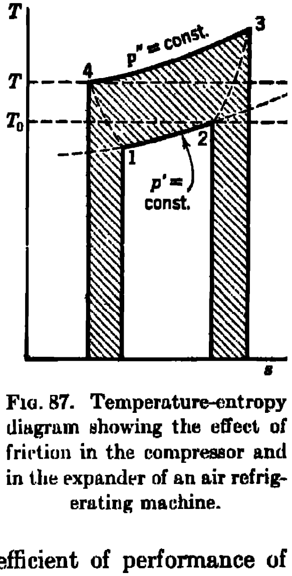


FIG. 87. Temperature-entropy diagram showing the effect of friction in the compressor and in the expander of an air refrigerating machine.

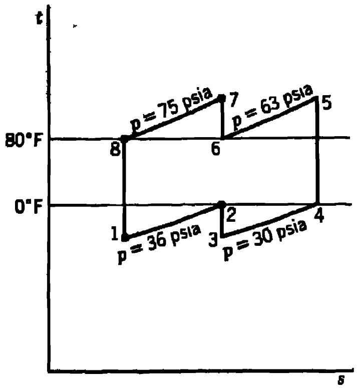
PROBLEMS

341. Derive Eq. 55.

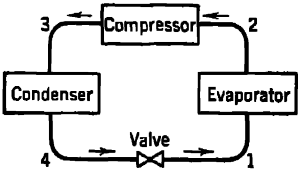
342. A refrigerating machine operates on the air refrigeration cycle. The air leaving the refrigerator is at 30 psia and 0°F, and the air leaving the cooler is at 75 psia and 80°F. Determine (*a*) the ideal coefficient of performance of the machine, (*b*) the horsepower ideally required per ton of refrigeration, and (*c*) the cubic feet of air which must be compressed per minute per ton of refrigeration.

Ans. (*a*) 3.35; (*b*) 1.41 hp per ton; (*c*) 104.5 cu ft per min per ton.

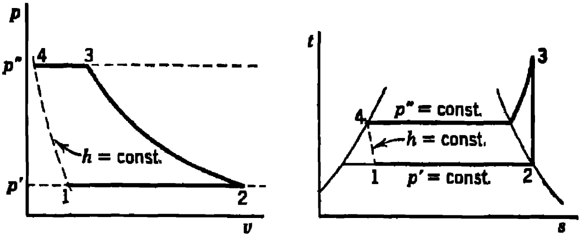
343. The ratio of the work which would be required to drive a compressor if it operated isentropically to the work actually required is called the *compressor efficiency* (see Art. 116), and the ratio of the work actually delivered by an engine to the work it would deliver if it operated isentropically is called the *engine efficiency* (see Art. 85). Suppose that a refrigerating machine operates on the air refrigeration cycle and that (*a*) the air leaves the refrigerator at 30 psia and 0°F, (*b*) it leaves the cooler at 75 psia and 80°F, (*c*) the compressor has an efficiency of 75%, and (*d*) the engine has an efficiency of 80%. Determine (1) the coefficient of performance of the machine, (2) the horsepower required per ton of refrigeration, and (3) the cubic feet of air which must be compressed per minute per ton of refrigeration. Assume that the compressor and the engine operate adiabatically and that the velocities of the air entering and leaving each are low.



344. Suppose that a refrigerating machine uses air as its refrigerant and operates on the steady-flow cycle shown on the accompanying *t*-*s* diagram. Determine the ideal coefficient of performance of the machine and the horsepower ideally required per ton of refrigeration. *Ans.* 3.83; 1.23 hp per ton.



(*a*) Flow-diagram



(*b*) Pressure-volume diagram (*c*) Temperature-entropy diagram

FIG. 88. The vapor compression refrigeration cycle.

103. The Vapor Compression Refrigeration Cycle. The refrigeration cycle most commonly used today is the **vapor compression cycle**, described in Art. 56. Figure 88*a* shows a flow-diagram for a refrigerating machine operating on this cycle, and Figs. 88*b* and 88*c* show the states through which the refrigerant would pass if the machine operated in an ideal manner. Let us review briefly the ideal manner of operation. The refrigerant enters the *evaporator* as a mixture of saturated liquid and saturated vapor (state 1). In the evaporator the liquid refrigerant changes at constant pressure into a vapor and absorbs heat from the region to be refrigerated. The refrigerant may leave the evaporator as a saturated vapor, as shown in Fig. 88c, or it may leave as a mixture of saturated liquid and vapor or as a slightly superheated vapor. It next enters the *compressor*, where it is compressed isentropically to the condenser pressure. It then flows through the *condenser* at constant pressure and rejects heat to the atmosphere (or to the cooling water). Finally, the refrigerant leaving the condenser as a liquid flows through the *expansion valve* back to the evaporator, part of it flashing into vapor during the process.

We can calculate the ideal coefficient of performance of the vapor compression refrigeration cycle by Eq. 37*a*. We must first determine the heat absorbed by each pound of refrigerant flowing through the evaporator and the heat rejected by each pound flowing through the condenser. To determine these two quantities, we must apply the steady-flow energy equation (Eq. 13*a*) to the evaporator and to the condenser. Assuming that the refrigerant enters and leaves each of these parts of the plant with a negligible velocity, we obtain

$$Q_{\text{in}} = h_2 - h_1$$

and

$$Q_{\text{out}} = h_3 - h_4.$$

Note that, since the flow through the expansion valve is a throttling process, $h_1 = h_4$.

Let us suppose, for example, that the evaporator pressure p' and the condenser pressure p'' are given, that the refrigerant leaves the evaporator as a saturated vapor, and that it leaves the condenser as a saturated liquid (as shown in Fig. 88*c*). We can read the values of h_1 $(= h_4)$, h_2, and s_2 directly from a table of properties of the refrigerant. Ideally, the compression process from 2 to 3 is isentropic. Therefore, we can also read the value of h_3 directly from the tables because we know the pressure p'' at state 3 and the entropy s_3 $(= s_2)$. Having determined the enthalpies at states 1, 2, 3, and 4, we can calculate Q_{in} and Q_{out} by the equations given in the preceding paragraph. Finally, we can calculate the ideal coefficient of performance by Eq. 37*a*.

It is of interest to consider what effect the use of an expansion valve, instead of an expander engine, has on the ideal coefficient of performance for the cycle. Let us consider the temperature-entropy diagrams shown in Fig. 89 for an ideal vapor compression cycle with (*a*) an ideal expander engine and (*b*) an expansion valve. Since for each cycle the heat Q_{in} is represented by the area under the constant pressure line from 1 to 2, and since the heat Q_{out} is represented by the area under the constant pressure line from 3 to 4, the ideal coefficient of performance is represented by the ratio of the area under line 1–2 to the shaded area. It is evident that using an expansion valve instead of an expander engine reduces the ideal coefficient of performance, but that, since a constant enthalpy line in the left part of the mixture region is nearly vertical, the reduction is small. For this reason the use of a relatively complex and expensive expander engine, in place of a simple and inexpensive expansion valve, is not justified.

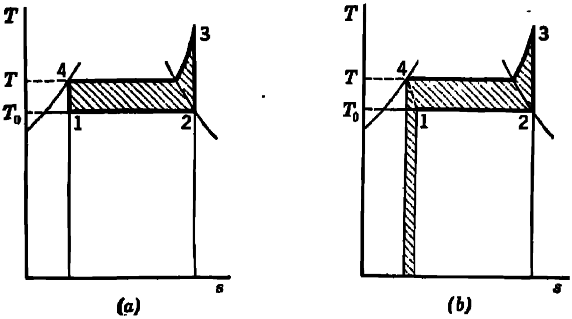


FIG. 89. Temperature-entropy diagrams for the vapor compression refrigeration cycle with (*a*) an ideal expander engine and (*b*) an expansion valve.

The loss due to the use of an expansion valve becomes even smaller if the liquid refrigerant is subcooled before it enters the valve. Although much subcooling is not possible, at least some can be obtained if the heat rejected in the condenser is absorbed by cooling water. Thus, the cooling water can be used while its temperature is lowest to subcool the liquid refrigerant in a separate heat exchanger, and it can then be sent to the condenser. As a result, point 4 on the temperature-entropy diagram for the cycle is moved down the liquid line, and the coefficient of performance is improved. Also, the refrigeration produced by each pound of refrigerant is increased, and consequently a smaller compressor is needed.

If *wet compression*, discussed in Art. 101, is used in place of the *dry compression* shown in Figs. 88*c* and 89, the ideal coefficient of performance is increased. The reason for this is evident from these diagrams. By using wet compression, the vapor does not become superheated in the compressor, and consequently the portion of the shaded area above the atmospheric temperature line is eliminated. The primary reason for using wet compression is not to improve the coefficient of performance, however. It is used to lower the average temperature of the vapor during the compression process and thereby make proper lubrication of the compressor easier.

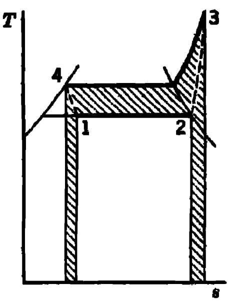


FIG. 90. Temperature-entropy diagram showing the effect of friction in the compressor.

Friction in the compressor affects the temperature-entropy diagram for the vapor compression cycle as shown in Fig. 90. Since the actual compression process is approximately adiabatic, the entropy of the refrigerant must increase during the process (in accordance with the *Entropy Principle* discussed in Art. 80). As a result, the coefficient of performance, represented by the ratio of the area under the constant pressure line 1–2 to the shaded area, is decreased. Moreover, the temperature of the refrigerant leaving the compressor (state 3) is raised. This adds to the difficulty of lubricating the compressor properly and requires that a larger condenser be provided.

PROBLEMS

345. A refrigerating machine operates on the vapor compression refrigeration cycle and uses ammonia as its refrigerant. The ammonia leaves the condenser at 200 psia and 90°F and leaves the evaporator as saturated vapor at 30 psia. Determine (*a*) the ideal coefficient of performance of the machine, (*b*) the power ideally required per ton of refrigeration, and (*c*) the cubic feet of vapor which must be compressed per minute per ton of refrigeration.

Ans. (*a*) 3.85; (*c*) 3.94 cu ft per min per ton.

346. If the compressor of the refrigerating machine described in the preceding problem operated with an efficiency of 80% (see Problem 343), what would be the coefficient of performance of the machine and how many horsepower would be required per ton of refrigeration? Assume that the compressor would operate adiabatically and that the velocity of the vapor entering and leaving it would be low.

347. Suppose that the expansion valve of the refrigerating machine described in Problem 345 were replaced by an engine which operated adiabatically and without friction. By how much would the power ideally required per ton of refrigeration be decreased?

348. Suppose that the ammonia entering the compressor of the refrigerating machine described in Problem 345, instead of being saturated, is a mixture such that the vapor leaving the compressor is saturated. Determine (*a*) the ideal coefficient of performance of the machine, (*b*) the power ideally required per ton of refrigeration, and (*c*) the volume of vapor to be compressed per minute per ton of refrigeration.

349. If the ammonia flowing through the condenser of the refrigerating machine described in Problem 345 rejects heat to cooling water, and if the cooling water enters the condenser at 75°F and leaves at a temperature 2 deg below that at which the ammonia condenses, how many cubic feet are required per hour per ton of refrigeration? Assume that the compressor operates isentropically and that no heat is lost from the condenser to the surroundings. *Ans.* 12.5 cu ft per hr per ton.

350. Suppose that a vapor compression refrigerating machine uses ammonia as its refrigerant and that the compressor is able to compress 25 cu ft per min of vapor (measured on the intake side). Suppose further that the liquid ammonia leaving the condenser is at 80°F and that the vapor leaving the evaporator is saturated. Determine what the capacity of the plant will be if the temperature of the ammonia in the evaporator is (*a*) −20°F, (*b*) 0°F, and (*c*) +20°F. Does the capacity vary linearly with the evaporator temperature?

351. It is proposed to use a vapor compression refrigerating machine as a "heat pump" to heat a building during the winter. To do this the evaporator will be placed out-of-doors, and the condenser will be inside the building. It is estimated that the temperature of the refrigerant in the evaporator will be 10 deg below the outdoor temperature and that the building can be heated satisfactorily if the refrigerant condenses at 100°F. To heat the building with coal when the outdoor temperature is 20°F requires that 100 lb of coal having a heating value of 12,000 Btu per lb be burned per day. The coal costs $9.00 per ton, and electricity costs 1 cent per kwhr. If both the coal-burning plant and the refrigerating machine operated in an ideal manner, how much would it cost per day to heat the building (*a*) with coal, and (*b*) with the heat pump? Assume that the refrigerant would be ammonia, that it would leave the evaporator as saturated vapor, and that it would leave the condenser as saturated liquid.

104. Refrigerants. To be suitable for use as the refrigerant in a vapor compression refrigeration plant, a fluid must have a freezing point lower than the temperature at which the evaporator is to operate. If its freezing point were higher, it would leave the expansion valve as a mixture of solid and vapor, and the evaporator could not function properly. The fluid must also have a critical temperature above atmospheric temperature (or above the temperature of the available cooling water) in order that it can be liquefied in the condenser. The freezing points and the critical temperatures of some of the more commonly used refrigerants are listed in Table V.

The vapor pressures of the refrigerant at the evaporator and condenser temperatures are important. It is usually desirable that they be above atmospheric pressure so that air leakage into the plant cannot occur. Such leakage is objectionable for the following reasons: Any air entering the plant will be carried with the refrigerant through the evaporator and compressor and into the condenser. Since the air will not condense at atmospheric temperature, and since only liquid can enter the expansion valve, the air will remain trapped in the condenser. As a result, the condenser pressure will be increased, and the work required to drive the compressor will thereby also be increased. Moreover, the presence of air in the condenser will lower the rate of heat

TABLE V

SOME PROPERTIES OF COMMON REFRIGERANTS

Refrigerant	Freezing Point, °F	Critical Temperature, °F	Saturation Pressure at 5°F, psia	Saturation Pressure at 86°F, psia
Ammonia	−107.9	271.2	34.27	169.2
Dichlorodifluoromethane ("Freon-12")	−252.0	232.7	26.51	107.9
Dichloromonofluoromethane ("Freon-21")	−211.0	353.3	5.24	31.2
Methyl chloride	−144.0	289.6	21.15	94.70
Methylene chloride (Carrene-1)	−142.0	421.0	1.17	10.60
Monochlorodifluoromethane ("Freon-22")	−256.0	204.8	43.02	174.5
Sulfur dioxide		314.8	11.81	66.45
Trichloromonofluoromethane ("Freon-11," Carrene-2)		388.4	2.93	18.28
Trichlorotrifluoroethane ("Freon-113")	−31.0	417.4	0.98	7.86

transfer through the condenser tubes and may prevent the plant from operating properly.

Although refrigerant pressures above atmospheric are usually desirable, they should not be too high because the danger of the refrigerant's leaking out of the plant increases as the pressure increases. The cost of the plant also increases if the pressures are too high because the various parts must be of heavier construction. The vapor pressures corresponding to a 5°F evaporator temperature and an 86°F condensing temperature for various refrigerants are listed in Table V.

It is sometimes advantageous to have the evaporator and condenser pressures be below atmospheric pressure. For example, an evaporator pressure below atmospheric is desirable in a refrigerating machine used as part of an air-conditioning plant if the air is blown directly across the evaporator surfaces. If the evaporator pressure is below atmospheric, any leakage will be *into* the evaporator, and the possibility of contaminating the air is avoided.

In selecting a refrigerant, the volume of vapor that must be compressed per minute per ton of refrigeration is of importance because it determines the type of compressor best used. A reciprocating compressor is usually most suitable if the volume is small, whereas a centrifugal compressor is likely to be best if the volume is large. It may be mentioned that, although centrifugal compressors can handle large volumes, they are usually less efficient than reciprocating compressors. For an evaporator temperature of 5°F and a condenser temperature of 86°F, Table VI shows for various refrigerants the volumes of vapor that must be compressed per minute per ton of refrigeration.

The ideal coefficient of performance and the ideal horsepower required per ton of refrigeration are also listed in Table VI for various refrigerants. Note that these quantities are nearly the same for all the refrigerants listed.

TABLE VI

COMPARISON[1] OF VARIOUS REFRIGERANTS

Refrigerant	Volume of Vapor To Be Compressed, cfm per ton	$(\text{c.p.})_{\text{ideal}}$	$(\text{hp per ton})_{\text{ideal}}$
Ammonia	3.44	4.75	0.99
Monochlorodifluoromethane ("Freon-22")	3.56	4.56	1.03
Dichlorodifluoromethane ("Freon-12")	5.82	4.72	1.00
Methyl chloride	6.09	4.90	0.97
Sulfur dioxide	9.08	4.73	1.00
Dichloromonofluoromethane ("Freon-21")	20.4	5.09	0.93
Trichloromonofluoromethane ("Freon-11," Carrene-2)	36.3	5.14	0.92
Methylene chloride (Carrene-1)	74.5	4.90	0.97
Trichlorotrifluoroethane ("Freon-113")	100.8	4.79	0.99

[1] An evaporator temperature of 5°F and a condensing temperature of 86°F are assumed.

If the volume of vapor to be compressed is large, and if the refrigerant need not be recovered, *steam jet ejectors* may be used to advantage. They are used in refrigerating machines employing water as the refrigerant, for example. A diagrammatic sketch of such a machine is shown in Fig. 91. The machine cannot be used to produce refrigeration below 32°F, but it is suitable for producing chilled water for air-conditioning purposes. A two- or three-stage ejector is usually used in practice instead of the single-stage ejector shown in the diagram. Although ejectors have a low efficiency, they are relatively inexpensive, and they require little maintenance because of the absence of moving parts.

In addition to having the various characteristics already discussed, a refrigerant should be non-poisonous, non-inflammable, non-corrosive,

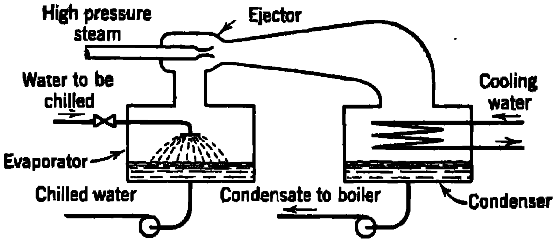


FIG. 91. Diagrammatic sketch of a water-vapor refrigerating machine.

chemically stable, and inexpensive. Since no known refrigerant has all these desirable characteristics, the choice of refrigerant is governed largely by the use to which the refrigeration plant is to be put.

PROBLEMS

352. Check the values for ammonia given in Table VI.

353. A vapor compression refrigeration plant uses ammonia as its refrigerant. The ammonia leaves the condenser as a liquid at 70°F and leaves the evaporator as saturated vapor at 0°F. If no air were present in the condenser, the condenser pressure would be 140 psia. Because air is present, however, the pressure is 150 psia. Determine what increase the air causes in the power required to drive the machine. Assume that the efficiency of the compressor (see Problem 343) is independent of the discharge pressure. *Ans.* 5.1%.

354. Suppose that the refrigerating machine shown in Fig. 91 is used to chill water from 65° to 40°F. How many cubic feet of water vapor must be compressed per minute per ton of refrigeration produced? Take the refrigeration produced by each pound of chilled water leaving the evaporator to be equal to the heat required to raise the temperature of this water from 40° back to 65°F.

105. Other Vapor Compression Refrigeration Cycles. The simple vapor compression refrigeration cycle with a single-stage compressor (Fig. 88) is suitable for temperatures down to about −20°F. It is not entirely satisfactory for temperatures lower than this principally because a single-stage compressor will not operate efficiently if the ratio of its discharge pressure (the condenser pressure) to its suction pressure (the evaporator pressure) is too high. The "volumetric efficiency" of the compressor (discussed in Art. 115) becomes quite low, and the temperature at which the vapor leaves the compressor becomes rather high. The high temperature makes proper lubrication of the compressor difficult and may even cause some decomposition of the refrigerant. Another difficulty is that the condenser pressure is likely to be extremely high if the evaporator pressure is moderate; or, conversely, the evaporator pressure is likely to be well below atmospheric pressure if the condenser pressure is moderate.

Figure 92 illustrates the fact that the temperature of the vapor leaving a single-stage compressor becomes quite high if the evaporator temperature is below −20°F. The figure shows the temperature-entropy diagrams for two ideal vapor compression refrigeration cycles with ammonia as the refrigerant. Both have the same condenser pressure, but one has an evaporator temperature of 0°F, and the other an evaporator temperature of −60°F. The ammonia leaves the compressor at a temperature of 222°F if the evaporator temperature is 0°F, whereas it leaves at a temperature above 400°F if the evaporator temperature is −60°F. With a temperature change as great as the latter, it would be almost impossible to build a single-stage compressor that would operate efficiently.

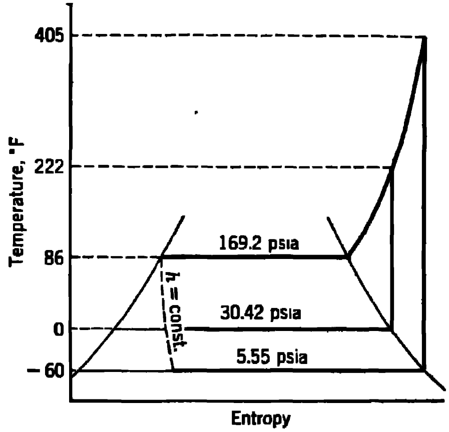
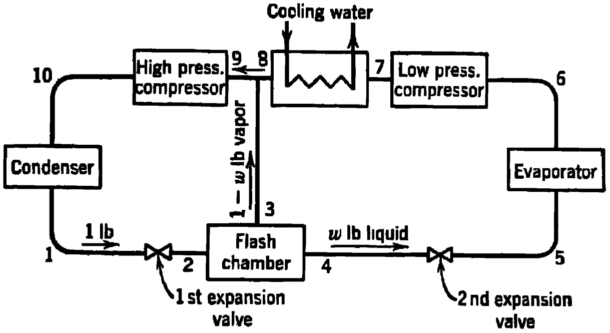
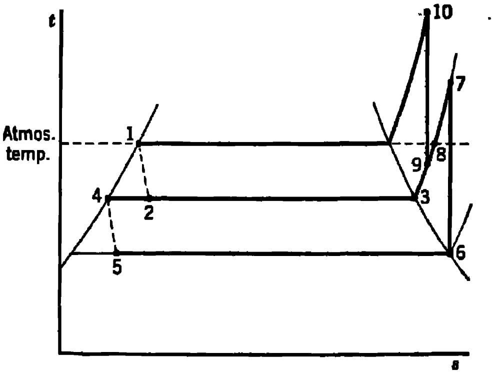


Fig. 92. Temperature-entropy diagrams for two ideal vapor compression refrigeration cycles with ammonia as the refrigerant.

One way of avoiding excessively high temperatures in the compressor is to use two compressors, two expansion valves, and a "flash chamber," as shown in Fig. 93*a*. The liquid refrigerant from the condenser is first throttled, not directly into the evaporator, but into an insulated tank in which the pressure is between that of the condenser and that of the evaporator. The vapor formed during this process is sent to the high pressure compressor, and the liquid remaining (which is now at a lower temperature) is throttled into the evaporator through the second expansion valve. The vapor leaving the evaporator is compressed to the flash chamber pressure in the low pressure compressor. On leaving this compressor, it is first cooled to atmospheric temperature in a water-cooled intercooler and is then cooled further by mixing with the cold vapor from the flash chamber. Consequently, the temperature at which it leaves the high pressure compressor is kept relatively low. Note that, since the vapor formed in the flash chamber does not pass through both compressors, the coefficient of performance of the machine is higher than it would be if the flash chamber were eliminated. If the machine operated in an ideal manner, the states of the refrigerant at the various sections numbered on the flow-diagram would be as shown on the temperature-entropy diagram in Fig. 93*b*.



(a) Flow-diagram



(b) Temperature-entropy diagram

FIG. 93. A refrigerating machine utilizing two compressors, two expansion valves, and a flash chamber.

The ideal coefficient of performance for the refrigerating machine just described can be calculated by Eq. 37*a*. In determining Q_{in}, it must be remembered that only the liquid part of the mixture leaving the first expansion valve is sent to the evaporator. In determining

Q_{out}, it must be remembered that the refrigerant rejects heat, not only in the condenser, but also in the intercooler which is used to cool the vapor leaving the low pressure compressor. Note also that, if all velocities are assumed to be negligible, the steady-flow energy equation (Eq. 14) applied to the adiabatic mixing process taking place between sections 8 and 9 gives us

$$h_9 = (1 - w)h_3 + wh_8.$$

Another way of producing refrigeration at temperatures below −20°F is to use the **cascade or multiple refrigerant** system. Instead of a single refrigerant two or more refrigerants are used, each undergoing a simple vapor compression refrigeration cycle. A flow-diagram for such a plant is shown in Fig. 94. The heat rejected by the low temperature refrigerant in its condenser is absorbed by the high temperature refrigerant in its evaporator. With such a system the refrigerants can be chosen so that the pressures in each evaporator and condenser and the volumes to be compressed are moderate.

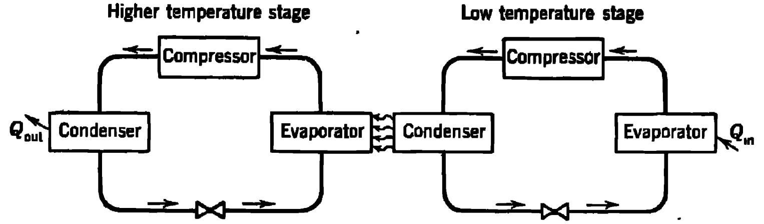


FIG. 94. Flow-diagram for a cascade refrigeration system.

It is frequently desired that a refrigerating machine produce refrigeration at more than one temperature. The simplest way of accomplishing this is to use a single compressor and condenser but to have a separate evaporator for each temperature, as shown in Fig. 95*a*. Since the intake pressure for the compressor must be the pressure in the second evaporator, a "back pressure" valve is necessary between the first evaporator and the compressor intake line. It is evident that the throttling of the vapor through this valve introduces a loss since the compressor must compress the vapor flowing through the valve, not from the pressure in the first evaporator to the condenser pressure, but from the lower pressure existing in the second evaporator.

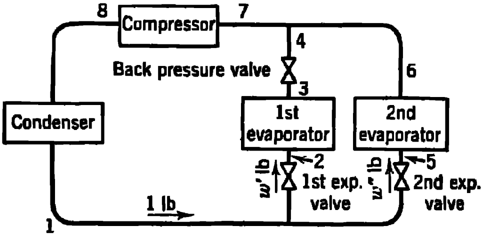
We can calculate the ideal coefficient of performance of the machine shown in Fig. 95 by Eq. 37*a*. We must first calculate the values of w' and w'' (the pounds of refrigerant flowing through each evaporator per pound flowing through the condenser). Since $w' + w'' = 1$, we can readily find the value of each after we have calculated the ratio of the two (w'/w'') by the equation

$$\frac{\text{load on first evaporator}}{\text{load on second evaporator}} = \frac{(h_3 - h_1)w'}{(h_6 - h_1)w''}.$$

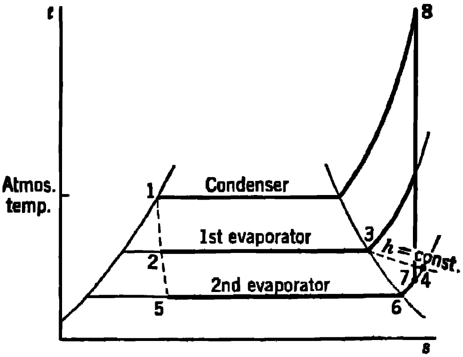
Note that, if we assume that the velocities at each numbered section are negligible, the steady-flow energy equation applied to the mixing process taking place between sections 4, 6, and 7 gives us

$$h_7 = w'h_4 + w''h_6.$$

After we have calculated the enthalpy at section 7, we can easily determine from the table of properties the value of the entropy at this section.



(a) Flow-diagram



(b) Temperature-entropy diagram

FIG. 95. Vapor compression refrigerating machine with one compressor and two evaporators.

Then, since the entropy at section 8 is the same as at section 7, we can find the enthalpy at section 8. With these various values, we can calculate Q_{in} and Q_{out} and then can calculate the ideal coefficient of performance.

Another way of producing refrigeration at two temperatures is to use a separate compressor for each evaporator, as shown in Fig. 96. This plant is in reality two separate vapor compression refrigerating machines using a common condenser. The coefficient of performance of the plant is better than that of the one shown in Fig. 95 because the loss caused by the back pressure valve does not occur. The cost of the plant is greater, however, because it requires two compressors.

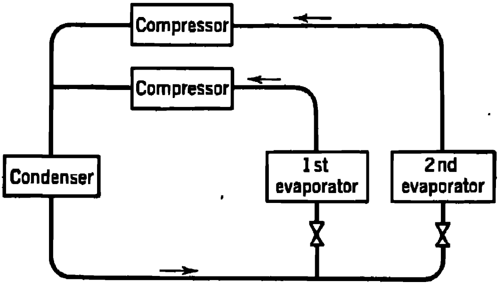


FIG. 96. Vapor compression refrigerating machine with two compressors and two evaporators.

If refrigeration at two temperatures is desired, and if two compressors are to be used, a better arrangement than that shown in Fig. 96 is the one shown in Fig. 97. It is similar to the one shown in Fig. 93, except that part of the liquid from the flash chamber is used in an evaporator operating at the flash chamber pressure. The ideal coefficient of performance of this plant can be calculated by following the same methods as for the other plants described.

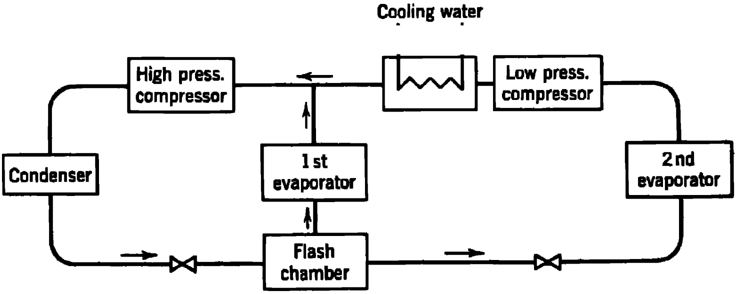


FIG. 97. Vapor compression refrigerating machine with two compressors, two evaporators, and a flash chamber.

Still other vapor compression cycles are possible, but those described illustrate the principles involved in the operation of all of them.

PROBLEMS

355. The following data are for the refrigerating machine shown in Fig. 93:

refrigerant = ammonia
condenser pressure = 170 psia
flash chamber pressure = 50 psia
evaporator temperature = −40°F
quality at section 6 = 100%
temperature at sections 1 and 8 = 80°F

Determine the ideal coefficient of performance of the machine and the horsepower ideally required per ton of refrigeration. *Ans.* 2.85; 1.66 hp per ton.

356. Suppose that the flash chamber and the vapor line leading from the flash chamber to the high pressure compressor were removed from the refrigerating machine shown in Fig. 93. If the operating conditions were the same as in the preceding problem, what would be the ideal coefficient of performance of the machine and the power ideally required per ton of refrigeration? Assume that the intercooler pressure is still 50 psia.

357. The following data are for the refrigerating machine shown in Fig. 95:

refrigerant = ammonia
condenser pressure = 170 psia
first evaporator pressure = 30 psia
second evaporator pressure = 10 psia
load on first evaporator = 10 tons
load on second evaporator = 5 tons
quality leaving evaporators = 100%
temperature leaving condenser = 80°F

Determine the power ideally required to drive the machine. *Ans.* 29.6 hp.

358. If the operating conditions for the refrigerating machine shown in Fig. 96 are those stated in the preceding problem, how many horsepower are ideally required to drive the machine?

359. Determine the power ideally required to drive the refrigerating machine shown in Fig. 97 if it operates under the conditions stated in Problem 357. Assume that only liquid enters the expansion valve for the second evaporator and that the vapor leaving the intercooler is at 80°F. *Ans.* 19.7 hp.

106. The Absorption Refrigeration Cycle. The **absorption refrigeration cycle** makes use of the fact that ammonia is more soluble in cold water than in warm water. At atmospheric pressure and 70°F, for example, 0.52 lb of ammonia will dissolve in 1 lb of water; but, at the same pressure and 150°F, only 0.13 lb will dissolve. Its solubility also depends upon the pressure and increases as the pressure increases. At any given pressure, however, its solubility always decreases as the temperature of the solution rises.

Figure 98 shows a flow-diagram for a simple absorption refrigerating machine. This diagram is similar to the one for the simple vapor compression cycle (Fig. 88a), except that the compressor is replaced by an *absorber*, a *pump*, and a *generator*. The method of operation is as follows: The liquid ammonia from the condenser is throttled through the expansion valve into the evaporator, just as in the vapor compression cycle. The ammonia vapor from the evaporator flows into the absorber which is at the evaporator pressure. In the absorber the ammonia vapor comes into contact with a "weak" solution of ammonia and water. The ammonia vapor dissolves in the weak solution and changes the weak solution into a "strong" solution. If this process were carried out adiabatically, the temperature of the solution would rise, and the solubility of the ammonia would decrease. The temperature is kept near atmospheric by cooling water, however, and the solubility is kept high. The strong solution from the absorber is pumped into the generator, which is at approximately the same pressure as the condenser. The strong solution is heated in the generator, and ammonia vapor (together with a small amount of water vapor) is driven off. The vapor is sent to the condenser; and the remaining solution, having become weak in ammonia, is returned to the absorber to repeat its cycle.

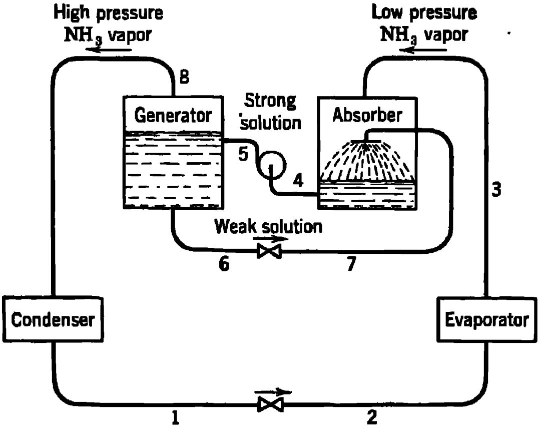


FIG. 98. Flow-diagram for a simple absorption refrigerating machine.

Although some work must be supplied to drive the strong-solution pump, the amount is quite small compared to that required to drive the compressor of a vapor compression machine. Most of the energy needed to drive an absorption machine is that supplied in the form of heat to the generator. Thus, except for the small amount of work supplied to the pump, the absorber and the generator of the absorption plant are equivalent to a heat-driven compressor.

Figure 98 shows the strong solution (which leaves the absorber at about atmospheric temperature) flowing directly into the generator and the hot weak solution from the generator flowing directly into the colder absorber. It is apparent that a heat exchanger can be used to good advantage to transfer heat from the weak solution to the strong. Such a heat exchanger is regularly used in practice.

Figure 98 also shows the vapor from the generator flowing directly into the condenser. If an actual plant operated in this way, some water vapor would be carried into the condenser with the ammonia vapor. The water would cause no trouble in the condenser, but it would freeze in the expansion valve or in the evaporator and might prevent these parts of the plant from operating properly. To reduce the amount of water vapor entering the condenser, a *rectifier* is usually placed in the system between the generator and the condenser. The strong solution from the absorber, instead of flowing directly into the generator, flows first through the rectifier, where it comes into contact with the vapor leaving the generator. As a result, most of the water vapor dissolves in the strong solution and returns to the generator. The rest of the vapor, which flows from the rectifier to the condenser, is almost pure ammonia.

The coefficient of performance of an absorption refrigerating machine may be defined as the ratio of the refrigeration produced to the heat supplied to the generator. This definition neglects the small amount of work required to drive the strong-solution pump.

We can calculate the ideal coefficient of performance of the simple absorption refrigerating machine shown in Fig. 98 as follows: Let us denote the concentration (pounds of ammonia per pound of solution) at each section by the letter x with a numerical subscript to indicate the section. Also, let us denote the weight of solution flowing past each section *per pound of ammonia flowing through the evaporator* by the letter w, again with a numerical subscript to indicate the section. The weight w_8 of the vapor mixture leaving the generator is evidently equal to $1/x_8$. Applying the steady-flow energy equation to the evaporator, we see that the refrigeration produced by this amount of solution is equal to $w_8(h_3 - h_2)$ or $w_8(h_3 - h_1)$, where each of the enthalpies is per pound of solution. Applying the steady-flow energy equation to the generator, we find that the heat supplied to the generator is $w_8h_8 + w_6h_6 - w_5h_5$. To determine w_5 and w_6, we note that

$$w_5 = w_6 + w_8$$

and that

$$x_5w_5 = x_6w_6 + x_8w_8.$$

It is evident that we can calculate the coefficient of performance by means of the foregoing expressions if we can find the concentrations and enthalpies of the various solutions. These depend upon the pressure and temperature of the solution and can be determined from tables of properties of ammonia-water solutions.*

It is interesting to consider what the best possible coefficient of performance is for a heat-driven refrigerating machine. Suppose that the temperature of the source of heat remains constant at T_1, that the temperature of the atmosphere is T, and that the temperature at which refrigeration is to be produced is T_0. The best conceivable plant is one consisting of a reversible heat engine driving a reversible refrigerating machine. Let us suppose that the heat engine and the refrigerating machine operate on Carnot cycles, as shown in Fig. 99. Since the work output from the heat engine is equal to the work input to the refrigerating machine,

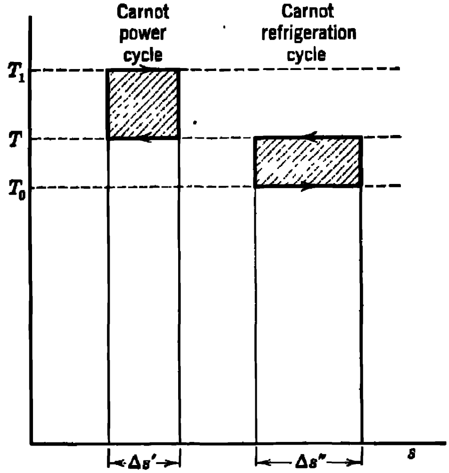


FIG. 99.

$$(T_1 - T)\,\Delta s' = (T - T_0)\,\Delta s'',$$

or

$$\frac{\Delta s''}{\Delta s'} = \frac{T_1 - T}{T - T_0}.$$

But, by definition,

$$(\text{c.p.})_{\text{ideal}} = \frac{\text{refrigeration produced by the refrigerating machine}}{\text{heat supplied to the heat engine}}$$

$$= \frac{T_0\,\Delta s''}{T_1\,\Delta s'}.$$

---

* See, for example, the *Refrigerating Data Book*, published by the American Society of Refrigerating Engineers.

Hence, combining the two foregoing equations, we obtain

$$(\text{c.p.})_{\text{ideal}} = \frac{T_0(T_1 - T)}{T_1(T - T_0)}. \tag{56}$$

Suppose, for example, that the evaporator temperature of a heat-driven refrigerating machine is 5°F, that atmospheric temperature is 86°F, and that the source of heat is steam condensing at atmospheric pressure (that is, at a temperature of 212°F). The best possible coefficient of performance for a machine operating under these conditions is

$$(\text{c.p.})_{\text{ideal}} = \frac{465(672 - 546)}{672(546 - 465)}$$
$$= 1.075.$$

107. The Electrolux Refrigerating Machine. Another kind of absorption refrigeration cycle is the one used in the Electrolux refrigerator. Figure 100 shows a simplified flow-diagram for this cycle. Liquid ammonia from the condenser flows into the evaporator, but it does not need to flow through an expansion valve because the same pressure exists in both evaporator and condenser. The evaporator contains hydrogen gas, and consequently the ammonia evaporates at a low *partial pressure* (discussed in Art. 136) and produces refrigeration. The resulting mixture of hydrogen and ammonia vapor flows from the evaporator into an absorber, where it comes into contact with a weak solution of

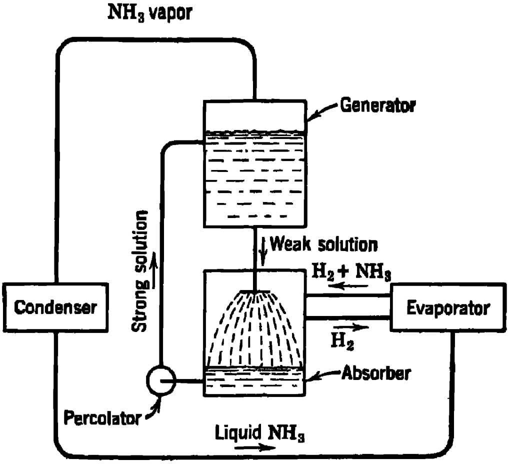


FIG. 100. Simplified flow-diagram for the Electrolux refrigerating machine.

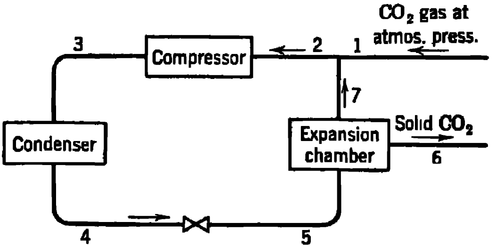
ammonia and water. The ammonia vapor dissolves in this solution, but the hydrogen does not. Instead, the hydrogen remains a gas and returns to the evaporator. The strong solution formed in the absorber is elevated to the generator by a heat-driven "percolator" pump. In the generator the strong solution is heated, and some of the ammonia is driven out of solution. The ammonia vapor is sent to the condenser, and the weak solution remaining is returned to the absorber. Note that the hydrogen gas is trapped in the evaporator and absorber, whereas the water in the system circulates only through the absorber and generator. Note also that the machine is driven by heat alone.

Since the Electrolux refrigerating machine has no moving parts and requires only a gas flame to operate it, it is particularly well suited for use in domestic refrigerators. The actual machine is somewhat more complex than the one shown in Fig. 100 because several heat exchangers and a rectifier are added to reduce the gas consumption.

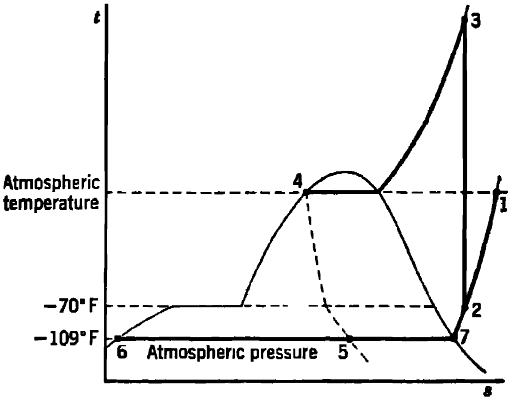
108. The Manufacture of Dry Ice. A *fluid* can sometimes be cooled most easily by using the fluid itself as the refrigerant in a refrigerating machine. One example is the producing of chilled water in the water-vapor refrigerating machine shown in Fig. 91. Another example is the producing of dry ice (solid carbon dioxide) in the machine shown in Fig. 101*a*. We shall discuss in this article the method of operation of the latter machine. First, however, let us consider the following facts:

If any vapor is cooled at a constant pressure *above* its triple point pressure, it eventually condenses and changes into a liquid. If it is cooled at a constant pressure *below* its triple point pressure, it changes from a vapor directly into a solid. For example, if water vapor (steam) is cooled at atmospheric pressure, it remains in the form of a vapor until its temperature drops to 212°F and then changes at constant temperature into a liquid. On the other hand, if it is cooled at a constant pressure of 0.0185 psia, it remains a vapor until its temperature drops to 0°F (see Table 5 of the *Steam Tables*) and then changes at constant temperature directly into ice. It follows that, since the triple point pressure of carbon dioxide is 75.0 psia, dry ice could be manufactured simply by cooling gaseous carbon dioxide at atmospheric pressure down to the temperature (−109°F) at which it changes from a vapor directly into a solid. To absorb the heat rejected by the gas during this process, a refrigerating machine operating with an evaporator temperature below −109°F would be necessary. Such a machine can be built, but it is simpler to produce dry ice in a machine using the carbon dioxide itself as the refrigerant.

A flow-diagram for a simple dry-ice-manufacturing machine is shown in Fig. 101*a*. Carbon dioxide gas at atmospheric pressure is compressed in the compressor to a pressure in the order of 1000 psia. At this pressure the gas can be liquefied in the condenser by cooling it to atmospheric temperature. The liquid carbon dioxide from the condenser flows through an expansion valve into an insulated chamber in which the pressure is atmospheric. Since this pressure is below the triple point pressure



(a) Flow-diagram



(b) Temperature-entropy diagram

Fig. 101. A simple dry-ice-manufacturing plant.

of carbon dioxide, the carbon dioxide issues from the expansion valve as a mixture of solid and gas. The solid part of the mixture is in the form of carbon dioxide snow and must be compressed into cakes before it is ready for commercial use. The gaseous part of the mixture is added to the supply of "make-up" gas and is returned to the compressor. The states of the carbon dioxide at the various sections numbered on the flow-diagram are shown on a temperature-entropy diagram in Fig. 101*b*.

The work required to drive the compressor can be decreased by using a multi-stage rather than a single-stage compressor. If, in addition, several expansion valves are used in series, and if each expansion valve is followed by a flash chamber, the vapor formed in each chamber can be sent directly to one of the higher pressure stages of the compressor. The work required to operate the plant is thereby decreased still further.

PROBLEMS

360. Suppose that the compressor of the simple dry-ice-manufacturing plant shown in Fig. 101 were driven by an electric motor and that the plant operated in an ideal manner. How many kilowatthours of electricity would be required to drive the motor per ton of solid CO_2 produced? Assume that the enthalpies of the CO_2 at certain of the sections numbered on the flow-diagram would be as follows:

$h_1 =$ 170.2 Btu per lb
$h_3 =$ 230.0 Btu per lb (if $s_3 = s_2$)
$h_4 =$ 63.7 Btu per lb
$h_6 = -113.2$ Btu per lb
$h_7 =$ 133.2 Btu per lb

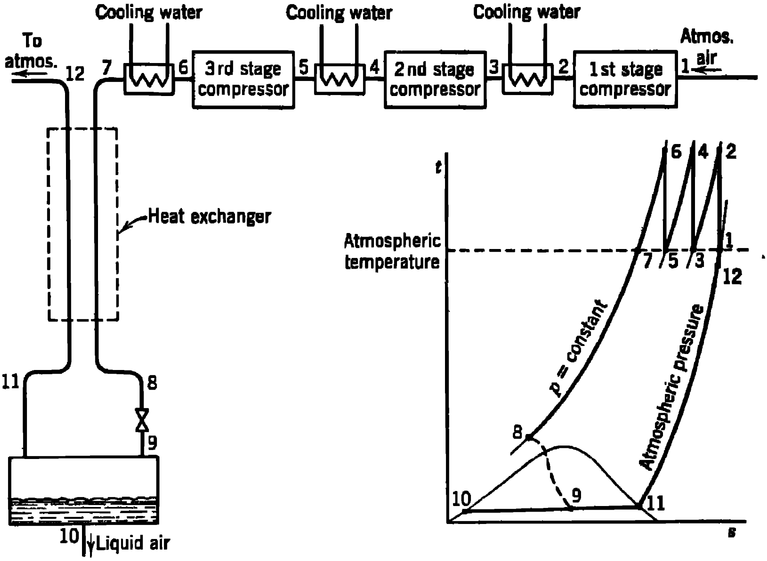
361. A plant similar to the one shown in Fig. 101 could in theory be used to produce ice. Instead of introducing water vapor into the plant at section 1, however, water would be added at section 4. Suppose that (*a*) the water supplied were at 70°F, (*b*) the temperature in the expansion chamber were 20°F, and (*c*) the pressure in the condenser were 1 psia. If the plant operated in an ideal manner, how much work would be required per pound of ice produced? (*Note:* The properties of saturated H_2O at temperatures below 32°F are given in Table 5 of the *Steam Tables.*)

Ans. 43.1 Btu.

362. If a refrigerating machine having an ideal coefficient of performance of 4.75 were used to change water at 70°F into ice at 20°F, how much work would be required per pound of ice produced? Compare this value with the value obtained in the preceding problem.

109. The Production of Liquid Air. Another example of cooling a fluid by using the fluid itself as the refrigerant is the producing of liquid air in the machine shown in Fig. 102*a*. Atmospheric air is first compressed to a pressure between 2000 and 3000 psia in a three- or four-stage compressor. After each stage the air is cooled back to atmospheric temperature in a water-cooled intercooler. The high pressure air from the last cooler flows first through a heat exchanger and then through an expansion valve. It is throttled in the valve to approximately atmospheric pressure. If the air behaved as a perfect gas, its temperature would remain constant during the throttling process (see Eq. 18). Actually, however, its temperature drops 50 or 60 deg, depending upon its initial pressure. This colder air at atmospheric pressure flows back through the heat exchanger and cools the incoming high pressure air. The high pressure air, now entering the expansion valve at a temperature below atmospheric, again drops in temperature

60 or more degrees on flowing through the valve; and this air cools the incoming high pressure air still further. As the machine continues to operate, the temperature of the high pressure air entering the expansion valve becomes lower and lower. Eventually its temperature becomes low enough that it issues from the valve as a mixture of liquid and gas. The gaseous part of the mixture continues to flow out through the heat exchanger, and the liquid portion is the "make." After the plant has begun to produce liquid air and has reached steady-flow conditions, the states of the air at the various sections numbered on the flow-diagram are as shown on the temperature-entropy diagram in Fig. 102*b*.



(*a*) Flow-diagram (*b*) Temperature-entropy diagram

FIG. 102. The Linde liquid-air machine.

PROBLEM

363. If the normal pressure and temperature of the atmosphere, instead of being in the order of 14.7 psia and 50°F, were 14.7 psia and 1400°F, water vapor could be liquefied by the apparatus shown in Fig. 102. Suppose that the high pressure water vapor entering the heat exchanger was at 5000 psia and 1400°F and that the low pressure vapor leaving was at 14.7 psia and 1350°F. How many pounds of water at 212°F would be produced per pound of water vapor compressed? Assume that the apparatus would be perfectly insulated. *Ans*. 0.0345 lb.

SYMBOLS

c_p	specific heat at constant pressure
c.p.	coefficient of performance
h	enthalpy per unit mass
J	mechanical equivalent of heat
k	ratio of c_p to c_v
p	pressure
Q	a quantity of heat
s	entropy per unit mass
t	temperature
T	absolute temperature
v	specific volume
w	weight
W'_{cycle}	net work supplied to system during one cycle
x	concentration

Chapter 13

COMPRESSORS

In this chapter some of the more important types of compressors are described briefly, equations for calculating the work required to drive ideal compressors are developed, and the various efficiencies used to measure the performance of actual compressors are discussed.

110. Types of Compressors. In general, a **compressor** is any device used to cause a gas or vapor to flow from a region at one pressure to another region at a higher pressure. If its intake pressure is above atmospheric, the compressor is sometimes called a *booster* compressor. Such compressors are used, for example, in natural gas transmission lines to maintain a high pressure in the line. If a compressor operates with an intake pressure below atmospheric, and if its discharge pressure is approximately equal to atmospheric pressure, it is usually called a *vacuum pump*. Vacuum pumps are used, for instance, to remove air from steam power plant condensers.

Most compressors may be classified either as (*a*) *positive displacement compressors* or (*b*) *centrifugal compressors*. Positive displacement compressors may in turn be classified as (*a*) *reciprocating compressors*, in which a piston moves back and forth inside a cylinder, (*b*) *positive displacement blowers*, in which two intermeshing gear-type elements rotate in opposite directions within a close-fitting casing, and (*c*) *rotary compressors*, in which a vaned rotor is mounted eccentrically inside a cylindrical casing. Each of these types of positive displacement compressors is superior to other types for certain applications. However, the reciprocating type has by far the widest field of application.

Centrifugal compressors may be classified according to their function. If they are used simply to move large quantities of a gas from one region to another at approximately the same pressure, they are called *blowers* or *fans*. If their primary purpose is to compress a gas from one pressure to some higher pressure, they are called *centrifugal compressors*.

Centrifugal compressors raise the pressure of a gas by first imparting a high velocity to it and then letting it flow through a *diffuser* in which its velocity decreases while its pressure rises. They are able to handle large volumes of gas since they operate at high rotative speeds. They are particularly well suited to applications in which a gas must be compressed only to moderate pressures.

111. The Ideal Single-Stage Reciprocating Compressor. The simplest kind of reciprocating compressor is one which accomplishes the entire compression process in a single cylinder. Such compressors are called **single-stage compressors.** They may be either *single-acting* or *double-acting*, depending upon whether the compression is effected in one end of the cylinder only or in both ends.

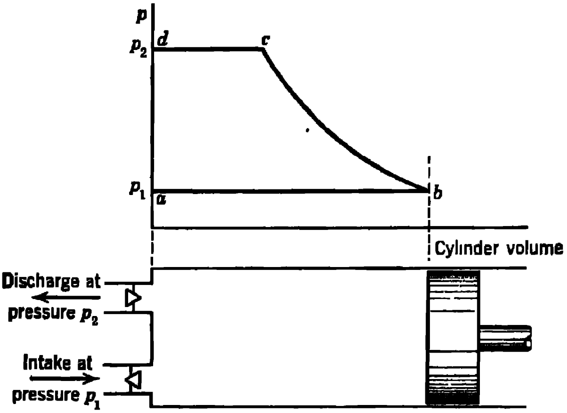


FIG. 103. Diagrammatic sketch of a single-stage reciprocating compressor with zero clearance volume, and the ideal indicator diagram for such a compressor.

A diagrammatic sketch of a single-stage compressor is shown in the lower part of Fig. 103. The intake and discharge valves may be simply check valves, as shown, or they may be mechanically operated. If they are of the first type, their opening and closing depends entirely upon a difference in pressure; if they are mechanically operated, their opening and closing is controlled usually by a cam on the crank shaft.

The volume swept out by the piston in one stroke (equal to the cross-sectional area of the piston times the stroke) is called the **piston displacement.** It will be designated by the symbol *PD*. Note that, for a double-acting compressor, the piston displacement of the "crank-end" is smaller than that of the "head-end" by an amount equal to the cross-sectional area of the piston rod times the stroke.

The volume within the cylinder when the piston is at the end of its inward travel plus the volume within the passages leading to the valves is called the **clearance volume.** It will be seen shortly that a clearance volume as small as possible is desirable. The clearance volume can be made small in an actual compressor, but it cannot be made zero since, for mechanical reasons, the piston face cannot be allowed to come into contact with the cylinder head. Besides, the passages leading to the valves will always contribute somewhat to the clearance volume.

If a reciprocating compressor operated in an *ideal* manner, no pressure drop would take place through its intake and discharge valves, and the pressure in both its intake and discharge lines would remain constant. If, in addition, the compressor had no clearance volume, its indicator diagram (see Art. 91) would appear as shown in the upper part of Fig. 103. When the piston was at the end of its inward travel (point *a* on the indicator diagram), the weight of gas inside the cylinder would be zero. As the piston began to move outward, the intake valve would open, and gas at the intake pressure p_1 would flow into the cylinder. The intake valve would remain open until the piston completed its outward travel (point *b*). As the piston began its inward travel, both the intake and discharge valves would be closed, and the pressure of the gas would rise as its volume decreased. This compression process might take place adiabatically, isothermally, polytropically, or in some other way. When the pressure of the gas reached the discharge pressure p_2, the discharge valve would open (point *c*); and, as the piston continued its inward travel, the gas would be forced out of the cylinder and into the discharge line. Note that the weight of gas inside the cylinder would change during the processes *a*–*b* and *c*–*d* but would remain constant during the process *b*–*c*.

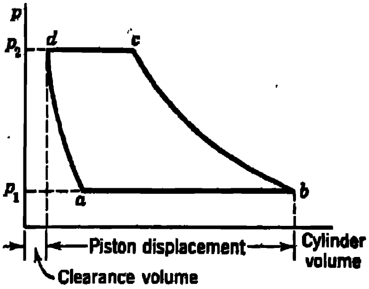


FIG. 104. Ideal indicator diagram for a single-stage reciprocating compressor with clearance volume.

Since no actual compressor can have a clearance volume of zero, it is of interest to consider how a compressor with clearance volume would function if it operated in an ideal manner. Its indicator diagram would appear as shown in Fig. 104. When the piston was at the end of its inward travel (point *d* on the diagram), the clearance volume would contain gas at the discharge pressure p_2. This gas would expand as the piston moved outward, and, when its pressure dropped to the intake pressure p_1, the intake valve would open (point *a*). The remainder of the cycle of operations would be the same as for the ideal compressor with zero clearance volume. Note that, if the clearance volume were zero, the volume of gas ideally drawn into the compressor during the intake stroke would be equal to the piston displacement, whereas, if the clearance volume is not zero, the volume drawn into the compressor (equal to the cylinder volume at b less the cylinder volume at a) is smaller. Clearance volume is therefore undesirable in that it reduces the capacity of the compressor.

Let us consider next how the indicator diagram for an actual single-stage reciprocating compressor compares with the diagram for an ideal compressor. An indicator diagram for an actual compressor is shown in Fig. 105; and for comparison the indicator diagram for an ideal compressor, *adiabatic compression from b to c and adiabatic expansion from d to a being assumed*, is superimposed upon it. In the actual compressor, when the piston is at the end of its inward travel, the pressure of the gas trapped in the clearance volume is likely to be higher than the discharge pressure p_2 because of the pressure drop which occurs in the discharge valve. Therefore, point d on the actual diagram is higher than point d on the ideal, and the actual expansion curve falls to the right of the ideal. This effect is increased still further if any heat flows from the cylinder walls *to* the gas during the process. Such a transfer of heat is likely to happen since the cylinder walls, though always either air- or water-cooled, are usually at a temperature above the temperature of the gas at point a. Because the actual expansion curve falls to the right of the ideal, the intake valve of the actual compressor opens later, and less gas is drawn into the cylinder.

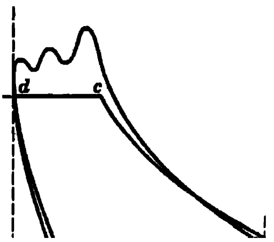


FIG. 105. Comparison of the actual and ideal indicator diagrams for a single-stage reciprocating compressor.

During the intake process a–b, the pressure inside the cylinder of the actual compressor is lower than the pressure p_1 in the intake line because of the pressure drop which occurs in the intake valve. Consequently, the *weight* of gas within the actual compressor at the end of the intake stroke is less than that within the ideal. The weight present is decreased still further if the gas is heated by the cylinder walls as it is drawn into the compressor.

The actual compression curve b–c starts below the ideal but may cross it, as shown. This effect may result if the gas is heated appreciably by the cylinder walls during the compression process.

Finally, the pressure inside the cylinder of the actual compressor during the discharge process c–d is above the pressure p_2 in the discharge line because of the pressure drop which occurs in the discharge valve. The cylinder pressure is also likely to fluctuate as shown because of the compressibility of the column of air in the discharge line. This air behaves much as a long, open-coil, helical spring would. If one end of such a spring is given a sudden displacement in an axial direction, the entire spring is not immediately displaced the same amount. Instead, vibrations are set up within it. Similarly, fluctuations in pressure occur within the column of air, and these affect the pressure inside the cylinder.

It is of interest now to consider how the net work done on the gas compressed during one complete cycle of operations compares for the actual and the ideal reciprocating compressors. To make this comparison, let us first show that the net work done on the gas is represented by the area enclosed within the indicator diagram for the compressor. We note that the work done by the piston *on the gas* while the piston moves inward an infinitesimal distance dx is $pA\,dx$, where p represents the pressure of the gas, and A the area of the piston face. But, since $A\,dx$ is equal to the change dV in the *cylinder volume*, the work done on the gas is equal to $p\,dV$. The total work done during the entire inward travel of the piston is therefore $\int p\,dV$ and is represented by the area on the indicator diagram under the compression and discharge curves (the area under b–c–d in Fig. 105). Similarly, the work done *by the gas* on the piston during the outward travel of the piston is represented by the area under the expansion and intake curves (the area under d–a–b). Hence, the net work done on the gas during one complete cycle of operations is represented by the area enclosed within the indicator diagram.

Now, comparing the actual and ideal diagrams shown in Fig. 105, we see that more work is done *per cycle of operations* by the actual compressor than by the ideal because the area enclosed by the actual diagram is larger than that enclosed by the ideal. Moreover, we see that the work done *per pound of gas compressed* is larger for the actual compressor because the amount of gas drawn into the cylinder per cycle is less for the actual than for the ideal compressor.

112. The Power Required to Drive an Ideal Single-Stage Reciprocating Compressor. Let us derive expressions for the power required to drive an ideal single-stage reciprocating compressor, first assuming that the gas being compressed flows through the compressor adiabatically, and then assuming that it flows through the compressor isothermally. For each manner of operation let us assume further (*a*) that the compressor operates in a steady-flow manner, (*b*) that the gas is a perfect gas, (*c*) that the specific heats c_v and c_p of the gas are constants, and (*d*) that the gas enters and leaves the compressor with negligible velocities.

The work done on each pound of gas can be determined by the steady-flow energy equation. Let us denote the work *supplied to the compressor* per pound of gas flowing through it by the symbol W'_{in}. Then, since $W' = -W'_{\text{in}}$, the steady-flow energy equation (Eq. 13*a*) may be written in the form

$$\frac{W'_{\text{in}}}{J} = h_2 - h_1 - Q + \frac{V_2^2 - V_1^2}{2gJ}. \tag{13b}$$

The velocities V_1 and V_2 with which the gas enters and leaves the compressor are assumed to be negligible, and therefore the preceding equation may be written in the simpler form

$$\frac{W'_{\text{in}}}{J} = h_2 - h_1 - Q. \tag{a}$$

Now let us consider the work ideally required to compress 1 lb of gas if the gas flows through the compressor *adiabatically*. For this condition Eq. *a* reduces to

$$\frac{W'_{\text{in}}}{J} = h_2 - h_1. \tag{b}$$

Therefore, it is necessary only to find the difference $h_2 - h_1$ between the enthalpies of the gas entering and leaving the compressor. Since the gas is assumed to be a perfect gas having a constant value of c_p, its change in enthalpy could be expressed in terms of its temperatures entering and leaving the compressor (see Eq. 18*a*). A more useful expression for W'_{in} is obtained, however, if $h_2 - h_1$ is expressed in terms of the pressures and specific volumes with which the gas enters and leaves. Such an expression was derived in Problem 90 (Eq. 18*b*). Introducing it into Eq. *b*, we get

$$W'_{\text{in}} = \frac{k}{k-1}(p_2v_2 - p_1v_1). \tag{c}$$

Since the gas is assumed to undergo an adiabatic process, and since the compressor is assumed to operate in an ideal manner (that is, without friction), the specific volume v_2 can be eliminated from this equation by making use of Eq. 28*c*:

$$v_2 = v_1\left(\frac{p_1}{p_2}\right)^{1/k}. \tag{28c}$$

Introducing this expression for v_2 into Eq. *c* and rearranging, we get

$$W'_{\text{in}} = \frac{k}{k-1} p_1 v_1 \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right]. \tag{d}$$

Finally, if the gas flows through the compressor at the rate of w pounds per minute, the power required to drive the compressor is

$$(\text{ideal horsepower})_{\text{adiabatic}} = \frac{k}{k-1} \times \frac{p_1 w v_1}{33{,}000} \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right].$$

Or, since wv_1 is equal to the volume of gas $\bar{V}_1$ entering the compressor per minute,

$$(\text{ideal horsepower})_{\text{adiabatic}} = \frac{k}{k-1} \times \frac{p_1 \bar{V}_1}{33{,}000} \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/k} - 1 \right], \tag{57}$$

where p_1 = the intake pressure, expressed in pounds per square foot absolute,

p_2 = the discharge pressure, expressed in pounds per square foot absolute, and

$\bar{V}_1$ = the volume of gas entering the compressor, expressed in cubic feet per minute.

Let us next derive an equation for the power required to drive an ideal single-stage reciprocating compressor if the gas flows through the compressor *isothermally*. Since the gas is now assumed to enter and leave at the same temperature, its change in enthalpy is zero (see Eq. 18*a*); and Eq. *a* reduces to

$$\frac{W'_{\text{in}}}{J} = -Q. \tag{e}$$

To see how Q can be determined, let us examine the series of processes taking place within the compressor. Referring to the indicator diagram for an ideal compressor shown in Fig. 104, we see that the state of the gas remains unchanged (that is, its pressure, specific volume, temperature, and other properties all remain constant) during the processes *a–b* and *c–d*. Therefore, no heat is transferred to or from the gas. During the processes *b–c* and *d–a*, however, the state of the gas does change, and a transfer of heat takes place. The amount of heat transferred can be determined readily since the intake and discharge valves both remain closed and therefore the gas actually undergoes two *non-flow* processes. By Eqs. 25*b* and 26, the heat added to the gas during each process is

$$JQ_{b-c} = p_b v_b \ln \left(\frac{p_b}{p_c} \right) = -p_1 v_b \ln \left(\frac{p_2}{p_1} \right)$$

and

$$JQ_{d-a} = p_a v_a \ln\left(\frac{p_d}{p_a}\right) = p_1 v_a \ln\left(\frac{p_2}{p_1}\right)$$

Note that, since v_a and v_b are not specific volumes, these expressions are not for 1 lb of gas but are for whatever weight of gas is confined within the cylinder. Introducing them into Eq. *e*, we get

$$\begin{aligned} W'_{\text{in}} &= -JQ \\ &= -J(Q_{b-c} + Q_{d-a}) \\ &= p_1(v_b - v_a) \ln\left(\frac{p_2}{p_1}\right), \end{aligned} \qquad (f)$$

where W'_{in} is the work required to compress $v_b - v_a$ cubic feet of gas (the volume that enters the compressor during the intake stroke). The work required to compress 1 cu ft (measured under inlet conditions) is evidently $p_1 \ln (p_2/p_1)$. Therefore, to compress $\overline{V}_1$ cubic feet per minute, the power required is

$$(\text{ideal horsepower})_{\text{isothermal}} = \frac{p_1 \overline{V}_1}{33{,}000} \ln\left(\frac{p_2}{p_1}\right), \qquad (58)$$

where each symbol has the same meaning as in Eq. 57.

It is interesting to note that, since the clearance volume does not appear in either Eq. 57 or 58, the power ideally required to compress a gas at any given rate is the same regardless of how large the clearance volume of the compressor may be. This is to be expected since the work done by the piston on the clearance-volume gas during the compression stroke is all returned to the piston when this gas reexpands.

PROBLEMS

364. Two hundred cubic feet of air per minute enter a single-stage reciprocating compressor at 14.7 psia and are compressed to 100 psia. How many horsepower would be required to drive the compressor if it operated in an ideal manner and the air flowed through it (*a*) adiabatically and (*b*) isothermally?

Ans. (*a*) 32.7 hp; (*b*) 24.6 hp.

365. For the conditions stated in the preceding problem, the ratio of the horsepower ideally required by a single-stage compressor operating adiabatically to the horsepower ideally required by a single-stage compressor operating isothermally is (32.7/24.6 =) 1.33. What would the ratio be if the discharge pressure were 200 psia instead of 100 psia?

366. A single-stage, single-acting reciprocating compressor has a piston displacement of 0.850 cu ft and a clearance volume of 0.035 cu ft. It runs at a speed of 200 rpm and compresses air from 14.5 to 120 psia. Determine what the horsepower ideally required to drive it would be if the air flowed through it (*a*) adiabatically and (*b*) isothermally. *Ans.* (*a*) 26.8 hp; (*b*) 15.9 hp.

367. Show that, if the processes from *b* to *c* and from *d* to *a* in Fig. 104 are polytropic processes, the horsepower ideally required to drive a single-stage reciprocating compressor may be determined by the equation

$$(\text{ideal horsepower})_{\text{polytropic}} \quad \frac{n}{n-1} \times \frac{p_1 V_1}{33{,}000}\left[\left(\frac{p_2}{p_1}\right)^{(n-1)/n} - 1\right]$$

(*Suggestion:* Determine the net heat added to each pound of air compressed by Eq. 32.)

113. Multistage Reciprocating Compressors.

In Problem 364 the power ideally required to compress 200 cfm of air from 14.7 to 100 psia is found to be 24.6 hp if the compressor operates isothermally and to be 32.7 hp if the compressor operates adiabatically. To explain this result, let us consider the indicator diagrams for two ideal compressors, shown superimposed in Fig. 106. One compressor operates adiabatically, and the other isothermally. Zero clearance volumes are assumed only for the sake of simplicity. Since the diagrams show equal volumes of air entering each compressor, and since the areas enclosed by the diagrams represent the work done on the air (see Art. 111), it is evident that *more work is always required if a compressor operates adiabatically rather than isothermally.* It is also evident that the ratio of the work required for adiabatic operation to the work required for isothermal operation increases as the discharge pressure p_2 rises.

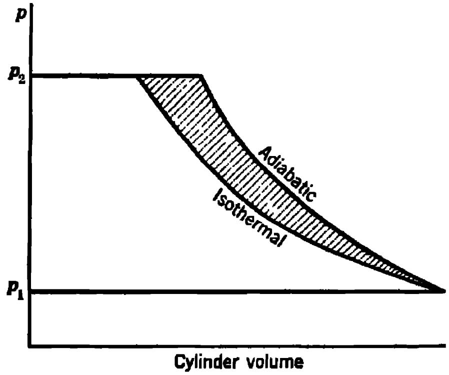


FIG. 106.

Let us now consider how an actual reciprocating compressor might be made to approach isothermal operation. It has already been explained in Art. 47 that all compressors tend to operate adiabatically because they run at relatively high speeds and so permit insufficient time for much heat transfer to take place. This suggests that isothermal operation might be achieved simply by (*a*) keeping the cylinder walls cool by water-jacketing them, and then (*b*) running the compressor at a very slow speed. This could be done, but it would not be practicable because the capacity of the compressor would be too small. Or, stated another way, a compressor would have to be very large to have a useful capacity. It is true that all reciprocating compressors have either fins or water jackets around their cylinders to help them dissipate heat to the atmosphere. However, the principal reason for keeping the cylinder walls cool is not to try to approximate isothermal operation but to make proper lubrication of the piston possible.

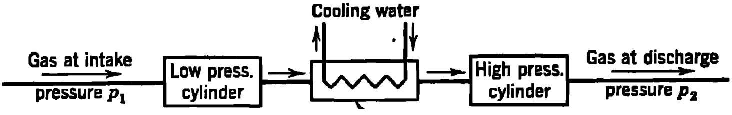


FIG. 107. Flow-diagram for a two-stage compressor.

The method used in practice to approximate isothermal operation is to compress the gas, not in a single cylinder, but successively in two or more cylinders and to cool the gas back to its initial temperature before it enters each successive cylinder. A compressor operating in this way is called a **multistage compressor.**

A flow-diagram for a multistage compressor having two cylinders (usually called a "two-stage" compressor) is shown in Fig. 107. The gas to be compressed is taken into the low pressure cylinder (the "first stage") at the intake pressure p_1, is compressed approximately adiabatically to an intermediate pressure p_i, and is discharged into an *intercooler*. In the intercooler the gas is cooled at approximately constant pressure by letting heat flow from it either to cooling water or to the atmosphere. It is then taken into the high pressure cylinder (the "second stage"), where it is compressed approximately adiabatically to the discharge pressure p_2.

The ideal indicator diagram for a two-stage compressor is shown in Fig. 108. It is drawn on the assumption (*a*) that each cylinder operates adiabatically, (*b*) that each has a clearance volume of zero, and (*c*) that the gas leaving the intercooler is at the same temperature as the gas entering the low pressure cylinder. The ideal indicator diagram for a single-stage compressor would be the figure *abc'h* if the compressor operated adiabatically and discharged at the pressure p_2. It is apparent

(a) that the work ideally required to compress the volume v_b from p_1 to p_2 is less for the two-stage compressor than for the single-stage compressor by an amount represented by the shaded area, and (b) that the work ideally required could be reduced still further by using more stages. In theory, the minimum work would be required if an infinite number of stages were used. It would equal that required by a compressor operating isothermally.

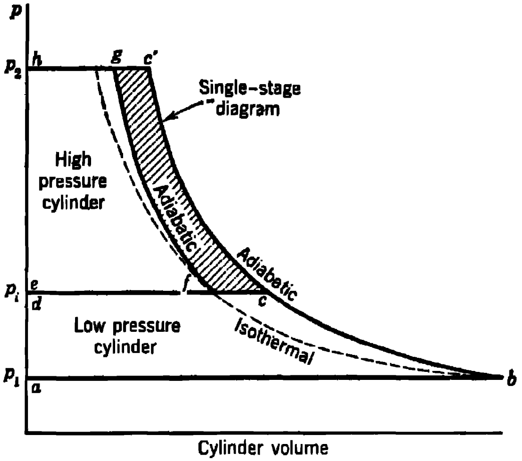


FIG. 108. Ideal indicator diagrams for a two-stage compressor with perfect intercooling and for a single-stage compressor operating adiabatically.

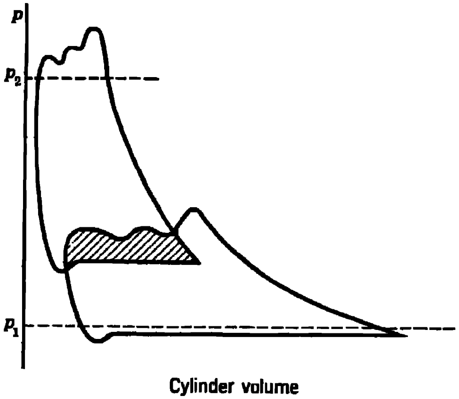


FIG. 109. Actual indicator diagrams for a two-stage compressor.

The number of stages actually used in any compressor is limited by two factors: First, the cost of the compressor is increased by increasing the number of its stages, and this may more than offset the saving in power. Second, although the power required is, in theory, decreased by adding more stages, it may actually be increased if too many stages are used. Consider, for instance, the indicator diagrams for an actual two-stage compressor shown in Fig. 109. The diagrams overlap on account of the pressure drop which occurs in the discharge valve of the low pressure cylinder, in the intercooler, and in the intake valve of the high pressure cylinder. The overlapping area (the shaded area) represents work that must be done twice. This work may be greater than the work saved by intercooling, and therefore the use of additional stages may increase rather than decrease the power actually required.

114. The Power Required to Drive an Ideal Multistage Reciprocating Compressor. The power required to drive an ideal multistage reciprocating compressor is equal to the sum of the power required to drive its separate stages. Let us first consider a two-stage compressor, and let us assume (*a*) that each of its stages operates adiabatically, (*b*) that it operates in a steady-flow manner, (*c*) that the gas compressed is a perfect gas, (*d*) that the specific heats c_v and c_p of the gas are constants, and (*e*) that the gas enters and leaves each stage with negligible velocities. Then, denoting the intake pressure of the compressor by p_1, its intercooler pressure by p_i, and its discharge pressure by p_2, we find that the power ideally required to drive it is, in accordance with Eq. 57,

$$(\text{ideal horsepower})_{\text{2-stage, adiabatic}} = \frac{k}{k-1} \times \frac{p_1\bar{V}_1}{33{,}000}\left[\left(\frac{p_i}{p_1}\right)^{(k-1)/k} - 1\right] + \frac{k}{k-1} \times \frac{p_i\bar{V}_i}{33{,}000}\left[\left(\frac{p_2}{p_i}\right)^{(k-1)/k} - 1\right], \quad (59)$$

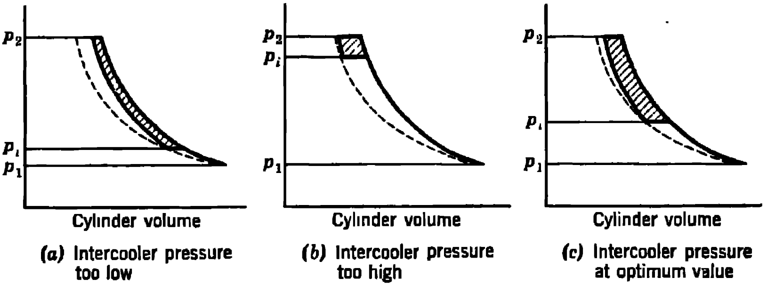
where $\bar{V}_1$ represents the cubic feet of gas entering the low pressure cylinder per minute, and $\bar{V}_i$ the cubic feet entering the high pressure cylinder per minute.

If the gas is cooled in the intercooler back to the temperature at which it enters the low pressure cylinder, the intercooling is said to be "perfect." It is evident from the characteristic equation $pv = RT$ that with perfect intercooling the product $p_i\bar{V}_i$ equals the product $p_1\bar{V}_1$. Hence, with perfect intercooling, Eq. 59 simplifies to

$$(\text{ideal horsepower})_{\substack{\text{2-stage, adiabatic}\\ \text{perfect intercooling}}} = \frac{k}{k-1} \times \frac{p_1\bar{V}_1}{33{,}000}\left[\left(\frac{p_i}{p_1}\right)^{(k-1)/k} + \left(\frac{p_2}{p_i}\right)^{(k-1)/k} - 2\right]. \quad (60)$$

It can be seen from Eq. 60 that for any given values of p_1, p_2, and $\bar{V}_1$, the power ideally required to drive a two-stage compressor with perfect intercooling depends upon the intercooler pressure p_i. It happens that there is an optimum value of p_i for which the power required is a minimum. That this is true can be seen from the ideal indicator diagrams shown in Fig. 110. The shaded area on each of these diagrams represents the work saved by intercooling. If the intercooler pressure p_i is either too low or too high, the saving is small; but, if p_i is as shown in Fig. 110*c*, the saving is a maximum.

The optimum intercooler pressure is the pressure at which the power required to drive the compressor is a minimum. Therefore, it can be found for an ideal two-stage compressor by differentiating the right side of Eq. 60 with respect to p_i, setting this derivative equal to zero, and solving for p_i. The optimum value of p_i is found to be



(*a*) Intercooler pressure too low

(*b*) Intercooler pressure too high

(*c*) Intercooler pressure at optimum value

FIG. 110.

$$p_i = \sqrt{p_1 p_2}. \tag{61}$$

Let us now consider the power required to drive an ideal two-stage compressor assuming (*a*) that each of its cylinders operates adiabatically, (*b*) that the intercooling is perfect, and (*c*) that the intercooler pressure is the optimum value. The ideal power required under these conditions can be found by introducing the optimum value of p_i, given by Eq. 61, into Eq. 60. The following equation is obtained:

$$(\text{ideal horsepower})_{\substack{\text{2-stage, adiabatic} \\ \text{perfect intercooling} \\ \text{opt. intercooler press.}}}$$

$$= \frac{2k}{k-1} \times \frac{p_1 \bar{V}_1}{33{,}000}\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/2k} - 1\right]. \tag{62}$$

It is interesting to consider how the intercooler pressure can be made to assume any desired value. Let us suppose that a two-stage compressor were built with its cylinders exactly the same size. For the sake of simplicity, let us also suppose that each cylinder had a clearance volume of zero. It is evident that equal volumes of gas per minute would enter the two cylinders of such a compressor. This could take place only if the gas entered each cylinder at the same pressure; that is, if no compression whatever occurred in the first stage. It is apparent, therefore, (*a*) that a rise in pressure can occur in the first stage only if the piston displacement of the second stage is smaller than that of the first, and (*b*) that the intercooler pressure is fixed by the relative sizes of the two stages.

For an ideal two-stage compressor with zero clearance volumes, the piston displacements of the two stages must be proportional to the volumes v_b and v_f shown on the indicator diagrams in Fig. 108. Note that, for perfect intercooling, $p_1 v_b = p_i v_f$; and, for minimum work, $p_i = \sqrt{p_1 p_2}$. Therefore, under the ideal conditions assumed,

$$\frac{PD_{\text{2nd stage}}}{PD_{\text{1st stage}}} = \frac{v_f}{v_b} = \frac{p_1}{p_i} = \sqrt{\frac{p_1}{p_2}}. \tag{63}$$

The foregoing discussion all pertains to a two-stage compressor. The method of determining the power ideally required to drive such a compressor and of determining the optimum intercooler pressure can be extended to compressors with more than two stages. The results of such an analysis are as follows: For a multistage compressor with N stages, each stage operating adiabatically and with perfect intercooling between stages, the optimum pressures for the successive intercoolers are

$$\begin{aligned} p'_i &= (p_1{}^{N-1} p_2)^{1/N}, \\ p''_i &= (p_1{}^{N-2} p_2{}^2)^{1/N}, \\ p'''_i &= (p_1{}^{N-3} p_2{}^3)^{1/N}, \end{aligned} \tag{64}$$

and so forth; and the power ideally required to drive such a compressor is

$$(\text{ideal horsepower})_{\substack{N\text{-stage, adiabatic} \\ \text{perfect intercooling} \\ \text{opt. intercooler press.}}} = \frac{Nk}{k-1} \times \frac{p_1 \bar{V}_1}{33{,}000} \left[\left(\frac{p_2}{p_1} \right)^{(k-1)/Nk} - 1 \right]. \tag{65}$$

PROBLEMS

368. Derive Eq. 61. If the intercooler pressure for an ideal two-stage compressor is equal to this optimum value, how does the work required to drive the first stage compare with the work required to drive the second stage?

369. In accordance with Eq. 64, the optimum intercooler pressures for a three-stage compressor with perfect intercooling are given by the expressions

$$p'_i = (p_1^2 p_2)^{1/3}$$

and

$$p''_i = (p_1 p_2^2)^{1/3}.$$

Derive these equations. (*Note:* If $z = f(x, y)$, z will be a minimum when $\partial z/\partial x = 0$ and $\partial z/\partial y = 0$.)

370. Determine the horsepower ideally required to compress 100 cfm of air from 14.5 to 150 psia if the compression is carried out (*a*) in a single-stage compressor operating adiabatically, (*b*) in a single-stage compressor operating isothermally, (*c*) in a two-stage compressor with each stage operating adiabatically, with perfect intercooling between stages, and with the optimum intercooler pressure, and (*d*) in a three-stage compressor with each stage operating adiabatically, with perfect intercooling between stages, and with the optimum intercooler pressures. What are the optimum intercooler pressures for the multistage compressors?

371. Determine the horsepower ideally required to compress 100 cfm of air from 14.5 to 150 psia in a two-stage compressor operating under the following conditions: (*a*) The air enters the first stage at 14.5 psia and 60°F; (*b*) it is compressed adiabatically to 40 psia in the first stage; (*c*) it is cooled at constant pressure to 90°F in the intercooler; and (*d*) it is compressed adiabatically to 150 psia in the second stage. Compare your answer with the answer to (*c*) in the preceding problem.

372. An ideal two-stage compressor compresses air from 14.5 to 150 psia. Each stage operates adiabatically, the intercooling is perfect, and the intercooler pressure is the optimum value. What must be the ratio of the piston displacements of the two stages if (*a*) the clearance volumes are zero and (*b*) the clearance volume of each stage is 5% of the piston displacement of the stage? *Ans.* (*a*) 0.311.

115. The Volumetric Efficiency of a Reciprocating Compressor.

The **volumetric efficiency** of a single-stage reciprocating compressor is defined as the volume of gas entering the compressor per minute (measured at the intake pressure and temperature) divided by the piston displacement of the compressor per minute; that is, for a *single-stage compressor*,

$$\text{volumetric efficiency} = \frac{\text{volume of gas entering compressor per minute}}{\text{piston displacement per minute}}. \tag{66}$$

The volumetric efficiency of a multistage compressor is defined in the same way except that the piston displacement is taken as that of the low pressure cylinder only; that is, for a *multistage* compressor,

$$\text{volumetric efficiency} = \frac{\text{volume of gas entering compressor per minute}}{\text{piston displacement of low pressure cylinder per minute}}. \tag{66a}$$

It is apparent that, if a compressor has a low volumetric efficiency, it will handle less gas than if it has a high volumetric efficiency. Or, stated another way, the size of the compressor needed to compress a gas at any given rate increases as the volumetric efficiency decreases.

If an ideal compressor had a clearance volume of zero, its volumetric efficiency would be 100%, as is evident from the indicator diagram shown in Fig. 103. The volume of gas drawn into the cylinder during the intake stroke would equal v_b, and the piston displacement would also equal v_b. If an ideal compressor did not have a clearance volume of zero, its volumetric efficiency would be less than 100%, as is evident from Fig. 104. Ideally, the volume of gas drawn into the cylinder during each intake stroke would be $v_b - v_a$, whereas the piston displacement would be $v_b - v_d$.

The volume of gas entering a compressor under ideal conditions divided by the piston displacement may be called the **ideal volumetric efficiency** of the compressor; that is,

$$\text{ideal volumetric efficiency} = \frac{v_b - v_a}{v_b - v_d} = \frac{v_b - v_a}{PD}, \tag{67}$$

where the subscripts refer to the points marked on Fig. 104. The ideal volumetric efficiency of any single-stage compressor can be calculated if the piston displacement of the compressor, the clearance volume, the intake and discharge pressures, and the nature of the expansion process d–a are known. It is necessary first to determine the volume v_a. If, for example, the expansion process is assumed to be adiabatic, v_a can be found by Eq. 28c:

$$v_a = v_d\left(\frac{p_2}{p_1}\right)^{1/k}; \tag{28c}$$

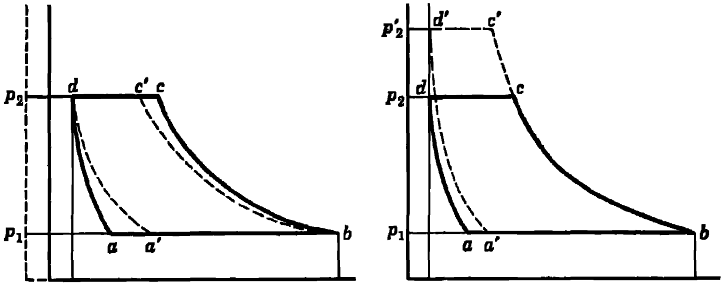
or, if the expansion process is assumed to be isothermal, v_a can be determined by the equation

$$v_a = v_d\left(\frac{p_2}{p_1}\right).$$

After v_a is determined, the ideal volumetric efficiency can be calculated by Eq. 67.

It is of interest to consider the factors that affect the ideal volumetric efficiency of a compressor because these factors also affect its actual volumetric efficiency. For any given intake pressure p_1 and discharge pressure p_2, both the ideal and actual volumetric efficiencies decrease as the ratio of the clearance volume to the piston displacement increases; and, for any given ratio of clearance volume to piston displacement, the volumetric efficiencies also decrease as the ratio p_2/p_1 increases. That these statements are true for the *ideal* volumetric efficiency is shown in Fig. 111. The dashed lines in Fig. 111*a* correspond to the larger ratio of clearance volume to piston displacement. Since $v_b - v_{a'}$ is smaller than $v_b - v_a$, the ideal volumetric efficiency decreases as the ratio of clearance volume to piston displacement increases. The dashed lines in Fig. 111*b* are for the larger ratio of p_2/p_1. Since $v_b - v_{a'}$ is again smaller than $v_b - v_a$, the ideal volumetric efficiency also decreases as the ratio of p_2/p_1 increases.

The *actual* volumetric efficiency of a compressor is also decreased by any leakage of gas past the piston or through the valves. If the gas is heated by the cylinder walls as it is drawn into the cylinder, or if there is an appreciable drop in pressure through the intake valve, the actual volumetric efficiency is decreased still further. This is true, as already explained in Art. 111, because raising the temperature or decreasing the pressure of the gas causes it to expand and thereby decreases the amount drawn into the compressor.



(*a*) Effect of increasing the clearance volume

(*b*) Effect of increasing the discharge pressure

FIG. 111.

PROBLEMS

373. A single-stage, double-acting reciprocating compressor takes in 250 cfm of air at 14.7 psia and discharges it at 75 psia. The diameters of the piston and piston rod are 12 and 1 in. respectively, the stroke is 12 in., and the compressor operates at 200 rpm. Determine the actual volumetric efficiency of the compressor.

Ans. 80.0%.

374. A two-stage, double-acting reciprocating compressor takes in 1000 cfm of air at 14.7 psia and discharges it at 125 psia. The diameter of the low pressure cylinder is 18 in., the diameter of the high pressure cylinder is 11 in., the diameter of each piston rod is 1½ in., the stroke of each piston is 10 in., and the compressor operates at 400 rpm. Determine the actual volumetric efficiency of the compressor.

375. A single-stage reciprocating compressor compresses air from 14.5 to 120 psia.

Determine what the ideal volumetric efficiency of the compressor is if the compressor operates adiabatically and its clearance volume is equal to 4% of its piston displacement. *Ans.* 86.0%

376. A two-stage reciprocating compressor compresses air from 14.5 to 120 psia. Determine what the ideal volumetric efficiency of the compressor is if (*a*) each stage operates adiabatically, (*b*) the intercooling is perfect, (*c*) the intercooler pressure is equal to the optimum value, and (*d*) the clearance volume of each stage is equal to 4% of the piston displacement of the stage. Compare with the preceding problem.

377. If the clearance volume of the compressor described in Problem 375 were reduced from 4% to 2% of the piston displacement, what would be the ideal volumetric efficiency of the compressor?

116. Other Reciprocating Compressor Efficiencies.

In this article certain other efficiencies used in comparing the performance of reciprocating compressors are defined. Each of these efficiencies involves what is known as the "indicated horsepower" of the compressor, and therefore it is necessary first to consider this term. It derives its name from the fact that it is determined by means of indicator diagrams.

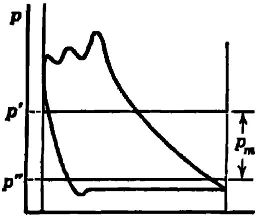


FIG. 112.

A typical indicator diagram for a single-stage compressor (or for one cylinder of a multistage compressor) is shown in Fig. 112. Let us denote by p' the *mean* (or average) pressure of the gas inside the cylinder while the piston is traveling inward. The work done *on* the gas during this process is equal to $p'AL$, where A denotes the cross-sectional area of the piston and L, the length of its stroke. Next, let us denote by p'' the mean pressure of the gas while the piston is traveling outward. The work done *by* the gas during this process is $p''AL$. The *net* work done on the gas when the crankshaft of the compressor makes one complete revolution is therefore $p'AL - p''AL$. Now, if the crankshaft is turning at the rate of n revolutions per minute, the horsepower delivered by the piston to the gas, called the **indicated horsepower** and denoted by the abbreviation ihp, is

$$\text{ihp} = \frac{(p' - p'')ALn}{33{,}000}.$$

The quantity $p' - p''$ is called the **mean effective pressure** and will be denoted by the symbol p_m. Introducing this symbol into the foregoing equation, we obtain, finally,

$$\text{ihp} = \frac{p_m LAn}{33{,}000}, \tag{68}$$

where p_m = the mean effective pressure, in pounds per square inch,
L = the length of stroke, in feet,
A = the cross-sectional area of the piston, in square inches, and
n = the crankshaft speed, in revolutions per minute.

The mean effective pressure p_m can be determined from the indicator diagram for the compressor by multiplying the average height of the diagram (its area divided by its length) times the "spring scale" for the diagram (the scale of its ordinate); that is,

$$p_m = \frac{\text{area of indicator diagram}}{\text{length of indicator diagram}} \times \text{spring scale,} \tag{69}$$

where the area is measured in square inches, the length in inches, and the spring scale in pounds per square inch per inch of height.

Now let us consider the definitions of certain reciprocating compressor efficiencies. The **compression efficiency** of a single-stage compressor, or of any one cylinder of a multistage compressor, is defined as the ratio of the ideal horsepower required for adiabatic compression (calculated by Eq. 57) to the indicated horsepower (determined from the actual indicator diagram for the cylinder); that is,

$$\text{compression efficiency} = \frac{(\text{ideal horsepower})_{\text{adiabatic}}}{\text{ihp}}. \tag{70}$$

The **equivalent compression efficiency** of a multistage compressor is defined as the ideal horsepower required to operate the compressor (adiabatic compression being assumed in each stage with perfect intercooling and optimum intercooler pressures)—calculated by Eq. 62 for a two-stage compressor or by Eq. 65 for an N-stage compressor—divided by the sum of the indicated horsepowers of the compressor cylinders; that is,

$$\text{equiv. compression efficiency} = \frac{(\text{ideal horsepower})_{\substack{N\text{-stage, adiabatic}\\ \text{perfect intercooling}\\ \text{opt. inter. press.}}}}{\Sigma\ \text{ihp}}. \tag{71}$$

If a reciprocating compressor is driven by a steam engine or by an internal-combustion engine, its **mechanical efficiency** is defined as the sum of the indicated horsepowers of the compressor cylinders divided by the sum of the indicated horsepowers of the steam or internal-combustion engine cylinders; that is, mechanical efficiency

$$= \frac{\Sigma \text{ ihp of compressor cylinders}}{\Sigma \text{ ihp of steam or internal-combustion engine cylinders}}. \qquad (72a)$$

If the compressor is driven by a motor, its mechanical efficiency is defined as the sum of the indicated horsepowers of the compressor cylinders divided by the horsepower delivered to the shaft of the compressor by the motor; that is,

$$\text{mechanical efficiency} = \frac{\Sigma \text{ ihp of compressor cylinders}}{\text{hp delivered to compressor shaft}}. \qquad (72b)$$

Finally, the **compressor efficiency** of a reciprocating compressor driven by a steam engine or an internal-combustion engine is defined as the ideal horsepower required to drive the compressor (calculated by Eq. 57, 62, or 65) divided by the sum of the indicated horsepowers of the steam or internal-combustion engine cylinders. If the compressor is driven by a motor, its compressor efficiency is defined as the ideal horsepower required to drive it divided by the horsepower supplied to its shaft by the motor. Note that the compressor efficiency is equal to the compression efficiency times the mechanical efficiency.

PROBLEMS

378. The following data were obtained from a test on a single-stage, double-acting air compressor driven by an electric motor:

intake pressure = atmospheric
intake temperature = 70°F
discharge pressure = 100 psi gage
free air compressed = 240 cfm
area of head-end indicator diagram = 1.18 sq in.
area of crank-end indicator diagram = 1.21 sq in.
length of indicator diagrams = 2.65 in.
spring scale = 80 psi per in.
diameter of piston = 10 in.
diameter of piston rod = 1 in.
stroke = 12 in.
speed = 300 rpm
power delivered by motor at shaft = 54.5 hp
barometric pressure = 29.5 in. Hg abs

Calculate (*a*) the compression efficiency of the compressor, (*b*) the mechanical efficiency, and (*c*) the compressor efficiency. *Ans.* (*a*) 83.5%; (*b*) 94.0%; (*c*) 78.5%.

379. A two-stage, double-acting air compressor is driven by a double-expansion steam engine. It is found in a test on the machine that, when the compressor is compressing 850 cfm of free air from atmospheric pressure to 120 psi gage, the sum of the indicated horsepowers of the compressor cylinders is 170 hp and the sum of the indicated horsepowers of the engine cylinders is 185 hp. Determine (*a*) the equivalent compression efficiency of the compressor, (*b*) the mechanical efficiency, and (*c*) the compressor efficiency. The barometric pressure during the test was 29.3 in. Hg abs.

117. The Power Ideally Required to Drive a Centrifugal Compressor, Blower, or Fan. If it is assumed (*a*) that a centrifugal compressor or blower operates in a steady-flow manner, (*b*) that the gas being compressed flows through the compressor adiabatically, (*c*) that the gas is a perfect gas having constant values of c_v and c_p, and (*d*) that the gas enters and leaves the compressor with negligible velocities, the power ideally required to drive the machine can be calculated by Eq. 57. That this equation applies to a centrifugal compressor or blower as well as to a reciprocating compressor is evident from the fact that the foregoing assumptions are those made in deriving the equation in Art. 112.

If a centrifugal compressor has two stages, and if the gas undergoes perfect intercooling between the stages, the power ideally required to drive the compressor may be calculated by Eq. 60. If the compressor has more than two stages, the power ideally required may be calculated by an expanded form of Eq. 60. If the compressor has three stages, for example, the power ideally required may be found by the equation

$$(\text{ideal horsepower})_{\substack{\text{3-stage, adiabatic}\\ \text{perfect intercooling}}} = \frac{k}{k-1} \times \frac{p_1\overline{V}_1}{33{,}000}\left[\left(\frac{p'_i}{p_1}\right)^{(k-1)/k} + \left(\frac{p''_i}{p'_i}\right)^{(k-1)/k} + \left(\frac{p_2}{p''_i}\right)^{(k-1)/k} - 3\right], \quad (60a)$$

where p'_i denotes the pressure in the first intercooler, and p''_i the pressure in the second. The form of the equation for more than three stages is apparent.

The power ideally required to drive a fan could be calculated by Eq. 57 if the velocity of the gas approaching and leaving the fan were small. Since these velocities are likely not to be negligible, however, an additional term must be added to allow for the increase in the kinetic energy of the gas. If such a term is added, Eq. 57 takes the form

$$(\text{ideal horsepower})_{\text{adiabatic}} = \frac{k}{k-1} \times \frac{p_1\overline{V}_1}{33{,}000}\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1\right] + \frac{w}{33{,}000}\left[\frac{V_2^2 - V_1^2}{2g}\right], \quad (57a)$$

where w denotes the pounds of gas flowing through the fan per minute.

Although Eq. 57*a* is correct for the conditions assumed, the first term usually cannot be easily evaluated. This is true because the pressure rise in a fan is likely to be very small, and therefore the ratio p_2/p_1 is likely to be almost unity. An alternative form of Eq. 57*a* in which the ratio p_2/p_1 does not appear can be derived. Before deriving this equation, let us consider the following:

Suppose that a system undergoes a non-flow process during which its state changes only an infinitesimal amount. For such a process the non-flow energy equation (Eq. 12) may be written in the form

$$dQ = du + \frac{dW}{J}. \tag{12a}$$

If the process is reversible, we may replace dW by $p\,dv$ and, in accordance with Eq. 41, we may replace dQ by $T\,ds$. Making these substitutions, we obtain

$$T\,ds = du + \frac{p\,dv}{J}. \tag{73}$$

But s, u, and v are all properties of the system; and for this reason the values of ds, du, and dv do not depend upon the nature of the process by which the system changes from its initial to its final state. Therefore, Eq. 73 gives the relation among the five properties p, v, T, u, and s of a system undergoing an infinitesimal change of state, regardless of how the change may take place. Now the enthalpy h of a system is defined as $u + pv/J$, and therefore

$$dh = du + \frac{p\,dv}{J} + \frac{v\,dp}{J}.$$

If we combine this equation with Eq. 73, we obtain, finally,

$$dh = T\,ds + \frac{v\,dp}{J}. \tag{74}$$

Now let us return to Eq. 13*b*, stated in Art. 112. If a fan operates in a steady-flow manner, and if gas flows through it adiabatically, we may write Eq. 13*b* in the form

$$\frac{W'_{\text{in}}}{J} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2gJ}$$

If the flow is also frictionless (that is, *ideal*), the entropy of the gas remains constant. Therefore, since integration of Eq. 73 for the condition that the entropy remains constant gives us

$$h_2 - h_1 = \frac{1}{J}\int v\,dp,$$

we may write

$$W'_{\text{in}} = \int v\,dp + \frac{V_2^2 - V_1^2}{2g}$$

But, if the gas undergoes only a slight increase in pressure, its specific volume v remains approximately constant; and therefore the preceding equation may be written

$$W'_{\text{in}} = v_1(p_2 - p_1) + \frac{V_2^2 - V_1^2}{2g}.$$

Hence, an approximate equation for the power ideally required to drive a fan is

$$(\text{ideal horsepower})_{\text{fan}} = \frac{w}{33{,}000}\left[v_1(p_2 - p_1) + \frac{V_2^2 - V_1^2}{2g}\right], \quad (75)$$

where w again denotes the pounds of gas flowing through the fan per minute.

Equation 75 can be derived also from Eq. 57*a* by performing a series expansion on the term $[(p_2/p_1)^{(k-1)/k} - 1]$ by means of the binomial theorem and then discarding all terms in the series except the first.

PROBLEMS

380. Two thousand cubic feet per minute of air at 14.0 psia and 70°F are compressed to 24.0 psia in a centrifugal compressor. The air enters the compressor with a velocity of 120 ft per sec and leaves with a velocity of 250 ft per sec. If the air flows through the compressor adiabatically, how many horsepower are ideally required to compress it?

381. A 5-stage centrifugal compressor compresses 30,000 cfm of air from 15 to 45 psia. If each stage operated adiabatically, if the intercooling were perfect, and if the ratio of the discharge pressure for each stage to the inlet pressure for the stage were the same for all the stages, how many horsepower would ideally be required to drive the compressor? *Ans.* 2230 hp.

382. Four thousand cubic feet of air at 14.7 psia and 60°F flow through a fan per minute. If the velocities of the air in the fan inlet and discharge ducts are 75 and 150 ft per sec respectively, and if the pressure in the discharge duct is 4.5 in. of water higher than in the inlet duct, how many horsepower are ideally required to operate the fan?

SYMBOLS

A	cross-sectional area of piston
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
g	acceleration that a force of 1 lb gives to a mass of 1 lb
h	enthalpy per unit mass
J	mechanical equivalent of heat
k	ratio of c_p to c_v
L	length of stroke
n	speed of rotation
N	number of stages
p	pressure
PD	piston displacement
Q	heat added to gas
R	gas constant
s	entropy per unit mass
T	absolute temperature
u	internal energy per unit mass
v	specific volume
V	cylinder volume
V	velocity
$\bar{V}$	volume rate of flow
w	mass rate of flow
W	work done by system; W' = shaft-work done by a system in steady flow; W'_{in} = shaft-work done on a system in steady flow
x	distance

Chapter 14

NOZZLES AND TURBINES

Nozzles are passages through which fluids can expand and acquire high velocities. They are an essential part of turbines, jet engines, and ejectors. In the present chapter certain peculiarities that characterize the flow of gases and vapors through nozzles are discussed, and equations are developed for calculating the velocities produced and the rates of discharge attained. Also, the more important types of *turbines* are described briefly, and the manner in which they utilize the high velocity jets produced by nozzles is explained.

118. Converging Nozzles. A longitudinal section through a **converging nozzle** is shown in Fig. 113. Transverse sections at various points along the length of the nozzle might be circular, rectangular, or any other shape. Regardless of their shape, however, the areas of these transverse sections decrease in the direction in which flow takes place. As marked on the figure, the section at which the transverse area begins to decrease is called the **entering section**, and the section where the fluid leaves the nozzle is called the **nozzle mouth.** Note that between these two sections the nozzle consists only of well-rounded surfaces and that its minimum transverse area is at the mouth. The region into which the nozzle discharges will be called the **discharge region.**

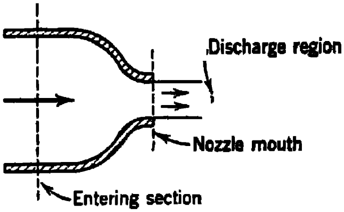


FIG. 113. Longitudinal section through a converging nozzle.

The flow of any gas or vapor through a converging nozzle is characterized by certain peculiarities. To illustrate, let us suppose that the fluid flowing through the nozzle is air, that its pressure p_1 and temperature t_1 at the entering section remain constant at 100 psia and 500°F, and that the area of the entering section is large enough so that the velocity V_1 is negligibly small. We shall consider how the pressure in the discharge region, denoted by p_d, affects (*a*) the pressure drop that takes place within the nozzle, (*b*) the velocity attained at the nozzle mouth, and (*c*) the rate at which the air discharges from the nozzle.

If the pressure p_d in the discharge region is the same as the pressure p_1 at the entering section, no flow takes place. If, however, the pressure p_d is lowered to, say, 80 psia, flow occurs. The pressure of the air decreases continuously as the air flows through the nozzle and drops to 80 psia (equal to the pressure in the discharge region) at the nozzle mouth. The velocity of the air increases and reaches its maximum value at the nozzle mouth. If the pressure in the discharge region is lowered from 80 to 60 psia, the air still undergoes a continuous drop in pressure within the nozzle and reaches a pressure of 60 psia at the nozzle mouth. Both its velocity at the nozzle mouth and its rate of discharge are higher than when the pressure in the discharge region is 80 psia.

Now let us suppose that the discharge region pressure is lowered to 40 psia. The velocity of the air at the nozzle mouth and the rate of discharge are higher than when the discharge region pressure is 60 psia. However, instead of the air undergoing a drop in pressure from 100 psia at the entering section to 40 psia at the nozzle mouth, it undergoes a drop in pressure only to 52.8 psia at the nozzle mouth. The remainder of the pressure drop (from 52.8 to 40 psia) takes place in the discharge region *after the air has left the nozzle.* Another peculiarity is that the velocity of the jet leaving the nozzle and the velocity of sound (in air at the nozzle mouth temperature) are now exactly equal.

If the pressure in the discharge region is changed to 20 psia (or to any other pressure less than 52.8 psia), the air still undergoes a drop in pressure from 100 psia at the entering section to 52.8 psia at the nozzle mouth, and the expansion from 52.8 psia to the discharge region pressure again takes place after the air has left the nozzle. Also, the velocity of the jet leaving the nozzle and the rate of discharge are still the same as when the discharge region pressure was 40 psia.

To summarize, if the discharge region pressure is any value between 100 and 52.8 psia, the nozzle mouth pressure is equal to the discharge region pressure, and both the nozzle mouth velocity and the rate of discharge increase as the discharge region pressure decreases. The velocity and the rate of discharge reach maximum values when the discharge region pressure is 52.8 psia. If the discharge region pressure is any value less than this, the nozzle mouth pressure remains constant at 52.8 psia, and the nozzle mouth velocity and the rate of discharge remain constant and equal to their maximum values.

The foregoing peculiarities are exhibited, not only by air, but also by any other gas or vapor flowing through a converging nozzle. The nozzle mouth pressure at which the rate of discharge reaches its maximum value (52.8 pisa in the preceding example) is called the **critical pressure.** It will be denoted by the symbol p_c. The ratio of this pressure to the pressure at the entering section is called the **critical pressure ratio** ($= p_c/p_1$). Methods for determining this ratio will be developed in Art. 120.

If the ratio of the discharge region pressure to the entering section pressure, p_d/p_1, is equal to or higher than the critical pressure ratio p_c/p_1, the entire expansion takes place while the gas or vapor is still inside the nozzle; and for this reason the gas issues from the nozzle mouth in a smooth jet, as shown in Figs. 114*a* and 114*b*. If the ratio p_d/p_1 is less than the critical pressure ratio, part of the expansion takes place in the discharge region; and the jet expands radially on leaving the nozzle mouth, as shown in Fig. 114*c*. On account of the inertia of the gas, this radial expansion continues past the point where the pressure within the jet is equal to the pressure in the discharge region; that is, a rarefaction occurs within the jet. Consequently, a contraction of the jet in a radial direction follows; and, again on account of the inertia of the gas, the jet contracts past the point where the pressure within it is equal to the discharge region pressure. These alternate rarefactions and compressions continue for some distance after the jet has left the nozzle but eventually are dissipated by frictional forces.

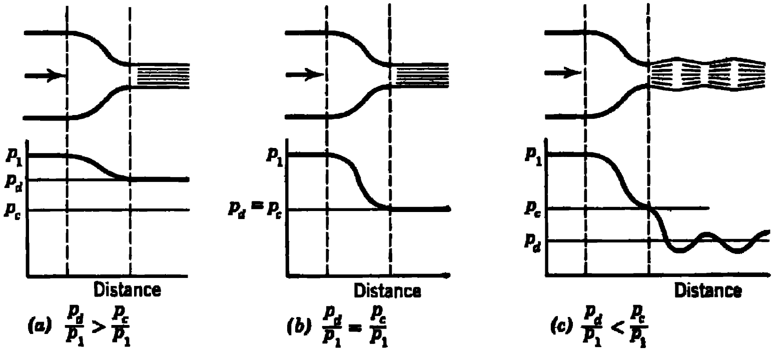

p_1
p_d
p_c
Distance
(a) $\frac{p_d}{p_1} > \frac{p_c}{p_1}$
p_1
$p_d = p_c$
Distance
(b) $\frac{p_d}{p_1} = \frac{p_c}{p_1}$
p_1
p_c
p_d
Distance
(c) $\frac{p_d}{p_1} < \frac{p_c}{p_1}$

FIG. 114. The effect of p_d/p_1 on the pressure drop within a converging nozzle and on the shape of the jet in the discharge region.

119. Isentropic Flow through Converging Nozzles. As explained in Art. 49, the flow of any gas or vapor through a nozzle is always nearly adiabatic because the time required for each element of the fluid to flow through the nozzle is too short to permit much heat transfer to take place. If the flow were also frictionless, the expansion would take place isentropically. Let us consider how we can calculate the velocity in the mouth of a converging nozzle and the rate of discharge from the nozzle if we make the idealizing assumption that the flow through the nozzle is isentropic.

We note first that the shaft-work W' supplied to any nozzle is zero. If the heat Q supplied is also zero, the steady-flow energy equation (Eq. 13*a*) takes the form

$$h_1 + \frac{V_1^2}{2gJ} = h + \frac{V^2}{2gJ},$$

where h_1 and V_1 are the enthalpy and velocity of the fluid at the entering section, and h and V are its enthalpy and velocity at the nozzle mouth. Solving for the mouth velocity V, we get

$$V = \sqrt{2gJ(h_1 - h) + V_1^2}. \tag{76}$$

Note that this equation requires that the flow be adiabatic but does not require that it be frictionless (that is, it need not be *isentropic*).

Now let us suppose that we know the pressure p_1, the temperature t_1, and the velocity V_1 at the entering section and that we know the pressure p at the mouth of the nozzle. If we were not assuming that the flow is isentropic, these data alone would not allow us to find the value of the enthalpy change $h_1 - h$ which appears in Eq. 76; and therefore we could not calculate the mouth velocity V. Since we are making this assumption, however, we are able to determine $h_1 - h$ as follows: If the fluid is a perfect gas having constant values of c_v and c_p, we can determine its temperature T at the nozzle mouth by Eq. 28*b*,

$$T = T_1\left(\frac{p}{p_1}\right)^{(k-1)/k} \tag{28b}$$

and can then determine the value of $h_1 - h$ by Eq. 18*a*,

$$h_1 - h = c_p(T_1 - T). \tag{18a}$$

If the fluid is not a perfect gas, but if a table of its thermodynamic properties is available, we can find its values of h_1 and s_1 by means of the tables. Then, since we are assuming that the entropy s at the nozzle mouth is equal to s_1, and since we know the mouth pressure p, we can find the enthalpy h at the nozzle mouth. We can then calculate the desired value of $h_1 - h$. Let us illustrate the foregoing by numerical examples.

Suppose that the fluid flowing through a converging nozzle is air, that its pressure and temperature at the entering section are 100 psia and 500°F, that its velocity at the entering section is negligibly small, and that its pressure in the nozzle mouth is 60 psia. If the air flows through the nozzle isentropically, its temperature T in the nozzle mouth is, by Eq. 28*b*,

$$T = 960(\tfrac{60}{100})^{0.4/1.4}$$

$$= 830\text{°F abs.}$$

The change $h_1 - h$ in its enthalpy between the entering section and the nozzle mouth is, by Eq. 18*a*,

$$h_1 - h = 0.241(960 - 830)$$

$$= 31.3 \text{ Btu per lb.}$$

Hence, the nozzle mouth velocity V is, by Eq. 76,

$$V = \sqrt{2 \times 32.2 \times 778 \times 31.3}$$

$$= 1250 \text{ ft per sec.}$$

Now let us suppose that the fluid is steam, that its pressure and temperature at the entering section are 100 psia and 500°F, that its velocity at the entering section is negligible, and that its pressure in the nozzle mouth is 60 psia. From Table 3 of the *Steam Tables*, $h_1 =$ 1279.1 Btu per lb and $s_1 = 1.7085$ Btu per lb deg F. The enthalpy h in the nozzle mouth corresponding to $p = 60$ psia and $s = 1.7085$ Btu per lb deg F is found, by interpolation in Table 3, to be 1228.7 Btu per lb. Hence, for isentropic flow the nozzle mouth velocity is, by Eq. 76,

$$V = \sqrt{2 \times 32.2 \times 778(1279.1 - 1228.7)}$$

$$= 1588 \text{ ft per sec.}$$

Let us consider next how the rate of discharge from a converging nozzle can be calculated if isentropic flow is again assumed. This can be done by means of the continuity equation (Eq. 15) if, in addition to the data required to calculate the nozzle mouth velocity, the cross-sectional area of the nozzle mouth is given. Thus, since the continuity equation states that $w = Va/v$,

$$w = \frac{a\sqrt{2gJ(h_1 - h) + V_1^2}}{v}. \tag{77}$$

It is necessary to determine the specific volume of the fluid in the nozzle mouth. If the fluid is a perfect gas, this can be done by the characteristic equation $pv = RT$. If it is not a perfect gas, its specific volume can be found from the table of thermodynamic properties just as its enthalpy is found. Let us illustrate by continuing the foregoing numerical examples.

Suppose that the cross-sectional area a at the mouth of the nozzle in the preceding example is 1 sq in. The air after expanding isentropically to a pressure of 60 psia was found to be at a temperature of 830°F abs. Hence, its specific volume at the nozzle mouth is

$$v = \frac{RT}{p} \quad \frac{53.3 \times 830}{60 \times 144} \quad 5.11 \text{ cu ft per lb,}$$

and its rate of discharge is, by the continuity equation,

$$w = \frac{Va}{v} = \frac{1250 \times 1}{5.11 \times 144} = 1.70 \text{ lb per sec.}$$

The steam leaving the nozzle at 60 psia is assumed to have an entropy of 1.7085 Btu per lb deg F. By interpolation in Table 3 of the *Steam Tables*, its specific volume is found to be 8.252 cu ft per lb. Hence, its rate of discharge is

$$w = \frac{1588 \times 1}{8.252 \times 144} = 1.335 \text{ lb per sec.}$$

Note that the continuity equation (or Eq. 77) can be used, not only to find the rate of discharge when the nozzle mouth area is known, but also to find the mouth area required when a desired rate of discharge is given. If the rate of discharge from the foregoing steam nozzle is to be 1 lb per sec, for example, the area of the nozzle mouth must be

$$a = \frac{wv}{V} = \frac{1 \times 8.252}{1588} = 0.0052 \text{ sq ft } (= 0.75 \text{ sq in.}).$$

Finally, let us consider how the critical pressure ratio discussed in the preceding article enters into the calculation of the nozzle mouth velocity and rate of discharge from a converging nozzle. Let us again suppose that the pressure and temperature at the entering section are 100 psia and 500°F, that the velocity in the entering section is negligibly small, that the cross-sectional area of the nozzle mouth is 1 sq in., and that the flow is isentropic. Then, *if we assume that the pressure in the nozzle mouth is always equal to the pressure in the discharge region*, calculations like the foregoing ones will yield the results shown in Fig. 115. For air, the rate of discharge is found to reach a maximum when the discharge region pressure is 52.8 psia; and, for steam, it is found to reach a maximum when the discharge region pressure is about 57.0 psia. At pressures lower than these, the rates of discharge are found to decrease and to approach zero as the pressure in the discharge region approaches zero. Thus, if a perfect vacuum existed in the discharge region, the obviously absurd result that no flow through the nozzle would take place is obtained. The fallacy, of course, lies in the assumption that the nozzle mouth pressure is always equal to the discharge region pressure. As explained in the preceding article, experiments show that the nozzle mouth pressure and the discharge region pressure are equal only down to the mouth pressure at which the rate of discharge reaches its maximum value. At discharge region pressures lower than this, the nozzle mouth pressure remains constant and equal to the value at which the discharge reaches a maximum. Consequently, the mouth velocity and rate of discharge also remain constant at their maximum values.

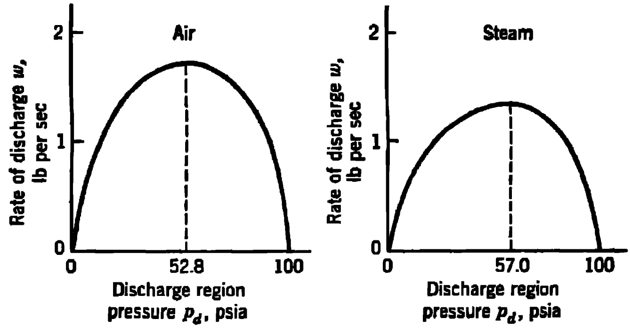


FIG. 115.

PROBLEMS

383. Show that, if (*a*) the fluid flowing through a nozzle is a perfect gas, (*b*) the velocity of the gas at the entering section is negligible, and (*c*) the gas expands isentropically, the velocity of the gas at a section where its pressure is p may be determined by the equation

$$V = \sqrt{\frac{2gk}{k-1} \times RT_1 \left[1 - \left(\frac{p}{p_1}\right)^{(k-1)/k}\right]}, \tag{76a}$$

where p_1 and T_1 denote the pressure and temperature of the gas at the entering section. (Note again that, if the nozzle is a converging nozzle, at no section can the pressure p drop below the critical pressure p_c.)

384. Show that, under the conditions stated in the preceding problem, the rate of flow through a nozzle may be determined by the equation

$$w = ap_1\sqrt{\frac{2gk}{(k-1)RT_1}\left[\left(\frac{p}{p_1}\right)^{2/k} - \left(\frac{p}{p_1}\right)^{(k+1)/k}\right]}, \tag{77a}$$

where a denotes the cross-sectional area of the nozzle at the section where the pressure of the gas has dropped to the value p.

385. Air at 60 psia and 400°F enters a converging nozzle. It expands isentropically within the nozzle and leaves at 40 psia (a pressure greater than the critical). The cross-sectional area of the nozzle mouth is 0.500 sq in. Determine the rate at which the air flows through the nozzle if its velocity at the entering section is (*a*) negligible and (*b*) 300 ft per sec. *Ans.* (*a*) 0.521 lb per sec.

386. Suppose that the fluid in the preceding problem were steam instead of air. At what rate would the steam flow through the nozzle if its velocity at the entering section were (*a*) negligible and (*b*) 300 ft per sec? *Ans.* (*a*) 0.410 lb per sec.

120. The Critical Pressure Ratio. As stated in Art. 118, the **critical pressure ratio** for a converging nozzle is defined as the nozzle mouth pressure at which the rate of discharge reaches its maximum value divided by the pressure in the entering section. It can be determined for isentropic flow by plotting curves such as those shown in Fig. 115 and finding the ratio of p/p_1 at which the rate of discharge w is a maximum. If the velocity V_1 in the entering section is negligibly small, the critical pressure ratio for isentropic flow can also be found as follows:

When V_1 is negligible, Eq. 77 in the preceding article reduces to

$$w = \frac{a\sqrt{2gJ(h_1 - h)}}{v} = a\sqrt{2gJ}\sqrt{\frac{h_1 - h}{v^2}}.$$

Hence, to determine the critical pressure ratio, we need only to find the pressure ratio at which the quantity $(h_1 - h)/v^2$ is a maximum.

Let us suppose first that the fluid is a *perfect gas* and that its values of c_v and c_p are constants. By Eq. 18*a*,

$$h_1 - h = c_p(T_1 - T), \tag{18a}$$

and, by Eqs. 28*b* and 28*c*,

$$T = T_1\left(\frac{p}{p_1}\right)^{(k-1)/k} \tag{28b}$$

and

$$v = v_1\left(\frac{p_1}{p}\right)^{1/k}. \tag{28c}$$

By making use of these three equations, we can show that

$$\frac{h_1 - h}{v^2} = \frac{c_pT_1}{v_1^2}\left[\left(\frac{p}{p_1}\right)^{2/k} - \left(\frac{p}{p_1}\right)^{(k+1)/k}\right].$$

Hence, to find the value of p/p_1 at which this expression is a maximum, we need only to differentiate the term in brackets with respect to p

(or with respect to p/p_1), set this derivative equal to zero, and solve for p/p_1. The result is

$$\frac{p_c}{p_1} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \tag{78}$$

Note that this ratio depends, not upon the condition of the gas at the entering section, but only upon the value of k for the gas. The value $p_c/p_1 = 0.528$ for air is obtained by substituting 1.40 for k into Eq. 78.

Let us suppose next that the fluid is something other than a perfect gas and that a table of its thermodynamic properties is available. We can find graphically the value of p/p_1 at which $(h_1 - h)/v^2$ reaches a maximum. For any given values of p_1 and t_1 at the entering section, we must first find the values of h_1 and s_1. We can then determine the values of h and v at the nozzle mouth corresponding to several values of p and $s = s_1$. Finally, by calculating $(h_1 - h)/v^2$ and plotting this quantity against p/p_1, we can find the critical pressure ratio. This procedure is, of course, equivalent to plotting the rate of discharge w against the pressure ratio p/p_1 but is somewhat easier to carry out. Note that in general the critical pressure ratio for compressible fluids other than perfect gases depends upon the condition of the fluid at the entering section. For steam, for example, p_c/p_1 is found to have a value of about 0.58 if the steam is initially saturated, whereas, if the steam is initially highly superheated, p_c/p_1 is found to be closer to 0.55.

PROBLEMS

387. Derive Eq. 78.

388. Show that, if (*a*) the fluid flowing through a converging nozzle is a perfect gas, (*b*) the velocity of the gas at the entering section is negligible, and (*c*) the gas expands isentropically, the critical temperature ratio T_c/T_1 (the nozzle mouth temperature at the maximum rate of discharge divided by the entering section temperature) is given by the expression

$$\frac{T_c}{T_1} = \frac{2}{k+1}. \tag{78a}$$

389. Show that, if (*a*) the fluid flowing through a converging nozzle is a perfect gas, (*b*) the velocity of the gas at the entering section is negligible, (*c*) the gas expands isentropically, and (*d*) the discharge region pressure p_d is equal to or less than the critical pressure p_c, the velocity of the gas in the nozzle mouth is given by the expressions

$$V = \sqrt{2gRT_1\left(\frac{k}{k+1}\right)} = \sqrt{kgRT_c},$$

where T_1 is the temperature of the gas at the entering section and T_c, the temperature at the nozzle mouth.

390. Show that, under the conditions stated in the preceding problem, the rate of flow through the nozzle is given by the expression

$$w = ap_1 \sqrt{\frac{2gk}{(k+1)RT_1}\left(\frac{2}{k+1}\right)^{2/(k-1)}}.$$

391. Prove that, if (*a*) the fluid flowing through a converging nozzle is a perfect gas, (*b*) the velocity of the gas at the entering section is V_1, and (*c*) the gas expands isentropically, the critical pressure ratio p_c/p_1 is given by the expression

$$\frac{p_c}{p_1} = \left[\frac{2}{k+1} + \frac{k-1}{k+1}\left(\frac{V_1}{V_{s,1}}\right)^2\right]^{k/(k-1)}, \qquad (78b)$$

where $V_{s,1}$ denotes the velocity of sound in the gas at its entering section temperature. (*Note:* It can be shown that the velocity of sound in a perfect gas at temperature T is $V_s = \sqrt{kgRT}$.)

392. Air enters a converging nozzle at 50 psia and 70°F and flows through the nozzle isentropically. Determine what the critical pressure p_c is if the velocity of the air at the entering section is (*a*) negligible and (*b*) 300 ft per sec.

393. A perfect gas flows isentropically through a converging nozzle. At the entering section the pressure of the gas is 100 psia, its temperature is 80°F, and its velocity is negligible. The cross-sectional area of the mouth of the nozzle is 1.00 sq in., and the pressure in the discharge region is 50 psia. Determine the rate at which the gas flows through the nozzle if the gas is (*a*) carbon dioxide and (*b*) helium. Assume that c_p and c_v for these gases are constant and equal to the values given in Table IV on page 95. *Ans.* (*a*) 2.74 lb per sec; (*b*) 0.903 lb per sec.

394. A converging nozzle is supplied with saturated steam at 100 psia. The steam enters at a low velocity and expands isentropically within the nozzle. Determine what the pressure in the mouth of the nozzle is if the pressure in the discharge region is 20 psia.

121. Converging-Diverging Nozzles. As explained in Art. 118, if a gas or vapor flows through a *converging* nozzle, and if the pressure p_d in the discharge region is equal to or greater than the critical pressure p_c, the entire expansion from the initial pressure p_1 to the discharge pressure p_d takes place within the nozzle. The fluid reaches the pressure p_d in the nozzle mouth and issues from the nozzle in a smooth jet. If the discharge pressure p_d is less than the critical pressure p_c, the entire expansion does not take place within the nozzle. Instead, the fluid expands only to the critical pressure p_c at the nozzle mouth, and the expansion from p_c to p_d takes place in the discharge region. Since this last part of the expansion is unrestrained, it is always accompanied by friction and so cannot be reversible. Therefore, isentropic expansion of a gas or vapor to pressures less than the critical is impossible in a converging nozzle.

Isentropic expansion of a gas or vapor to pressures less than the critical is possible if a diverging section is added to a converging nozzle, as shown in Fig. 116. Such a nozzle is called a **converging-diverging**

nozzle. As for a converging nozzle, the section where the cross-sectional area begins to decrease is called the **entering section,** and the section where the fluid leaves the nozzle is called the **nozzle mouth.** The section at which the cross-sectional area is a minimum is called the **throat.**

In the converging part of a converging-diverging nozzle, the fluid behaves exactly as it would if the diverging part were not present. The fluid expands to the critical pressure p_c at the throat (which would be the mouth of the nozzle if the diverging part were not present), and its velocity in the throat is exactly the velocity of sound. In the diverging part of the nozzle the fluid expands to pressures below the critical, and its velocity increases above the velocity of sound. If the nozzle mouth is the proper size, the fluid reaches the discharge pressure p_d at the mouth and issues from the nozzle in a smooth jet.

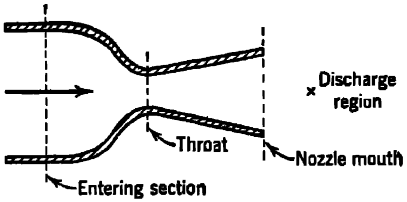


FIG. 116. Longitudinal section through a converging-diverging nozzle.

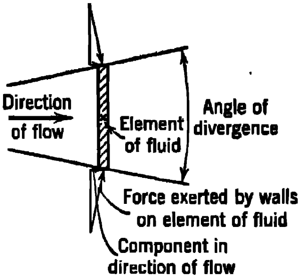

Direction of flow
Element of fluid
Angle of divergence
Force exerted by walls on element of fluid
Component in direction of flow

FIG. 117.

It is of interest to consider how the expansion in the diverging part of the nozzle takes place. Because the fluid expands in a radial direction, it exerts a force on the nozzle walls. The nozzle walls, in turn, exert a resisting force on the fluid. As shown in Fig. 117, the force exerted by the nozzle walls on an element of the fluid bounded by parallel planes has a component in the direction of flow. This component causes the element to accelerate; so the velocity of the fluid continues to increase as the fluid flows through the diverging part of the nozzle.

The diverging part of the nozzle shown in longitudinal section in Fig. 116 appears as two straight lines. The diverging part need not have this shape, but, if it does not, it must have no abrupt changes in curvature. The angle between the diverging walls of the nozzle is called the **angle of divergence.** If this angle is too large, the fluid will not be able to expand fast enough in a radial direction to stay in contact with the nozzle walls, and turbulence and friction will result. If the angle of divergence is too small, the amount of friction is increased because of the increased wall surface over which the fluid must flow. An angle of divergence between 6° and 12° is commonly used.

122. Isentropic Flow through Converging-Diverging Nozzles. If the pressure, temperature, and velocity of a gas or vapor at the entering section of a converging-diverging nozzle are known, if the pressure at the mouth of the nozzle is known, and if the flow through the nozzle is assumed to be isentropic, the velocity at the nozzle mouth can be easily calculated. The procedure is exactly the same as that used to calculate the velocity at the mouth of a converging nozzle.

Suppose, for example, that the pressure p_1 and the temperature t_1 at the entering section of a converging-diverging nozzle are 100 psia and 500°F, that the velocity V_1 at the entering section is negligible, and that the pressure p in the nozzle mouth is 20 psia (a pressure that could not be attained if the nozzle were a converging nozzle). If the fluid is air, its temperature T after an isentropic expansion to 20 psia will be, by Eq. 28*b*,

$$T = T_1\left(\frac{p}{p_1}\right)^{(k-1)/k} \qquad (28b)$$

$$= 960\left(\tfrac{20}{100}\right)^{0.4/1.4}$$

$$= 606°\text{F abs};$$

and the change $h_1 - h$ in its enthalpy between the entering section and the mouth will be, by Eq. 18*a*,

$$h_1 - h = c_p(T_1 - T) \qquad (18a)$$

$$= 0.241(960 - 606)$$

$$= 85.4 \text{ Btu per lb.}$$

Therefore, its velocity at the nozzle mouth will be, by Eq. 76,

$$V = \sqrt{2 \times 32.2 \times 778 \times 85.4}$$

$$= 2065 \text{ ft per sec.}$$

If the fluid is steam, its enthalpy and entropy at the entering section will be 1279.1 Btu per lb and 1.7085 Btu per lb deg F respectively (from Table 3 of the *Steam Tables*). After the steam expands isentropically to 20 psia, it will be a mixture of saturated water and saturated steam (assuming that "supersaturation," discussed in the next article, does does not occur). Its quality can be calculated by Eq. 44*b*:

$$s = s_g - (1 - x)s_{fg}, \tag{44b}$$

or

$$1.7085 = 1.7319 - (1 - x)1.3962,$$

whence

$$(1 - x) = 0.168.$$

Its enthalpy can now be calculated by Eq. 20b:

$$\begin{aligned} h &= h_g - (1 - x)h_{fg} \qquad (20b) \\ &= 1156.3 - 0.168 \times 960.1 \\ &= 995 \text{ Btu per lb.} \end{aligned}$$

Therefore, the velocity of the steam at the nozzle mouth will be, by Eq. 76,

$$\begin{aligned} V &= \sqrt{2 \times 32.2 \times 778(1279 - 995)} \\ &= 3770 \text{ ft per sec.} \end{aligned}$$

If the area of the nozzle mouth is also known, the ideal rate of discharge (that is, the rate attained with isentropic flow) can be calculated by making use of the continuity equation (or Eq. 77). Suppose, for instance, that the area of the nozzle mouth in the foregoing example is 1 sq in. The specific volume of the air at the nozzle mouth will be

$$v = \frac{RT}{p} = \frac{53.3 \times 606}{20 \times 144} = 11.2 \text{ cu ft per lb,}$$

and its ideal rate of discharge will be

$$w = \frac{Va}{v} = \frac{2065 \times 1}{11.2 \times 144} = 1.28 \text{ lb per sec.}$$

The specific volume of the steam at the nozzle mouth will be, by Eq. 5a,

$$v = xv_g = 0.832 \times 20.089 = 16.7 \text{ cu ft per lb;}$$

and its ideal rate of discharge will be

$$w = \frac{Va}{v} = \frac{3770 \times 1}{16.7 \times 144} = 1.57 \text{ lb per sec.}$$

Let us also consider briefly the inverse problem of finding what the cross-sectional area of the throat and of the nozzle mouth must be if we assume isentropic flow and know (*a*) the pressure, temperature, and velocity at the entering section, (*b*) the pressure at the nozzle mouth (which must be lower than the critical pressure if the nozzle is to be a converging-diverging nozzle), and (*c*) the rate of discharge. To determine the throat area, we must first determine the throat pressure. It will be equal to the critical pressure and can be determined by the methods developed in Art. 120. Then, knowing the throat pressure, we can calculate the velocity and specific volume at the throat by the usual methods. Finally, we can determine the throat area by the continuity equation. We can determine the mouth area by similar calculations since we know the mouth pressure.

PROBLEMS

395. Air at 60 psia and 400°F enters a converging-diverging nozzle. It expands isentropically within the nozzle and leaves at 15 psia. The cross-sectional area of the nozzle mouth is 0.500 sq in. Determine the rate at which the air flows through the nozzle if its velocity at the entering section is (*a*) negligible and (*b*) 300 ft per sec.

396. Suppose that the fluid in the preceding problem were steam instead of air. At what rate would the steam flow through the nozzle if its velocity at the entering section were (*a*) negligible and (*b*) 300 ft per sec?

Ans. (*a*) 0.320 lb per sec; (*b*) 0.322 lb per sec.

397. Air is to enter a converging-diverging nozzle at 200 psia and 500°F and is to leave at 40 psia. The cross-sectional area of the entering section will be large so that the velocity of the air at this section will be low. If the air were to expand isentropically within the nozzle, what would the cross-sectional areas of the throat and mouth have to be in order for the air to flow through the nozzle at the rate of 1 lb per sec?

398. If the fluid in the preceding problem were steam instead of air, what would the cross-sectional areas of the throat and mouth have to be in order for the steam to flow through the nozzle at the rate of 1 lb per sec? Take the value of p_c/p_1 to be 0.56.

399. Air at 100 psia and 440°F enters a converging-diverging nozzle. The cross-sectional area of the entering section of the nozzle is large. The cross-sectional areas of the throat and mouth are 0.700 and 1.000 sq in. respectively. What would the pressure at the mouth be if the air expanded isentropically within the nozzle?

123. Actual Flow through Nozzles. Although the actual flow of any gas or vapor through a nozzle is virtually adiabatic, it is not isentropic because some friction inevitably occurs. How nearly the actual flow approaches isentropic flow is indicated by the **nozzle efficiency.** This term is defined as the ratio of the actual kinetic energy possessed by each pound of fluid leaving the nozzle to the kinetic energy it would have if the expansion to the discharge pressure p_d took place isentropically; that is,

$$\eta_n = \frac{\bar{V}^2/2g}{V^2/2g} = \frac{\bar{V}^2}{V^2}, \tag{79}$$

where η_n denotes the nozzle efficiency, $\bar{V}$ the actual velocity leaving the nozzle, and V the ideal velocity. For a converging nozzle, the nozzle efficiency is usually between 95% and 100%, whereas, for a converging-diverging nozzle, it is usually between 90% and 100%.

Let us see how to determine what the actual cross-sectional area $\bar{a}$ must be at a section where the pressure has dropped to the value p if we know (*a*) the pressure, temperature, and velocity at the entering section, (*b*) the rate of discharge, and (*c*) the nozzle efficiency. We can determine the ideal velocity V at the section by the methods explained in Art. 119 or 122. Next, we can calculate the actual velocity $\bar{V}$ by Eq. 79. Then, to determine the actual specific volume of the fluid at this section, we may proceed as follows: Since the actual flow through any nozzle is virtually adiabatic, we may write the steady-flow energy equation (Eq. 13*a*) in the form

$$h_1 + \frac{V_1^2}{2gJ} = \bar{h} + \frac{\bar{V}^2}{2gJ},$$

where $\bar{h}$ denotes the actual enthalpy of the fluid at the section where the pressure has dropped to the value p. We can find the value of $\bar{h}$ (or of $h_1 - \bar{h}$) by this equation. Now, if the fluid is a perfect gas having a constant specific heat c_p, we can calculate its actual temperature $\bar{T}$ by Eq. 18*a*,

$$h_1 - \bar{h} = c_p(T_1 - \bar{T}), \tag{18a}$$

and can then calculate its actual specific volume $\bar{v}$ by the characteristic equation $p\bar{v} = R\bar{T}$. Finally, we can find the desired value of $\bar{a}$ by the continuity equation

$$w = \frac{\bar{V}\bar{a}}{\bar{v}}. \tag{15}$$

If the fluid is not a perfect gas, but, if a table of its thermodynamic properties is available, we can determine the value of $\bar{v}$ from the tables since we know the values of p and $\bar{h}$. We can then calculate $\bar{a}$ by the continuity equation.

To determine whether a nozzle must be converging or converging-diverging, we must find the actual critical pressure $\bar{p}_c$. If the initial velocity V_1 is negligible, we can determine $\bar{p}_c$ by calculating $(h_1 - \bar{h})/\bar{v}^2$ for several values of p and plotting this quantity against p. The value of p at which the quantity reaches a maximum is the actual critical pressure $\bar{p}_c$. The actual critical pressure usually differs very little from the ideal, however, and therefore the ideal value may ordinarily be used.

If the pressure p_d in the discharge region is equal to or higher than the critical pressure p_c, the nozzle must be converging; and its mouth pressure and the discharge region pressure will be equal. Therefore, to find the mouth area required, we need only to calculate the value of $\bar{a}$ corresponding to $p = p_d$. If the discharge region pressure p_d is less than the critical pressure p_c, the nozzle must be converging-diverging. Since its throat pressure will be p_c and its mouth pressure p_d, to find the throat and mouth areas required we need only to calculate the values of $\bar{a}$ corresponding to these two pressures.

It has been assumed in the foregoing articles that a vapor initially saturated or slightly superheated will remain in a stable state as it flows through a nozzle. For example, it has been assumed that steam initially in the state represented by point 1 in Fig. 118 changes into a mixture of saturated water and saturated steam if it expands isentropically to the pressure p. Actually, because a finite interval of time is required for condensation to take place, and because the time required for the steam to expand to the pressure p may be less than this, no condensation may occur. Instead, the steam may reach what is known as a state of **supersaturation,** a state in which the steam exists as a vapor at a pressure higher than the saturation pressure corresponding to its temperature. Such a state is unstable, and a sudden condensation in the form of a large number of extremely small droplets follows. Approximate values of the velocity and rate of flow attained by a vapor that becomes supersaturated may be calculated by assuming that the vapor behaves as a perfect gas.

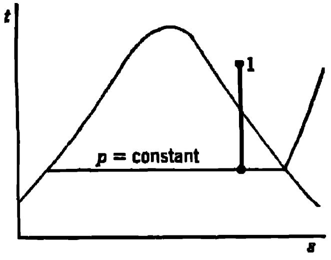


FIG. 118.

PROBLEMS

400. Air at 60 psia and 400°F enters a converging-diverging nozzle. Its velocity at the entering section is 300 ft per sec, and its pressure at the nozzle mouth is 15 psia. If the cross-sectional area of the nozzle mouth is 0.500 sq in., and if the nozzle operates with an efficiency of 94%, what is the actual rate at which the air flows through the nozzle? Compare your answer with the answer to part *b* of Problem 395.

401. If the fluid in the preceding problem were steam instead of air, what would be the actual rate at which the steam flowed through the nozzle? Assume that supersaturation does not occur. Compare your answer with the answer to part *b* of Problem 396.

402. Air is to enter a converging-diverging nozzle at 200 psia and 500°F and is to leave at 40 psia. The cross-sectional area of the entering section will be large, so that the velocity of the air at this section will be low. The nozzle is expected to have an efficiency of 95%. What must the cross-sectional areas of the throat and mouth be in order for the air to flow through the nozzle at the rate of 1 lb per sec? Assume that the actual critical pressure ratio is the same as the ideal. Compare your answers with those for Problem 397. *Ans.* 0.301 sq in.; 0.412 sq in.

403. If the fluid in the preceding problem were steam instead of air, what would the cross-sectional areas of the throat and mouth have to be in order for the steam to flow through the nozzle at the rate of 1 lb per sec? Assume that the actual critical pressure ratio is 0.56 and that supersaturation does not occur. Compare your answers with those for Problem 398. *Ans.* 0.384 sq in.; 0.573 sq in.

124. Single-Stage Impulse Turbines.

Since one of the most important uses made of nozzles is in turbines, it is logical to consider at this point the flow of fluids through turbines. Turbines may be classified as *impulse* turbines, as *reaction* turbines, or, if part of the turbine operates on the impulse principle and part on the reaction principle, as *combination* turbines. The principle upon which a single-stage impulse turbine operates is explained in this article.

A single-stage impulse turbine consists of one or more stationary nozzles and a single rotor. A row of *blades*, or *buckets*, is mounted around the periphery of the rotor, as shown in Fig. 119. The gas or vapor flowing through the turbine enters the nozzles at a relatively high pressure and low velocity. It expands in the nozzles to a lower pressure and acquires a high velocity. This high velocity stream enters the passages between the blades, flows through them at constant pressure, and undergoes a reversal in its direction of flow.

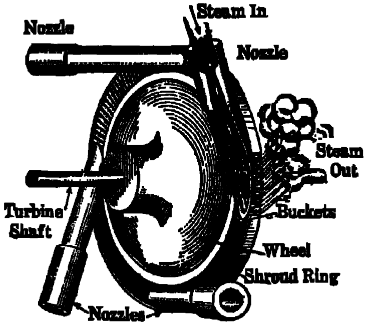


Fig. 119. The elements of a single-stage impulse turbine. (Reproduced by permission from *Heat Power Engineering*, Part I, by W. N. Barnard, F. O. Ellenwood, and C. F. Hirshfeld, published by John Wiley & Sons, 1926.)

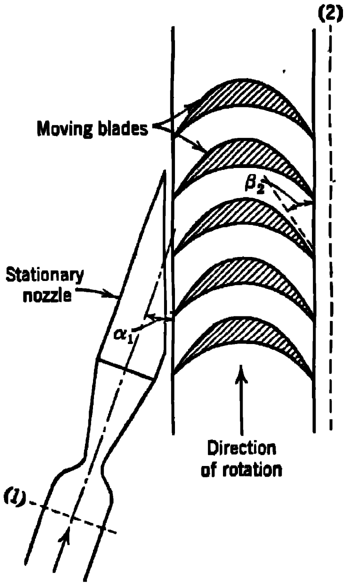


Fig. 120. Sectional view through one of the nozzles and several adjacent blades of an impulse turbine.

To understand how the turbine is able to deliver work at its shaft, let us consider a sectional view through one of its nozzles and several of the blades adjacent to the nozzle at any instant, as shown in Fig. 120. It is evident that the angle α_1 marked on this figure can be made small but can never be made zero in an actual turbine because the stream leaving the nozzle would not enter the passages between the blades. For the sake of simplicity, however, let us suppose that this angle can be made zero and that the angle β_2 marked on the figure can also be made zero. If the blades of such an *ideal zero-angle turbine* were moving at exactly half the velocity of the stream leaving the nozzle, and if no friction occurred within the blade passages, the velocity of the fluid leaving the blades would be zero. Suppose, for example, that the fluid left the nozzle with a velocity of 2000 ft per sec and that the blades were moving with a velocity of 1000 ft per sec. The velocity of the fluid *relative to the blades* would be (2000 − 1000 =) 1000 ft per sec. Since the fluid is assumed to enter and flow through the blade passages without friction, it would leave them with a velocity, relative to the blades, of 1000 ft per sec. But, since it would be flowing in a direction opposite to that in which the blades were moving, its velocity, *relative to the stationary parts of the turbine*, would be zero. Thus, at the mouth of the nozzle the fluid would have a high kinetic energy, whereas, at the section where it left the blades, its kinetic energy would be zero. The kinetic energy lost by the fluid between these two sections would leave the turbine in the form of work at the rotor shaft. Note that, if no friction occurred, all the properties of the fluid except its velocity would remain constant as it flowed through the blade passages.

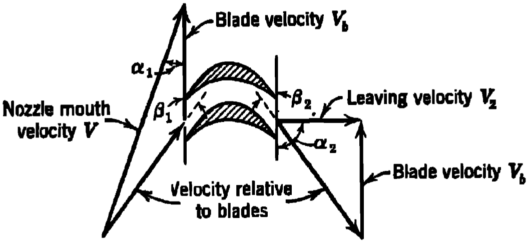


FIG. 121. Velocity diagram for a single-stage impulse turbine.

If the angles α_1 and β_2 in Fig. 120 are not zero, the velocity of the fluid leaving the blades, relative to the stationary parts of the turbine, cannot be zero. Consider, for example, the velocity diagram shown in Fig. 121. For any given nozzle mouth velocity V and nozzle angle α_1, the blade angle β_1 is fixed by the value selected for the blade velocity

V_b. Let us suppose that V_b is equal to $(V/2) \cos \alpha_1$. Then, if β_2 were made equal to β_1, and if no friction occurred within the blade passages, the angle α_2 would be a right angle. The velocity V_2 of the fluid leaving the blades, relative to the stationary parts of the turbine, would be a minimum and would be equal to $V \sin \alpha_1$. To illustrate, for an α_1 of 15°, V_2 would be a minimum if V_b were $0.483V$ and if β_1 and β_2 were each about 28°. The value of V_2 would be $0.259V$. The kinetic energy possessed by the fluid leaving the turbine ($= V_2^2/2gJ$, per pound of fluid) is called the **leaving loss** because it represents energy that could be delivered as work at the rotor shaft. It is zero only for an ideal zero-angle turbine.

Because of the short time required for each element of fluid to flow through the turbine, all turbines operate nearly adiabatically. If the flow is assumed to be perfectly adiabatic, and if the velocity of the fluid entering the turbine is negligible, the steady-flow energy equation (Eq. 13*a*) may be written

$$\frac{W'}{J} = h_1 - \left(h_2 + \frac{V_2^2}{2gJ}\right),$$

where h_1 denotes the enthalpy of the fluid entering the turbine, and h_2 and V_2 the enthalpy and velocity leaving. If V_2 is zero or negligible (that is, if the leaving loss is zero), the foregoing equation reduces to

$$\frac{W'}{J} = h_1 - h_2.$$

Note that these equations are correct regardless of whether friction occurs within the turbine. If no friction occurred, the flow would be reversible, and the entropy of the fluid would remain constant. The value of h_2 would then depend only upon the exhaust pressure.

PROBLEMS

404. Suppose that the steam supplied to an ideal, single-stage, zero-angle impulse turbine is at 50 psia and 300°F, that it enters the turbine at a low velocity, and that it leaves at a pressure of 15 psia. Suppose further that the steam flows through the turbine at the rate of 10,000 lb per hr. Determine (*a*) the ideal blade velocity V_b, and (*b*) the horsepower ideally delivered by the turbine.

Ans. (*a*) 1055 ft per sec; (*b*) 350 hp.

405. Suppose that the steam supplied to an ideal single-stage impulse turbine is at 50 psia and 300°F, that it enters the turbine at a low velocity, and that it leaves at a pressure of 15 psia. Suppose further that the nozzle angle α_1 is 20°, that the blade angles β_1 and β_2 are equal, and that the steam flows through the turbine at the rate of 10,000 lb per hr. Determine (*a*) the ideal blade velocity V_b (that is, the blade velocity which will make the leaving loss a minimum), (*b*) the blade angles β_1 and β_2, and (*c*) the horsepower ideally delivered by the turbine. Compare your answers with those for the preceding problem. *Ans.* (*a*) 992 ft per sec; (*b*) 36°; (*c*) 309 hp.

406. If an ideal turbine operates under the conditions stated in the preceding problem, but if its blades move at a velocity of 900 ft per sec (instead of at the velocity which makes the leaving loss a minimum), (*a*) what must the blade angles β_1 and β_2 be, and (*b*) what horsepower does the turbine deliver?

407. The efficiency of a turbine is defined in Art. 85 as the work delivered by the turbine per pound of steam flowing through it divided by the difference between the enthalpy of the steam entering the turbine and the enthalpy after an isentropic expansion to the exhaust pressure. It is apparent that, even if the steam expands isentropically, the leaving loss cannot be avoided if the blade angle β_2 is other than zero, and therefore the turbine efficiency cannot be 100%.

Suppose that the steam supplied to an ideal, single-stage, impulse turbine is at 50 psia and 300°F, that it enters the turbine at a low velocity, and that it leaves at a pressure of 15 psia. Suppose further that the blade angles β_1 and β_2 are equal and that the blade velocity is such that the leaving loss is a minimum. Determine what the efficiency of the turbine would be if the steam expanded isentropically and if the nozzle angle were (*a*) 0°, (*b*) 10°, (*c*) 20°, and (*d*) 30°.

125. Multistage Impulse Turbines.

Although the single-stage impulse turbine is probably as simple a turbine as can be devised, it is not practicable if the enthalpy drop is large because the blade velocities required are too high to be safe. The blade velocities in any turbine must be such that the stresses within the rotor, the attachment of the blades to the rotor, and the blades themselves are within the limits of structural safety. Blade velocities in the order of 500 or 600 ft per sec are common, although velocities as high as 1200 ft per sec are sometimes used.

Let us illustrate by a numerical example the fact that velocities higher than the foregoing are required in a single-stage turbine if the enthalpy drop is large. Suppose that the fluid flowing through an ideal, single-stage, zero-angle impulse turbine is steam, that it enters at 100 psia and 500°F, and that it exhausts at 1 in. Hg abs. To calculate the blade velocity required, we must first determine the nozzle mouth velocity V. From Table 3 of the *Steam Tables*, $h_1 = 1279.1$ Btu per lb and $s_1 = 1.7085$ Btu per lb deg F. Since the entropy of the steam leaving the nozzle is ideally the same as its entropy entering, and since the pressure drop through the blade passages is zero, we know both the entropy and the pressure of the steam at the nozzle mouth. Hence, we can determine first its quality and then its enthalpy at this section. By Eq. 44*b*,

$$s = s_g - (1 - x)s_{fg} \tag{44b}$$

or

$$1.7085 = 2.0387 - (1 - x)1.9473;$$

whence

$$1 - x = 0.170.$$

Its enthalpy can now be calculated by Eq. 20*b*:

$$h = h_g - (1 - x)h_{fg} \tag{20b}$$

$$= 1096.3 - 0.170 \times 1049.2$$

$$= 918.3 \text{ Btu per lb.}$$

Therefore, the nozzle-mouth velocity is, by Eq. 76,

$$V = \sqrt{2 \times 32.2 \times 778(1279.1 - 918.3)}$$

$$= 4250 \text{ ft per sec.}$$

Finally, since the blade velocity V_b for an ideal, single-stage, zero-angle impulse turbine is one-half the nozzle mouth velocity,

$$V_b = 2125 \text{ ft per sec.}$$

This velocity is considerably above the limit of safety. It would be somewhat smaller for an actual turbine because of the effect of friction on the velocity of the fluid and because the nozzle angle α_1 would be greater than zero. It would not, however, be reduced enough to fall within the limit of safety.

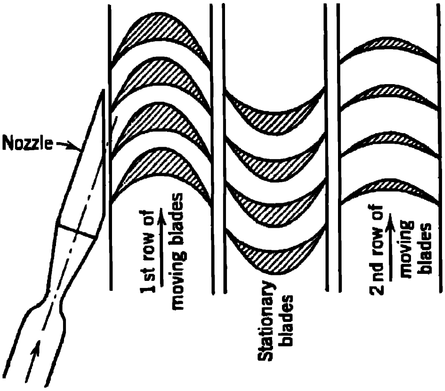


FIG. 122. Cross-sectional view through one of the nozzles and several adjacent blades of an impulse turbine with two velocity stages.

The blade velocities required for an impulse turbine can be reduced by making use either of (*a*) *velocity staging* (also called *velocity compounding*), or of (*b*) *pressure staging*. Let us first consider velocity staging.

The high velocity stream of fluid from the nozzles of an impulse turbine using *velocity staging* flows at constant pressure, not through a single row of moving blades, but through two or more rows that have stationary, reversing blades between them, as shown in Fig. 122. Let us suppose that two rows of moving blades are used and that, for the sake of simplicity, the nozzle angle and the entering and leaving angles of each row of blades are all zero. If the nozzle mouth velocity were, say, 2000 ft per sec, and if the moving blades were traveling at *one-quarter* this velocity (500 ft per sec), the fluid would enter the first row of moving blades with a velocity, relative to the blades, of (2000 − 500 =) 1500 ft per sec. If no friction occurred within the blade passages, the fluid would leave them at the same velocity, but its direction of flow would be reversed. Its velocity, relative to the stationary parts of the turbine, would be (1500 − 500 =) 1000 ft per sec. The fluid would flow next through the row of stationary blades, and its direction would again be reversed. If no friction occurred, it would leave these blades with the velocity at which it entered. It would flow next into the second row of moving blades, and its velocity relative to them would be (1000 − 500 =) 500 ft per sec. Since, with no friction, it would also leave at this velocity but would be flowing in a direction opposite to that in which the blades were moving, its final velocity relative to the stationary parts of the turbine would be zero.

It is evident from the foregoing example that the blade velocity V_b required for an ideal, zero-angle, impulse turbine with two rows of moving blades is one-fourth of the nozzle mouth velocity V. If three rows of moving blades are used, the blade velocity ideally required is one-sixth of the nozzle mouth velocity; and, if N rows are used, it is $1/2N$ times the nozzle mouth velocity. In theory, therefore, the blade velocity could be reduced to any desired value by using enough velocity stages. In practice, however, the efficiency of a turbine decreases as the number of velocity stages is increased; and for this reason more than two rows of moving blades are seldom used. The decrease in efficiency is due partly to the fact that the blade surface over which the fluid must flow increases as the number of stages is increased, and therefore the frictional losses increase. In addition, the velocity of the fluid relative to the blades is higher in the first row of moving blades than in the blades of a single-stage turbine, and this further increases the frictional loss. Note, for example, that, with a nozzle mouth velocity of 2000 ft per sec, the velocity of the fluid relative to the blades is 1000 ft per sec for an ideal, zero-angle, *single-stage* turbine; whereas it is 1500 ft per sec in the first row of moving blades for an ideal, zero-angle turbine with *two* velocity stages. Finally, since some turbulence and attendant frictional losses occur each time a high velocity stream of fluid enters a row of blades, the efficiency of a multistage turbine is still further reduced as the number of stages is increased.

It is of interest to note that, if the nozzle mouth velocity is 2000 ft per sec, the velocity of the fluid relative to the blades in an ideal, zero-angle turbine with two velocity stages is 1500 ft per sec in the first row of moving blades, 1000 ft per sec in the stationary blades, and 500 ft per sec in the second row of moving blades. Therefore, since under ideal conditions the specific volume of the fluid remains constant as it flows through these various rows of blades, it is necessary that the cross-sectional area of the passages between the blades be greater for the stationary blades than for the first row of moving blades and that it be greater for the second row of moving blades than for the stationary blades. This increase in area is obtained by making the blades in each succeeding row longer than those in the one before.

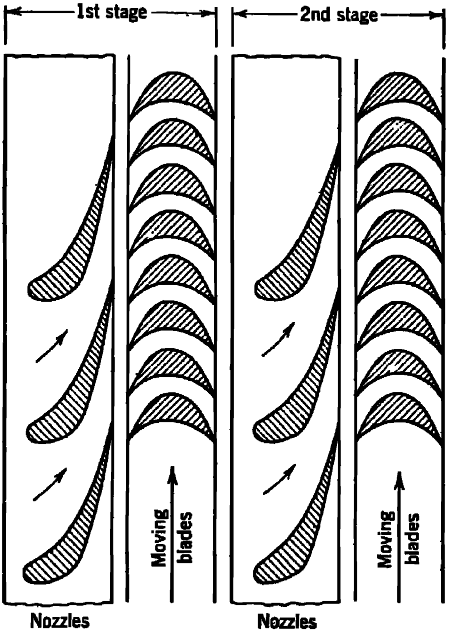


FIG. 123. Cross-sectional view through several of the nozzles and blades of an impulse turbine with two pressure stages.

Let us consider next how the required blade velocity V_b can be reduced by *pressure staging*. An impulse turbine using pressure staging consists, in effect, of a series of single-stage impulse turbines. The fluid leaving the blades of the first stage enters the nozzles of the second stage, the fluid from the second stage enters the nozzles of the third stage, and so forth, as shown in Fig. 123. Since the total enthalpy drop is divided among the various stages, the nozzle mouth velocities are lower than for a single-stage turbine, and consequently the blade velocities required are also lower.

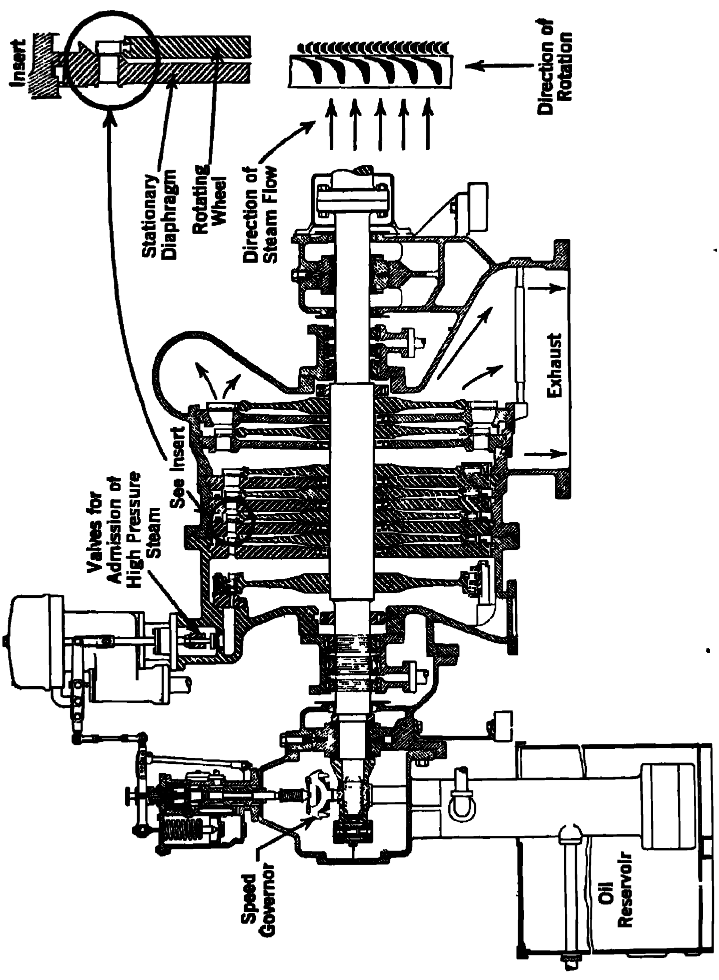


FIG. 124. Cross-sectional view through a typical multistage impulse turbine. (Reproduced by permission from *Modern Turbines*, by Louis E. Newman, Allen Keller, John M. Lyons, and Lawrence B. Wales, published by John Wiley & Sons, 1944.)

It was found in the second paragraph of this article that, if steam initially at 100 psia and 500°F expands isentropically to 1 in. Hg abs, the difference between its initial and final enthalpies is (1279.1 − 918.3 =) 360.8 Btu per lb. Let us determine the blade velocities required for an ideal, zero-angle impulse turbine if the turbine has, say, 5 pressure stages, and if the total drop in enthalpy is divided equally among the stages (that is, if the enthlapy drop in each stage is 72.2 Btu per lb). The nozzle mouth velocity for each stage would be, by Eq. 76,

$$V = \sqrt{2 \times 32.2 \times 778 \times 72.2}$$
$$= 1900 \text{ ft per sec,}$$

and therefore the blade velocity would have to be 950 ft per sec. This velocity is within the limits of safety.

Besides reducing the blade velocity required, pressure staging has certain other advantages: First, it reduces the leaving loss from the turbine because (*a*) the kinetic energy of the fluid leaving each stage, except the last, is carried into the nozzles of the next stage and so is not lost, and (*b*) the nozzle mouth velocity for the last stage is much lower than for a single-stage turbine, and therefore the velocity leaving this stage is lower. Pressure staging also reduces the velocity of the fluid relative to the various blade passages, which in turn reduces the magnitude of the frictional forces. This advantage is partially offset, however, by the fact that the fluid must flow over more blade surface and must enter more than a single row of blades.

Since the specific volume of the fluid increases as the pressure of the fluid drops, more cross-sectional area must be provided in the blade passages at the low pressure end than at the high pressure end of a turbine using pressure staging. This increase in area can be obtained (*a*) by increasing the number of nozzles and thereby increasing the number of blade passages to which steam is admitted, (*b*) by increasing the length of the blades, and (*c*) by increasing the diameter of the rotor.

A cross-sectional view through a typical multistage impulse turbine is shown in Fig. 124. Note that the turbine has seven pressure stages and that the first of these has two velocity stages. The nozzles for each of the pressure stages are mounted in stationary *diaphragms* which fit closely around the rotor shaft in order to reduce the leakage of steam from one stage to the next. Note also that the length of the blades at the low pressure end of the turbine is greater than at the high pressure end.

PROBLEMS

408. Suppose that an ideal, zero-angle, impulse turbine has two velocity stages, that the steam supplied to the turbine is at 60 psia and 400°F, that the steam enters at a low velocity, and that it leaves at a pressure of 2 in. Hg abs. Suppose further that the steam flows through the turbine at the rate of 10,000 lb per hr. Determine (*a*) the ideal blade velocity V_b, and (*b*) the horsepower ideally delivered by the turbine.
Ans. (*a*) 930 ft per sec; (*b*) 1090 hp.

409. Suppose that an ideal impulse turbine has two velocity stages, that the steam supplied to the turbine is at 60 psia and 400°F, that the steam enters at a low velocity, and that it leaves at a pressure of 2 in. Hg abs. Suppose further that the nozzle angle α_1 is 20°, that for each row of blades the blade angles β_1 and β_2 are equal, and that the steam flows through the turbine at the rate of 10,000 lb per hr. Determine (*a*) the ideal blade velocity V_b (that is, the blade velocity which will make the leaving loss a minimum), (*b*) the blade angles β_1 and β_2 for each row of blades, and (*c*) the horsepower ideally delivered by the turbine. Compare your answers with those for the preceding problem.

410. Suppose that an ideal, zero-angle, impulse turbine has four pressure stages, that the steam supplied to the turbine is at 60 psia and 400°F, that the steam enters at a low velocity, and that it leaves at a pressure of 2 in. Hg abs. Suppose further that the total enthalpy drop is divided equally among the four stages, that all the rows of blades move at the same blade velocity V_b, and that steam flows through the turbine at the rate of 10,000 lb per hr. Determine (*a*) the ideal value for V_b, and (*b*) the horsepower ideally delivered by the turbine.

Ans. (*a*) 930 ft per sec; (*b*) 1090 hp.

411. Suppose that an ideal impulse turbine has four pressure stages, that the steam supplied to the turbine is at 60 psia and 400°F, that the steam enters at a low velocity, and that it leaves at a pressure of 2 in. Hg abs. Suppose further that the total enthalpy drop is divided equally among the four stages, that for each stage the nozzle angle α_1 is 20° and the blade angles β_1 and β_2 are equal, and that steam flows through the turbine at the rate of 10,000 lb per hr. Determine (*a*) the ideal blade velocity for each stage (that is, the blade velocity which will make the work delivered by the stage a maximum), and (*b*) the horsepower ideally delivered by the turbine. Compare your answers with those for Problem 409.

126. Reaction Turbines. In an impulse turbine the velocity of the fluid leaving the nozzles is high, not only relative to the nozzles, but also relative to the moving blades. For this reason some friction inevitably occurs when the fluid enters the blade passages, and some as it flows through these passages. The friction occurring within the blade passages is appreciably greater than it would be in a straight passage of equal length because changing its direction of flow causes the fluid to follow, not the contour of the passage, but two spiral paths, as shown in Fig. 125.

It is evident that low fluid velocities relative to the moving blades are desirable at least (*a*) when the fluid enters the blade passages, and (*b*) while its direction of flow is changing within these passages. It would be possible to realize these low velocities if a turbine could be built to operate as follows: First, the blade velocity V_b would have to equal $V \cos \alpha_1$, where V denotes the nozzle mouth velocity, and α_1 the nozzle angle. The fluid would then enter the blade passages in a direction normal to the plane of rotation, and its velocity relative to the moving blades would be $V \sin \alpha_1$, as shown in Fig. 126. Hence, the entering velocity could be kept low by making the nozzle angle α_1 as small as possible. Second, the cross-sectional area of that portion of the blade passage where the direction of flow changed would have to

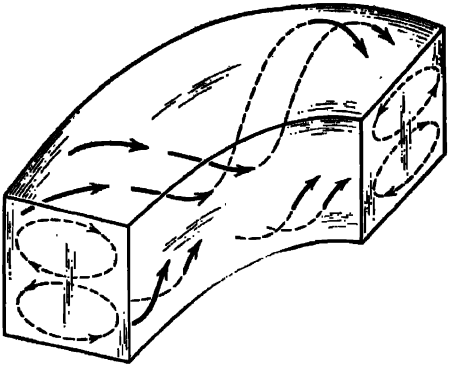


FIG. 125. Sketch showing the two spiral paths followed by the particles of a fluid flowing at high velocity through a curved passage.

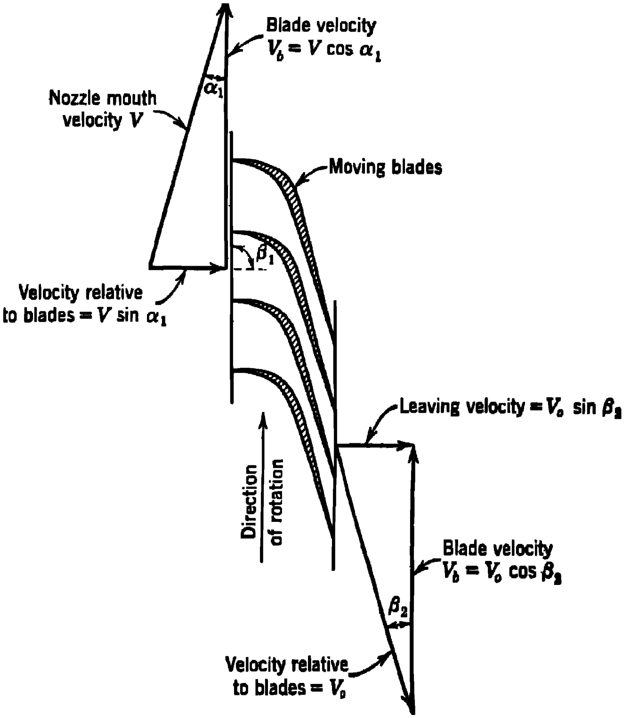

Blade velocity
$V_b = V \cos \alpha_1$
Nozzle mouth velocity V
α_1
Moving blades
β_1
Velocity relative to blades = $V \sin \alpha_1$
Leaving velocity = $V_o \sin \beta_2$
Direction of rotation
Blade velocity
$V_b = V_o \cos \beta_2$
β_2
Velocity relative to blades = V_o

FIG. 126. Velocity diagram for an ideal single-stage reaction turbine.

be large. The velocity of the fluid would thereby be kept low while its direction of flow changed. Finally, instead of letting it flow through the blade passages at constant pressure, the fluid could be caused to undergo a drop in pressure by having the pressure on the discharge side of the blades lower than on the entrance side. The fluid would then expand as it flowed through these passages and would leave at some velocity V_0 higher than the velocity $V \sin \alpha_1$ with which it entered. By having the proper pressure on the discharge side of the blades, the velocity V_0 could be made such that $V_0 \cos \beta_2$ equaled the blade velocity V_b. Relative to the stationary parts of the turbine, therefore, the fluid leaving the blades would be flowing in a direction normal to the plane of rotation, and its velocity would be $V_0 \sin \beta_2$, as shown in Fig. 126. It is apparent that, by making the angle β_2 as small as possible, the velocity of the fluid leaving the blades could be kept low.

A turbine operating in the foregoing manner might be called an **ideal, single-stage, reaction turbine.** Let us consider whether such a turbine is practicable. For the sake of simplicity, let us suppose that the nozzle angle α_1 and the leaving blade angle β_2 could each be made equal to zero; and let us determine what blade velocity V_b would be required if the fluid flowing through the turbine were steam, if it entered the turbine at 100 psia and 500°F, and if it expanded isentropically to a pressure of 1 in. Hg abs. With α_1 equal to zero, the nozzle mouth velocity V and the blade velocity V_b would be equal. Likewise, with the blade angle β_2 equal to zero, the velocity V_0 of the steam leaving the blades and the blade velocity V_b would also be equal. Therefore, the nozzle mouth velocity V and the velocity V_0 would have to be equal, and this would require that the enthalpy drop in the stationary nozzles and the enthalpy drop in the moving blades be equal. It was found in Art. 124 that steam initially at 100 psia and 500°F expanding isentropically to 1 in. Hg abs undergoes a drop in enthalpy of (1279.1 − 918.3 =) 360.8 Btu per lb. Hence, the enthalpy drop in the nozzles would have to be 180.4 Btu per lb; and the nozzle mouth velocity would be, by Eq. 76,

$$V = \sqrt{2 \times 32.2 \times 778 \times 180.4}$$

$$= 3000 \text{ ft per sec.}$$

This is also the required blade velocity V_b. As explained in Art. 125, for structural reasons it is impossible to build a turbine with blade velocities as high as this. We conclude, therefore, that a turbine of the type under consideration actually cannot be built.

It is evident that the blade velocity required can be reduced by having the enthalpy drop take place, not in a single set of nozzles and moving blades, but in a number of such sets. Still assuming that the angles α_1 and β_2 could be made zero, let us determine the number of sets required if the blade velocity is to be uniform for all the rows of moving blades and is to be equal to, say, 600 ft per sec. For adiabatic flow with a negligible initial velocity, the enthalpy drop Δh required to produce this velocity is, by the steady-flow energy equation,

$$\Delta h = \frac{V^2}{2gJ} = \frac{(600)^2}{50{,}000} = 7.2 \text{ Btu per lb.}$$

Hence, the enthalpy drop *per stage* (that is, for one row of nozzles and the adjacent row of moving blades) is (2 × 7.2 =) 14.4 Btu per lb; and the total number of stages required is (360.8/14.4 =) 25. Such a multistage turbine is called simply a **reaction turbine.**

Note that, for a nozzle mouth velocity of 600 ft per sec, a converging rather than a converging-diverging nozzle is required. Also, for a velocity V_0 of 600 ft per sec, converging rather than converging-diverging blade passages are required. Since converging nozzles and blade passages are required even for the highest permissible blade velocities, and since the angles α_1 and β_2 actually cannot be made zero, a cross-section through a typical row of nozzles and blades in an actual reaction turbine appears as shown in Fig. 127. Note that the entering blade angle is less than a right angle. The proper nozzle mouth velocity for any given blade velocity V_b is thereby increased, and the enthalpy drop in the nozzles is also increased. Fewer stages are required, therefore, and the length of the turbine is decreased. Such blades make use of both the impulse and the reaction principle.

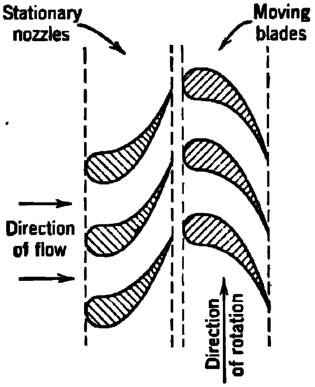


FIG. 127.

Since a pressure drop occurs in the blade passages of a reaction turbine, some provision must be made to prevent leakage around the ends of the blades. Such by-passing of the blade passages is prevented by having the clearance between the ends of the blades and the turbine casing quite small. Likewise, by-passing of the fluid around the ends of the stationary nozzles is prevented by having the clearance between the ends of the nozzles and the rotor small. To prevent damage to the turbine if any rubbing should take place, the ends of the nozzles and blades are made thin. This allows them to wear away quickly and thus provide the necessary clearance. Another way to prevent damage is to have a *shroud ring* around the ends of the nozzles and blades and to have a thin sealing strip mounted on this ring, as shown in Fig. 128. Rubbing wears away part of this sealing strip and provides the necessary clearance. Because of the pressure drop across each row of moving blades, an axial thrust occurs on the rotor, and some means of balancing this must be provided.

It is of interest to compare briefly impulse and reaction turbines. As already explained, reaction turbines have an advantage over impulse turbines in that the velocity of the fluid relative to the blades is low when the fluid enters the blade passages and when its direction of flow changes. On the other hand, reaction turbines have a disadvantage in that the enthalpy drop permissible in any one stage is much smaller than in an impulse turbine. For this reason more stages are required, and reaction turbines must be longer. Impulse turbines have an advantage in that no pressure drop occurs across the blades, and no provision need be made to prevent leakage of steam around the ends of the blades. Impulse turbines have a disadvantage in that the pressure drop through the nozzles of any one stage is much greater than through the nozzles and blades of one stage of a reaction turbine; and therefore leakage through the clearance space between the diaphragms and the shaft may be as great as the leakage around the ends of the nozzles and blades in a reaction turbine.

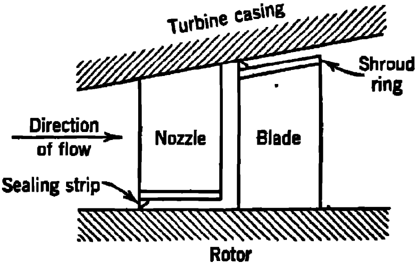


FIG. 128.

PROBLEMS

412. A reaction turbine will be supplied with steam at 100 psia and 500°F. The steam will enter the turbine with a velocity of 200 ft per sec and will leave at a pressure of 1 in. Hg abs. The nozzle angle α_1 will be 20° for each of the rows of nozzles, the entering blade angle β_1 will be 90° for each of the rows of blades, the leaving blade angle β_2 will be 20°, and the velocity of the blades will be 600 ft per sec. If the steam were to flow through the turbine isentropically, and if the total enthalpy drop were divided equally among the stages, how many stages would be required?

Ans. 25.

413. Suppose that a reaction turbine is to operate under the conditions stated in the preceding problem except that the blade velocity is to be increased to a value such that the turbine requires only 20 stages. If the steam were to flow through the turbine isentropically, if all the blades were to move at the same velocity, and if the total enthalpy drop were divided equally among the stages, what blade velocity would be required?

127. The Stage Efficiency, Condition Curve, and Reheat Factor.

The efficiency η_t of a turbine has already been defined in Art. 85 as the work W'_t actually delivered by the turbine divided by the work it would deliver if the expansion were adiabatic and if no friction occurred; that is,

$$\eta_t = \frac{W'_t}{J(\Delta h)_t}, \tag{48}$$

where $(\Delta h)_t$ denotes the difference between the enthalpy of the gas or vapor entering the turbine and its enthalpy after expanding isentropically to the exhaust pressure. Note that the work delivered to the *turbine blades* is not all delivered at the turbine shaft. Some of it is consumed in overcoming friction in the bearings and in the various seals within the turbine. In an impulse turbine, some work is consumed in overcoming fluid friction between the disks on which the blades are mounted and the fluid surrounding these disks. If some of the blades are idle (that is, if the nozzles do not extend completely around the periphery of the rotor), work is also consumed in moving these idle blades through the surrounding fluid. Work consumed in overcoming the friction from these various sources is called the **rotation loss** of the turbine.

If the turbine is a multistage turbine, the designer is interested in the efficiency of each of its individual stages as well as in the efficiency of the turbine as a whole. The efficiency of any one stage is called the **stage efficiency** and will be denoted by the symbol η_s. It is defined as the work W'_s the stage actually delivers at the turbine shaft divided by the work it would deliver if the expansion through the stage were isentropic and there were no rotation losses; that is,

$$\eta_s = \frac{W'_s}{J(\Delta h)_s}, \tag{80}$$

where $(\Delta h)_s$ denotes the isentropic drop in enthalpy for the stage.

The turbine designer is also interested in what are called the *condition curve* and the *reheat factor* for the turbine. The **condition curve** may be drawn on any diagram of properties by passing a curve through the points that represent the state of the fluid at the entrance to each stage and the point that represents its state at the turbine exhaust. The condition curve for an ideal turbine is an isentropic line through the point that represents the state of the fluid at the turbine entrance. The condition curve for an actual turbine always passes from this point to points of successively higher entropy. That this must be true follows from the Entropy Principle, discussed in Art. 80. This principle states that, whenever any system undergoes an adiabatic process, its entropy always either increases or remains constant. Since the actual flow through any turbine is nearly adiabatic, the Entropy Principle applies; and, since some friction inevitably occurs, the entropy of the fluid must increase.

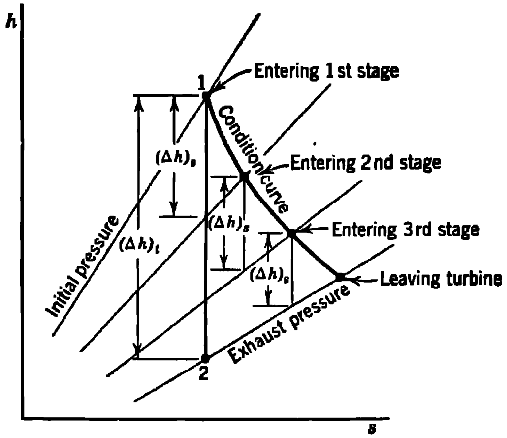


FIG. 129. Condition curve for a three-stage turbine.

The **reheat factor** $\mathcal{R}$ for a turbine is defined as the sum of the isentropic drops in enthalpy for the individual stages divided by the isentropic drop in enthalpy for the turbine as a whole; that is,

$$\mathcal{R} = \frac{\Sigma(\Delta h)_s}{(\Delta h)_t}. \qquad (81)$$

The reheat factor for an ideal turbine is unity, whereas for any actual turbine it is always larger than unity. That this must be true can be seen from the condition curve drawn on an enthalpy-entropy diagram. A condition curve for a three-stage turbine is shown in Fig. 129. Because constant pressure lines on an h-s diagram diverge (see Art. 79), and because the condition curve for an actual turbine always passes through points of successively higher entropy, it is evident that the sum of the isentropic drops in enthalpy for the individual stages must always be greater than the isentropic drop for the entire turbine. Thus, the work ideally obtainable from the last stages (equal to the isentropic drop in enthalpy for these stages) is increased by the friction occurring in the first stages. The reason for this is that the work that fails to leave the turbine because of friction "reheats" the fluid flowing through the turbine.

If the stage efficiency η_s is uniform for all the stages of a multistage turbine, a simple relation exists among its turbine efficiency, stage efficiency, and reheat factor. Thus, since $W'_t = \Sigma W'_s$, by Eq. 80

$$\frac{W'_t}{J} = \frac{\Sigma W'_s}{J} = \Sigma\eta_s(\Delta h)_s = \eta_s\Sigma(\Delta h)_s.$$

Substituting the last expression into Eq. 48, we obtain

$$\eta_t = \frac{\eta_s\Sigma(\Delta h)_s}{(\Delta h)_t},$$

and therefore, by making use of Eq. 81, we get

$$\eta_t = \eta_s\mathcal{R}. \qquad (82)$$

This relation is used by the turbine designer to determine an approximate value for the efficiency of the turbine from estimated values of the average stage efficiency and the reheat factor.

PROBLEMS

414. It is estimated that the average stage efficiency of a multistage turbine will be 75% and that the reheat factor will be 1.05. If steam will be supplied to the turbine at 200 psia and 500°F and will be expanded to 2 in. Hg abs, what will the rate of steam flow have to be in order for the turbine to deliver 5000 hp?

Ans. 44,600 lb per hr.

415. A four-stage impulse turbine is supplied with saturated steam at 100 psia. The steam flows through the turbine at the rate of 12,000 lb per hr, and the turbine delivers 1000 hp at its shaft. The steam leaves the first stage at 35 psia, the second stage at 10 psia, the third stage at 2.5 psia, and the last stage at 1 in. Hg abs. Assuming that no heat is lost from the turbine and that the condition curve on an h-s diagram is a straight line, determine (a) the efficiency of the turbine, (b) the efficiency of each stage, and (c) the reheat factor.

SYMBOLS

a	cross-sectional area
c_p	specific heat at constant pressure
g	acceleration that a force of 1 lb gives to a mass of 1 lb
h	enthalpy per unit mass; h_f = enthalpy per unit mass of saturated liquid; h_g = enthalpy per unit mass of saturated vapor; $h_{fg} = h_g - h_f$
J	mechanical equivalent of heat
k	ratio of c_p to c_v
N	number of velocity stages
p	pressure; p_1 = entering section pressure; p_c = critical pressure; p_d = discharge region pressure
R	gas constant
$\mathcal{R}$	reheat factor
s	entropy per unit mass; s_f = entropy per unit mass of saturated liquid; s_g = entropy per unit mass of saturated vapor; $s_{fg} = s_g - s_f$
t	temperature
T	absolute temperature
v	specific volume; v_g = specific volume of saturated vapor
V	velocity
w	mass rate of flow
W'	shaft-work done by a system in steady flow
x	quality, weight of saturated vapor per unit weight of mixture
η	efficiency; η_n = nozzle efficiency; η_s = stage efficiency; η_t = turbine efficiency

Chapter 15

VENTURI METERS, FLOW NOZZLES, ORIFICES, AND PITOT TUBES

Of the many kinds of meters that may be used to measure the flow of gases and vapors, probably the most important ones industrially are *venturi meters*, *flow nozzles*, and *orifices*. These meters all measure rates of flow. Another instrument of considerable importance is the *pitot tube*. Pitot tubes are used to measure the velocity of a fluid at any point. If pitot tube measurements are taken at a number of points across one section of a passage, the average velocity of the fluid can be determined, and from this its rate of flow can be calculated.

In this chapter equations are developed for determining the ideal rates of flow through converging passages from pressure and temperature measurements. The coefficients by which these ideal rates must be multiplied to determine the actual rates are discussed. Typical venturi meters, flow nozzles, and orifices together with their characteristics are described. Finally, equations are developed for calculating the velocity of a moving fluid from pitot tube measurements.

128. Measuring the Pressure and Temperature of a Moving Fluid. The pressure of a moving fluid could be readily measured by, say, a Bourdon tube gage if it were possible to have the gage move with the fluid. It is evident that the gage would indicate the true pressure of the fluid because the gage and the fluid would be at rest relative to each other. The descriptive name *static pressure* is frequently used in the literature on fluid meters to denote the pressure that such a moving gage would indicate. This name will also be used in the present chapter, although it should be recognized as only another name for what has hitherto been called simply the *pressure* of the fluid.

It is not practicable to measure the static pressure of a moving fluid by a gage that moves with the fluid. For this reason it is customary to measure its pressure by a gage connected to a small hole drilled perpendicularly through the wall of the passage through which the fluid is flowing. The inside edge of the hole is made free of burrs or other irregularities that might cause either an impact or a suction to occur. That pressures measured in this way are the same as pressures measured with a moving gage has never been verified experimentally. It is reasonable to assume that the two are nearly the same, however, since this assumption leads to no unexplainable discrepancies between theoretical and experimentally determined rates of flow through the passage.

If the fluid is a gas or a vapor, and if it is flowing through a straight passage, the location of the static pressure hole around the periphery of the passage at any one section has been found to have little effect on the gage reading. Sometimes, to insure a true average reading, several similar holes spaced uniformly around the periphery are connected to a ring-shaped pressure chamber, called a *piezometer ring*, and a single gage is used to measure the pressure in this chamber.

Just as the pressure of a moving fluid could be easily measured if it were possible to have a pressure gage move with the fluid, its temperature could also be easily measured (if it is assumed that the walls of the passage are at the same temperature as the fluid) if it were possible to have a thermometer move with the fluid. Since this is not practicable, it is necessary to hold the thermometer stationary (relative to the passage) and to allow the fluid to flow over it. It happens, conveniently, that the temperature indicated by such a stationary thermometer differs little from the true temperature of the fluid if the velocity of the fluid is not very high. To illustrate, let us suppose that air flows past a stationary thermometer with a velocity of 100 ft per sec. The greatest error in the temperature determination will occur if the air striking the thermometer comes almost to rest, and if it undergoes this deceleration adiabatically. For these conditions the steady-flow energy equation (Eq. 13*a*) applied to the air striking the thermometer takes the form

$$h_1 + \frac{V_1^2}{2gJ} = h_2,$$

where subscript 1 refers to the air approaching the thermometer, and subscript 2 to the almost stationary air in contact with the thermometer. Since $h_2 - h_1 = c_p(t_2 - t_1)$ for air (Eq. 18*a*), we may write

$$\frac{100^2}{50{,}000} = 0.241(t_2 - t_1),$$

whence

$$t_2 - t_1 = 0.83^\circ\text{F}.$$

Thus, the temperature indicated by the thermometer will be higher than the true temperature of the air by less than 1°F.

The value of c_p for most gases is in the same order of magnitude as the value for air. It follows, therefore, that the temperatures of most gases can be measured with reasonable accuracy by stationary thermometers if the gases flow past the thermometers at velocities no higher than about 100 ft per sec. If they are flowing at velocities substantially higher than this, an approximate correction may be made by the method just illustrated.

129. Isentropic Flow of a Gas or Vapor through a Converging Passage. Let us consider the passage shown in longitudinal section in Fig. 130. In cross-section it might be circular, or it might be of some other shape. Regardless of its shape, however, its cross-sectional area is constant up to section 1, and it decreases gradually between sections 1 and 2.

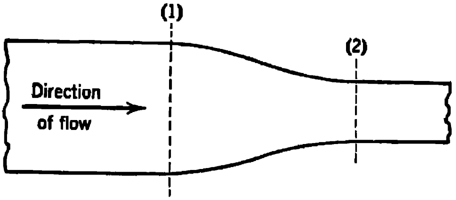


FIG. 130.

If sections 1 and 2 are not far apart, the flow of any fluid from one section to the other takes place nearly adiabatically because the time required for each element of the fluid to flow between the two sections is short. Also, because the passage is smoothly convergent, the fluid flows from one section to the other with little friction. Therefore, the fluid undergoes an approximately isentropic process. We shall assume in this article that any fluid flowing through the passage undergoes a process that is exactly isentropic, and we shall denote its mass rate of flow under this ideal condition by the symbol w_{ideal}.

Let us suppose that we know the cross-sectional areas a_1 and a_2 at sections 1 and 2, and that we can measure the following properties of the fluid: (*a*) its static pressure p_1 at section 1, (*b*) its temperature t_1 at section 1, and (*c*) its static pressure p_2 at section 2. With these data we can calculate the ideal mass rate of flow of the fluid by means of equations derived as follows:

(1) *Any Gas or Vapor.* Let us suppose first that the fluid is any gas or vapor for which a table of thermodynamic properties is available. Since, by assumption, the gas or vapor flows from section 1 to section 2 adiabatically, and since the shaft-work between these sections is zero, the steady-flow energy equation (Eq. 13*a*) takes the form

$$h_1 + \frac{V_1^2}{2gJ} = h_2 + \frac{V_2^2}{2gJ}.$$

The continuity equation (Eq. 15) states that

$$\frac{V_1a_1}{v_1} = \frac{V_2a_2}{v_2}. \tag{15}$$

Eliminating V_1 from these two equations and solving the resulting equation for V_2, we obtain

$$V_2 = \sqrt{2gJ(h_1 - h_2)} \times \frac{1}{\sqrt{1 - (a_2/a_1)^2(v_1/v_2)^2}}.$$

Finally, since $w = V_2a_2/v_2$, we may write

$$w = \frac{a_2}{v_2}\sqrt{2gJ(h_1 - h_2)} \times \frac{1}{\sqrt{1 - (a_2/a_1)^2(v_1/v_2)^2}}. \tag{83}$$

It is evident that, knowing the pressure p_1 and temperature t_1 of the fluid at section 1, we can find its values of v_1, h_1, and s_1 from the table of properties; and, knowing its pressure p_2 at section 2, we can find its values of v_2 and h_2 (corresponding to p_2 and $s_2 = s_1$). Therefore, we can calculate the value of w_{ideal} by Eq. 83.

(2) *Perfect Gases.* Let us suppose next that the fluid flowing through the passage is a perfect gas. An equation for calculating its ideal mass rate of flow, in terms of p_1, p_2, and T_1, can be derived from Eq. 83 by making use of the following perfect gas relations:

$$J(h_1 - h_2) = \frac{k}{k-1}(p_1v_1 - p_2v_2), \tag{18b}$$

$$p_1v_1^k = p_2v_2^k, \tag{28c}$$

and

$$p_1v_1 = RT_1. \tag{3}$$

Eliminating $h_1 - h_2$, v_1, and v_2 from these three equations and from Eq. 83, we obtain

$$w_{\text{ideal}} = a_2p_1\sqrt{\frac{2gk}{(k-1)RT_1}\left[\left(\frac{p_2}{p_1}\right)^{2/k} - \left(\frac{p_2}{p_1}\right)^{(k+1)/k}\right]} \times \frac{1}{\sqrt{1 - (a_2/a_1)^2(p_2/p_1)^{2/k}}}. \tag{83a}$$

(3) *Perfect Gases, p_2/p_1 Almost Unity.* Equation 83*a* is always applicable if the fluid is a perfect gas, but, if the ratio p_2/p_1 is only slightly

less than one, the term in brackets is difficult to evaluate. A simpler equation, which may be used in place of Eq. 83a when p_2/p_1 is close to unity, may be derived from Eq. 83 by making use of Eq. 74, developed in Art. 117:

$$dh = T\,ds + \frac{v\,dp}{J}. \tag{74}$$

Since the entropy of the gas is assumed to remain constant, this equation reduces to

$$J\,dh = v\,dp;$$

and, since the specific volume v will remain approximately constant when p_2/p_1 is almost unity, integration yields

$$J(h_1 - h_2) = v_1(p_1 - p_2).$$

Introducing this relation into Eq. 83 and replacing v_2 by v_1, we obtain *

$$w_{\text{ideal}} = a_2\sqrt{\frac{2g(p_1 - p_2)}{v_1}} \times \frac{1}{\sqrt{1 - (a_2/a_1)^2}}.$$

Finally, since $v_1 = RT_1/p_1$, we may write

$$w_{\text{ideal}} = a_2\sqrt{\frac{2gp_1(p_1 - p_2)}{RT_1}} \times \frac{1}{\sqrt{1 - (a_2/a_1)^2}}. \tag{83b}$$

It must be remembered that this equation may be used for perfect gases only if p_2/p_1 is close to unity (that is, if the gas acts approximately like an incompressible fluid).

PROBLEMS

416. Derive Eqs. 83 and 83*a*.

417. Superheated steam flows through the converging passage shown in Fig. 130. At section 1 the pressure and temperature of the steam are 100 psia and 400°F, and at section 2 the pressure is 90 psia. If the cross-sectional area of the passage is 1.00 sq in. at section 1 and 0.80 sq in. at section 2, and if the steam flows isentropically from the one section to the other, what is its mass rate of flow? *Ans.* 1.05 lb per sec.

418. If the fluid in the preceding problem were air instead of steam, what would be its mass rate of flow?

419. Air flows through a converging passage like the one shown in Fig. 130. At section 1 its pressure is 14.7 psia and its temperature is 70°F, and at section 2 its pressure is 3 in. of water lower than at section 1. If the cross-sectional area of the passage is 100 sq in. at section 1 and 50 sq in. at section 2, and if the air flows through the passage isentropically, what is the mass rate of flow of the air?

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* Note that this is the so-called "theoretical hydraulic equation" for the flow of incompressible fluids.

Ans. 3.48 lb per sec.

420. Suppose that air flows through the passage shown in Fig. 130 and that the ratio of a_2 to a_1 is 0.50. If the ratio of p_2 to p_1 is 0.98, by what per cent will the value of w_{ideal} calculated by Eq. 83*b* exceed the value of w_{ideal} calculated by Eq. 83*a*? (*Suggestion:* Use logarithms.)

130. The Coefficient of Discharge. The equations developed in the preceding article permit us to determine the *ideal* mass rate of flow (that is, the mass rate if the flow is *isentropic*) through a converging passage. To determine the *actual* rate of flow, we must know the **coefficient of discharge** for the passage. This coefficient, denoted by the symbol c_d, is defined as the actual mass rate of flow through the passage divided by the ideal mass rate of flow; that is,

$$c_d = \frac{w_{\text{act}}}{w_{\text{ideal}}}, \tag{84}$$

where w_{act} denotes the actual mass rate of flow.

The coefficient of discharge for any passage must be determined experimentally. As is to be expected, it is found to depend upon the size and shape of the passage. If the passage is smoothly convergent, as is the one shown in Fig. 130, the flow of any gas or vapor through it is nearly isentropic; and the coefficient of discharge is close to unity. If, on the other hand, the passage is abruptly convergent, the flow is attended by considerable friction; and the coefficient of discharge is appreciably smaller than unity. It is also found that, for any one passage, the coefficient of discharge may depend upon the kind of fluid flowing through the passage and upon the pressure, temperature, and velocity of the fluid.

Let us consider next the shape of the passages in some of the more common kinds of fluid meters.

131. Venturi Meters. Of the more common kinds of fluid meters the one having the most smoothly convergent passage is the **venturi meter.** A longitudinal section through a typical venturi meter is shown in Fig. 131. The meter consists first of a short cylindrical entrance section the diameter of which is the same as that of the straight pipe preceding the meter. This section is followed by a conical converging section. The transition from the one section to the other is rounded off by a curved surface tangential to each. The conical section is usually made with an included angle of about 21° and is followed by a short cylindrical section called the *throat.* The transition between these two sections is also rounded off to avoid turbulence. The diameter of the throat section is usually between 0.25 and 0.50 that of the cylindrical entrance section. The throat section is followed by a conical diverging section the function of which is to restore the pressure of the fluid as nearly as possible to its value at the entrance section. The included angle of this diverging section is usually between 5° and 7°, and the diameter of its leaving end is the same as that of the cylindrical entrance section.

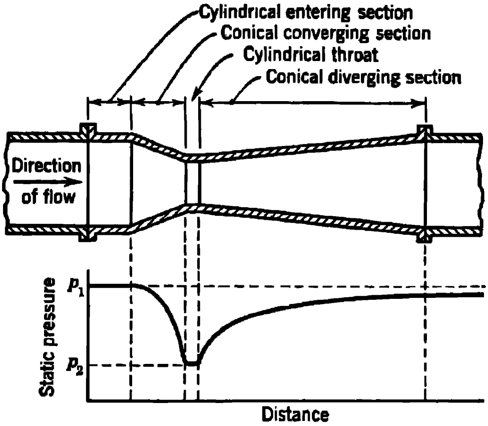
Cylindrical entering section
Conical converging section
Cylindrical throat
Conical diverging section
Direction of flow
Static pressure
p_1
p_2
Distance

FIG. 131. Longitudinal section through a venturi meter.

The static pressure of the fluid flowing through the meter is measured at the entrance and throat sections. If the pressure p_1 at the entrance section is well above atmospheric, it can be measured with a Bourdon tube gage. If it is near atmospheric pressure, it can be measured more accurately with a manometer. If the pressure p_2 at the throat section is well above atmospheric, it can also be measured with a Bourdon tube gage. If the pressure difference $p_1 - p_2$ is desired (see Eq. 83*b*), and if this difference is small (as it usually is), it cannot be determined accurately by subtracting one gage reading from another. For this reason it is usually measured with a manometer, one leg of which is connected to the entrance-section pressure hole, and the other, to the throat-section pressure hole.

Since a venturi meter is smoothly convergent between its entering and throat sections, it is logical to expect that its discharge coefficient will be close to unity. Experiments show that this is usually true, and values in the order of 0.98 or 0.99 are common. The fact that its discharge coefficient is so close to unity is one advantage of this kind of meter. Thus, a value of, say, 0.985 may usually be assumed with reasonable assurance that it will be in error by no more than plus or minus 1%.

Another advantage of a venturi meter is that the pressure loss across the entire meter is relatively low because the diverging section restores most of the pressure drop which takes place in the converging section. The way in which the static pressure of any fluid varies as it flows through the meter is shown in the lower part of Fig. 131. The pressure rise in the diverging section is usually equal to 80% or 90% of the pressure drop in the converging section. It may be noted that the sole purpose of the diverging section is to produce this rise in pressure and that the diverging section has no appreciable effect on the coefficient of discharge of the meter.

The principal disadvantage of venturi meters is that they are expensive (compared to orifices, for example). Their length is also a disadvantage if the flow through a short pipe is to be measured or if the meter is to be installed where only limited space is available.

PROBLEMS

421. The entrance of a venturi meter is 12 in. in diameter, and the throat is 6 in. Air enters the meter at 14.7 psia and 70°F and undergoes a drop in pressure of 2.50 in. of water between the entrance and the throat. If the meter has a coefficient of discharge of 0.985, what is the actual mass rate of flow of the air? *Ans.* 1.58 lb per sec.

422. Air flows through a venturi meter at the rate of 0.312 lb per sec. The air enters the meter at 20.0 psia and 80°F, and its pressure at the throat is 5.3 in. Hg lower than at the entrance. If the entrance is 3.065 in. in diameter and the throat is 1.128 in. in diameter, what is the coefficient of discharge for the meter?

423. It is estimated that an internal combustion engine uses 20 cu ft per min of air at 14.5 psia and 60°F. The exact rate is to be determined by means of a venturi meter. Assuming that the velocity of the air at the entrance section of the meter will be low (that is, that a_1 will be large) and that the coefficient of discharge will be 0.98, determine what the diameter of the throat of the meter must be in order to produce a pressure differential of 3 in. of water between the entrance and the throat. *Ans.* 0.735 in.

132. Flow Nozzles. Another fluid meter having a smoothly convergent passage is the **flow nozzle.** A longitudinal section through a simple form of flow nozzle is shown in Fig. 132. It consists of a curved entrance section leading tangentially into a short cylindrical *throat* section. It may be mounted between two pipe flanges, as shown.

The static pressure p_1 of the fluid approaching the nozzle is usually determined at a section about one pipe diameter ahead of the entrance plane of the nozzle. The static pressure p_2 in the throat may be determined by connecting a gage to a small hole drilled through the throat itself. If this is done, it is necessary to have a tube extend from the hole out through the wall of the pipe. This kind of connection is unnecessary, however, since approximately the same pressure is indicated by a gage connected to a hole drilled through the pipe opposite the throat of the nozzle, as shown in Fig. 132. As in a venturi meter, if the pressure p_1 is well above atmospheric, it may be determined by a Bourdon tube gage; if it is close to atmospheric, it may be determined by a manometer. The pressure difference $p_1 - p_2$ is most accurately determined by a manometer connected between the two pressure holes.

Since a flow nozzle has no smoothly diverging section following its throat, it is logical to expect that its pressure-recovery characteristics will be poor. It has been found that some pressure rise does take place in the pipe on the downstream side of the nozzle, however, that the pressure reaches its maximum value between 4 and 6 pipe diameters from the nozzle, and that it depends upon the ratio of throat diameter to pipe diameter. If this diameter ratio is 0.8, for example, the pressure rise in the discharge pipe is usually about 70% of the pressure drop $p_1 - p_2$; whereas, if the diameter ratio is 0.2, the pressure rise may be only about 5% of $p_1 - p_2$.

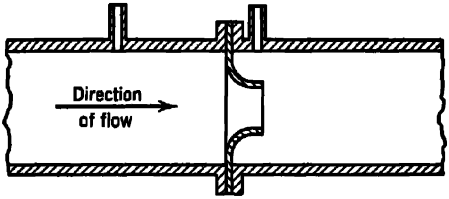

FIG. 132. Longitudinal section through a simple form of flow nozzle.

Flow nozzles, like venturi meters, have the advantage of having discharge coefficients close to unity. They have an advantage over venturi meters in that they are less expensive and are more easily installed. Their principal disadvantage, as explained in the preceding paragraph, is that they cause a greater over-all loss in pressure than do venturi meters.

Flow nozzles are sometimes installed at the end of a pipe or plenum chamber, instead of between pipe flanges, and the gas flowing through them is discharged into the atmosphere. They are frequently installed in this manner when used to measure the rate of flow through air compressors.

PROBLEMS

424. The air approaching a flow nozzle is at a pressure of 40.0 psia and at a temperature of 90°F. The tube through which the air is flowing is 3 in. in diameter, and the throat of the nozzle is 2 in. in diameter. If the pressure drop between a point one pipe diameter ahead of the nozzle and a point opposite the throat is 12.6 in. Hg, at what rate is the air flowing? Assume that the coefficient of discharge of the nozzle is 0.99. *Ans.* 2.28 lb per sec.

425. Air at a pressure of 100 psia and a temperature of 70°F is to be supplied to a certain apparatus at the rate of 3.0 lb per min. It is proposed to insert a flow nozzle into the supply line and to maintain the desired rate of flow by controlling the pressure on the upstream side of the nozzle. If the inside diameter of the supply line is 1.00 in., and if the diameter of the throat of the nozzle is 0.50 in., what pressure drop must be maintained across the nozzle? Assume that the coefficient of discharge for the nozzle is 0.99.

426. Suppose that air at 150°F approaches a flow nozzle at a low velocity. Suppose further that the diameter of the throat of the nozzle is 1.00 in. and that the air discharges from the nozzle into the atmosphere. What would the pressure on the upstream side of the nozzle have to be in order for 50 lb of air to flow through the nozzle per minute? Assume that the coefficient of discharge for the nozzle is unity.

133. Thin-Plate Orifices.

The fluid meter having the least smoothly convergent passage is the **thin-plate orifice.** Because of its simplicity, however, it is more widely used for industrial purposes than any other kind of meter. It consists ordinarily of a circular hole in a thin plate which is clamped between the flanges of two sections of straight pipe. The hole is usually, although not always, concentric with the pipe. A characteristic of most thin-plate orifices is that the edge of the orifice hole is left sharp and is not rounded off. For this reason these orifices are sometimes called *sharp-edged,* or *square-edged,* orifices. The ratio of the orifice diameter to the pipe diameter may have any value, although ratios less than 0.1 or greater than 0.8 are usually not used.

It was shown in Art. 129 that the ideal mass rate of flow of any gas or vapor through a converging passage can be calculated if the pressure p_1 and temperature t_1 at the entering section and the pressure p_2 at the point of minimum cross-sectional area are known. The pressure p_2 at the throat of a venturi meter or flow nozzle can be easily measured. For an orifice, however, the pressure p_2 at the point of minimum cross-sectional area (the orifice hole) cannot be easily measured. For this reason the static pressure is measured at some point on the downstream side of the orifice, and the ideal mass rate of flow is calculated *on the assumption that the pressure in the orifice hole is the same as this downstream pressure.* Since the static pressure on the downstream side of the orifice is different at various points, the value obtained depends upon where the pressure p_2 is measured. Consequently, the coefficient of discharge for the passage depends upon where the static pressures p_1 and p_2 are measured. Before discussing further the location of the pressure holes (or "taps") in the pipes leading to and from the orifice, let us consider the manner in which the static pressure varies on either side of the orifice.

The static pressure is found to increase slightly at the upstream face of an orifice, to decrease abruptly between the upstream and downstream faces, to decrease still further for a short distance beyond the orifice, and then gradually to increase for several pipe diameters. These changes are shown graphically in Fig. 133. Since the orifice has neither a smoothly convergent section nor a smoothly divergent section, it causes a much greater loss in pressure than a venturi meter and a slightly greater loss than a flow nozzle.

The locations most commonly used for the pressure holes are as follows: (*a*) The centers of the holes are located 1 in. from the upstream and downstream faces of the orifice plate. Holes located in this way are called *flange taps*. (*b*) The center of the upstream hole is located ½ to 2 pipe diameters from the upstream face of the orifice plate, and the downstream hole is located at the point where the downstream pressure is a minimum. Such holes are called *vena contracta taps*. (*c*) The upstream hole may be located 2½ pipe diameters from the upstream face of the orifice plate, and the downstream hole 8 pipe diameters from the orifice plate. Holes located in this way are called *pipe taps*. Other locations less common are as follows: (*d*) The holes are located in the orifice plate itself and are drilled perpendicular to the face of the plate. The plate must be thick enough so that radial holes can be drilled from its edge to join these longitudinal openings. Such taps are called *integral*, or *plate*, *taps*. (*e*) The holes are located at the corners between the pipe and the orifice plate. Such holes are called *corner taps*. (*f*) The center of the upstream hole is located 1 pipe diameter from the upstream face of the orifice plate, and the center of the downstream hole ½ pipe diameter from the downstream face. Holes located in this way are called *radius taps*.

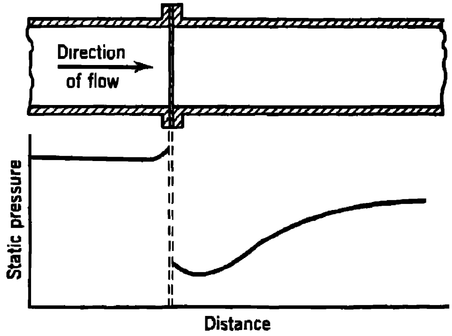

FIG. 133. Longitudinal section through a thin-plate orifice clamped between pipe flanges.

It is found that the coefficient of discharge for a thin-plate orifice depends upon the diameter of the pipe in which the orifice is located, upon the ratio of orifice diameter to pipe diameter, upon the kind of fluid flowing through the pipe, and upon the pressure, temperature, and velocity of the fluid. It also depends, of course, upon the location of the holes used in determining the static pressures p_1 and p_2. If flange or vena contracta taps are used, the coefficient of discharge is usually close to 0.60, whereas, if pipe taps are used, it may be between 0.60 and 0.90.

The principal advantages of thin-plate orifices are that they are inexpensive to make and are easy to install. Their principal disadvantage lies in the fact that, unless calibrated in place, they may be less accurate than venturi meters and flow nozzles. In addition, as has been explained, they cause a greater loss in pressure.

PROBLEMS

427. The rate at which air is flowing through a duct 8 in. in diameter is measured by a thin-plate orifice 6 in. in diameter. Manometers connected to flange taps show that the pressure on the upstream side of the orifice is 3.2 in. of water above atmospheric and on the downstream side is 1.4 in. of water above atmospheric. If the air is at 70°F, and if the coefficient of discharge for the orifice is 0.61, how many cubic feet of air per minute (measured at atmospheric pressure and 70°F) are flowing through the duct? The barometric pressure is 29.5 in. Hg abs. *Ans.* 788 cfm.

428. Superheated steam at 250 psia and 600°F is flowing through a pipe 2 in. in diameter (actual inside diameter = 2.067 in.). Its rate of flow is measured by a thin-plate orifice 1 in. in diameter. If a manometer connected between flange taps shows a pressure differential of 5.6 in. of mercury, and if the coefficient of discharge for the orifice is 0.60, how many pounds of steam are flowing through the pipe per minute?

134. The Velocity-of-Approach Factor. The ideal mass rate of flow through a converging passage can be calculated by Eq. 83, 83*a*, or 83*b*, developed in Art. 129. It can also be calculated by (*a*) determining what its value would be if the velocity V_1 at the entering section were negligible, and then (*b*) multiplying this value by a correction factor to allow for the fact that V_1 may not be negligible. This factor is called the **velocity-of-approach factor** and will be denoted by the symbol *F*. To derive expressions for it, we must first develop equations for finding w_{ideal} when V_1 is negligible.

(1) *Any Gas or Vapor.* Let us suppose that the fluid is any gas or vapor for which a table of thermodynamic properties is available. If the flow between the entrance section and the section of minimum cross-sectional area is adiabatic, and if the velocity V_1 is negligible, the steady-flow energy equation (Eq. 13*a*) reduces to

$$h_1 = h_2 + \frac{V_2^2}{2gJ},$$

whence

$$V_2 = \sqrt{2gJ(h_1 - h_2)}.$$

Therefore, since $w = V_2a_2/v_2$,

$$w = \frac{a_2}{v_2}\sqrt{2gJ(h_1 - h_2)}. \tag{85}$$

The values of h_1 and s_1 can be found from the table of properties if p_1 and t_1 are known; and the values of h_2 and v_2 can be found if p_2 is known (since, ideally, $s_2 = s_1$). Hence, *if V_1 is assumed to be negligible*, the ideal mass rate of flow can be calculated by Eq. 85. Comparing Eqs. 83 and 85, we see that the velocity-of-approach factor to be used with Eq. 85 is

$$F = \frac{1}{\sqrt{1 - (a_2/a_1)^2(v_1/v_2)^2}}. \tag{86}$$

Note that, if the area a_1 is considerably larger than the area a_2, V_1 will be negligible and the velocity-of-approach factor, calculated by the above equation, will be very nearly unity.

(2) *Perfect Gases.* If the fluid is a perfect gas, and if V_1 is negligible, an equation for w_{ideal} in terms of p_1, T_1, and p_2 can be derived from Eq. 85. It is necessary only to eliminate $h_1 - h_2$, v_1, and v_2 from this equation and from Eqs. 3, 18*b*, and 28*c* (see Art. 129). The following equation is obtained:

$$w_{\text{ideal}} = a_2p_1\sqrt{\frac{2gk}{(k-1)RT_1}\left[\left(\frac{p_2}{p_1}\right)^{2/k} - \left(\frac{p_2}{p_1}\right)^{(k+1)/k}\right]}. \tag{85a}$$

Comparing Eqs. 83*a* and 85*a*, we see that the velocity-of-approach factor to be used with Eq. 85*a* is

$$F = \frac{1}{\sqrt{1 - (a_2/a_1)^2(p_2/p_1)^{2/k}}}. \tag{86a}$$

(3) *Perfect Gases, p_2/p_1 Almost Unity.* As explained in Art. 129, if the fluid is a perfect gas, and if p_2/p_1 is almost unity, v_1 and v_2 will be nearly equal; and the isentropic enthalpy change, $J(h_1 - h_2)$, will be approximately equal to $v_1(p_1 - p_2)$. Introducing these relations into Eq. 85, we obtain

$$w_{\text{ideal}} = a_2\sqrt{\frac{2g(p_1 - p_2)}{v_1}};$$

or, since $v_1 = RT_1/p_1$,

$$w_{\text{ideal}} = a_2\sqrt{\frac{2gp_1(p_1 - p_2)}{RT_1}}. \tag{85b}$$

Comparing Eqs. 83*b* and 85*b*, we see that the velocity-of-approach factor to be used with Eq. 85*b* is

$$F = \frac{1}{\sqrt{1 - (a_2/a_1)^2}}. \tag{86b}$$

Note that, to determine *actual* rates of flow, we must multiply the ideal rates calculated by Eq. 85, 85*a*, or 85*b*, not only by the proper velocity-of-approach factor F, but also by the coefficient of discharge c_d. The last two quantities are sometimes combined into a single **flow coefficient** K, defined by the equation

$$K = Fc_d. \tag{87}$$

If the value of this coefficient is known, the actual mass rate of flow can be calculated simply by multiplying the ideal mass rate of flow, calculated by Eq. 85, 85*a*, or 85*b*, times K.

PROBLEMS

429. A venturi meter has an entrance section 1.50 in. in diameter and a throat 0.50 in. in diameter. Steam enters the meter at 10.5 psi gage and 300°F and undergoes a drop in pressure of 10.2 in. of mercury between the entrance and the throat. The coefficient of discharge for the meter is 0.99, and the barometric pressure is 29.5 in. Hg abs. Determine (*a*) the ideal rate of flow, assuming that the velocity of the steam at the entrance is negligible, (*b*) the velocity-of-approach factor F, and (*c*) the actual rate of flow.

430. Suppose that air flows through a flow nozzle and that the ratio of its pressure at the throat to its pressure at the entrance is 0.85. Determine the velocity-of-approach factor if the ratio of the throat diameter to pipe diameter is (*a*) 0.25, (*b*) 0.50, and (*c*) 0.75.

431. Suppose that the flow coefficient K for a thin-plate orifice 2.50 in. in diameter in a 4-in. pipe (actual inside diameter = 4.026 in.) is 0.672 if the pressure drop across the orifice is low. Determine the coefficient of discharge for the orifice.

Ans. 0.620.

135. Pitot Tubes. The velocity of a gas or vapor at any point in a passage can be determined by means of a **pitot tube.** In its simplest form a pitot tube consists of two elements, one for measuring the static pressure of the fluid, and the other for measuring its so-called *kinetic pressure.* The element used to measure the static pressure may consist of a gage connected to a small hole drilled perpendicularly through the wall of the passage, as shown in Fig. 134. The element used to measure the kinetic pressure may consist of a gage connected to a small tube the open end of which faces directly into the moving stream, as also shown in Fig. 134. The end of the tube must be cut off square, but the outside edge may be rounded off. This tube is usually called an *impact tube*.

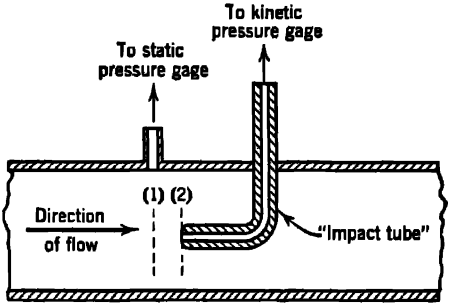

Fig. 134. Sectional view of an elementary type of pitot tube.

The static pressure is sometimes determined by a *static pressure tube* surrounding the impact tube, as shown in Fig. 135. The only openings into this tube are several small holes which are drilled perpendicular to the surface and are located far enough back from the tip so that they are not affected by any turbulence occurring at the tip. It has been found that static pressures determined with such a "combined type" of pitot tube are almost the same as those determined by means of static pressure holes drilled in the walls of the passage.

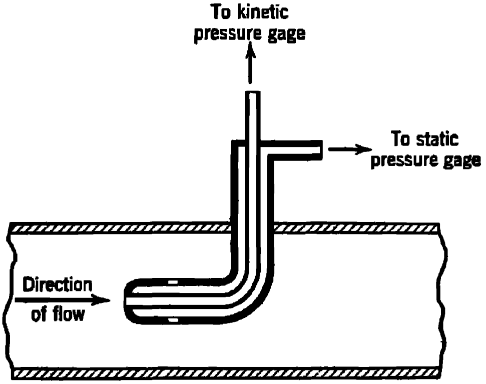

Fig. 135. Sectional view of a combined type of pitot tube.

Let us consider again the elementary type of pitot tube shown in Fig. 134. Let us denote as section 1 a section close to the mouth of the impact tube but far enough upstream so that the velocity at the section is not affected by the tube. Further, let us call the mouth of the impact tube section 2. The velocity of the fluid at this section is zero because the outside end of the impact tube is closed by the pressure gage, and therefore no flow through the tube can take place. Now, it has been found that *the deceleration of the fluid between sections 1 and 2 takes place very nearly isentropically.* This fact allows us to calculate the velocity V_1 from measurements of (*a*) the static pressure p_1 at section 1, (*b*) the temperature t_1 at section 1, and (*c*) the kinetic pressure p_2 at section 2.

(1) *Any Gas or Vapor.* Let us suppose first that the fluid is any gas or vapor for which a table of thermodynamic properties is available. To determine its velocity V_1, we must apply the steady-flow energy equation (Eq. 13*a*) to that portion of the fluid which has entered the mouth of the impact tube. Since this fluid has undergone an adiabatic process between sections 1 and 2, and since its velocity V_2 is zero, the steady-flow energy equation reduces to

$$h_1 + \frac{V_1^2}{2gJ} = h_2;$$

whence

$$V_1 = \sqrt{2gJ(h_2 - h_1)}. \tag{88}$$

Therefore, if we measure the pressure p_1 and temperature t_1 of the fluid, we can determine its enthalpy h_1 and entropy s_1 by means of the table of properties; and, if we measure its pressure p_2, we can determine its enthalpy h_2 (since we also know that $s_2 = s_1$). Knowing these values of h_1 and h_2, we can calculate the velocity V_1 of the fluid by Eq. 88.

(2) *Perfect Gases.* Let us suppose next that the fluid is a perfect gas. An equation for calculating the velocity V_1 of the gas, in terms of p_1, p_2, and T_1, can be derived from Eq. 88 by making use of Eqs. 3, 18*b*, and 28*c* (see Art. 129). Eliminating $h_2 - h_1$, v_1, and v_2 from these four equations, we obtain

$$V_1 = \sqrt{\frac{2gkRT_1}{(k-1)}\left[\left(\frac{p_2}{p_1}\right)^{(k-1)/k} - 1\right]}. \tag{88a}$$

(3) *Perfect Gases, p_2/p_1 Almost Unity.* As explained in Art. 129, if the fluid is a perfect gas, and if p_2/p_1 is almost unity, the specific volume of the fluid will remain approximately constant during the isentropic compression between sections 1 and 2; and therefore the enthalpy

change $J(h_2 - h_1)$ will be approximately equal to $v_1(p_2 - p_1)$. By introducing this expression into Eq. 88, we obtain *

$$V_1 = \sqrt{2gv_1(p_2 - p_1)};$$

or, finally, since $v_1 = RT_1/p_1$,

$$V_1 = \sqrt{\frac{2gRT_1(p_2 - p_1)}{p_1}}. \tag{88b}$$

The pressure difference $p_2 - p_1$ which appears in Eq. 88*b* is called the *impact pressure.* It is the difference between the kinetic and static pressures of the fluid and may be measured most accurately by a manometer, one leg of which is connected to the impact tube, and the other to the static pressure hole.

It should be noted that the velocity determined by means of a pitot tube is the velocity of the fluid only at a point directly upstream from the mouth of the impact tube. Since it is never uniform across any one section of a stream, the velocity must be measured at a number of points if the average velocity at the section is to be determined. These measurements should be made by dividing the cross-section of the stream into a number of equal areas and measuring the velocity at the center of each.

PROBLEMS

432. It is found by means of a pitot tube that the static pressure of a stream of air flowing through a pipe is 50.0 psi gage and that the impact pressure at the center of the pipe is 3.5 in. of water. If the air is at a temperature of 90°F, what is its velocity at the center of the pipe? The barometric pressure is 29.3 in. Hg abs.

Ans. 60.9 ft per sec.

433. Air discharges from a nozzle into the atmosphere. An impact tube at the mouth of the nozzle shows that the kinetic pressure of the air is 12.2 in. of mercury above atmospheric. A thermometer placed in the stream reads 70°F. If the barometric pressure is 29.9 in. Hg abs, what is the velocity of the air leaving the nozzle? (*Note:* The air in contact with the thermometer is virtually at rest, and therefore its pressure is equal to the kinetic pressure and its temperature is higher than that of the air stream. Estimate the true temperature of the air stream by assuming that the air in contact with the thermometer has undergone an isentropic compression from the static to the kinetic pressure.)

434. Steam at 100 psia and 500°F flows through a pipe. A pitot tube facing into the stream shows an impact pressure of 1.45 in. of mercury. Determine the velocity of the steam. (*Suggestion:* Consider the steam to be a perfect gas.)

Ans. 194 ft per sec.

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* Note that this is the "theoretical hydraulic equation" for determining the velocity of any incompressible fluid.

SYMBOLS

a	cross-sectional area
c_d	coefficient of discharge
c_p	specific heat at constant pressure
F	velocity-of-approach factor
g	acceleration that a force of 1 lb gives to a mass of 1 lb
h	enthalpy per unit mass
J	mechanical equivalent of heat
k	ratio of c_p to c_v
K	flow coefficient
p	pressure
R	gas constant
s	entropy per unit mass
t	temperature
T	absolute temperature
v	specific volume
V	velocity
w	mass rate of flow

Chapter 16

AIR CONDITIONING

The atmosphere consists of nitrogen, oxygen, argon, traces of certain other gases, and water vapor. The amount of water vapor present varies over a wide range, but the other consitutents are always present in very nearly constant proportions. For this reason the atmosphere may be considered to be made up of just two constituents, water vapor and air, the term *air* denoting the mixture of gases that would remain if all the water vapor were removed. This mixture is sometimes called *dry air*.

Engineers are frequently concerned with the problem of (*a*) raising or lowering the temperature of a mixture of air and water vapor, and (*b*) increasing or decreasing the amount of water vapor in the mixture. Such changes in the temperature and humidity may be necessary to produce an atmosphere that is comfortable for human beings or to make possible the satisfactory performance of certain manufacturing processes. The control of the temperature and humidity of the atmosphere in any enclosure is called **air conditioning.**

In this chapter certain properties of perfect gas mixtures are discussed first, and some of the properties of air-water vapor mixtures are considered next. The terms commonly used in air-conditioning work are then defined, and finally some of the more common kinds of air-conditioning processes are analyzed.

In the last article of the chapter some additional relations pertaining to mixtures of perfect gases are developed. Although these relations are not needed to solve air-conditioning problems, they are appended to this chapter because, like the material on air conditioning, they have to do with mixtures of gases.

136. Dalton's Law. Let us consider a mixture of gases consisting of m_x pounds of gas x, m_y pounds of gas y, and m_z pounds of gas z; and let us denote the mass of the mixture by m $(= m_x + m_y + m_z)$, its pressure by p, its specific volume by v, its total volume by V $(= mv)$, and its absolute temperature by T.

Now let us suppose that by some means we are able to separate the foregoing mixture into its constituents and that we place each constituent in a separate container. If the volume of each container is equal to the volume V of the original mixture, and if the temperature of each constituent is made the same as that of the original mixture, we shall find that (a) the pressures p_x, p_y, and p_z of the constituents are each less than the pressure p of the original mixture, but that (b) the sum of these pressures is approximately equal to that of the original mixture; that is,

$$p \approx p_x + p_y + p_z.$$

If we were to carry out this experiment for several values of p, we should find that the above relation is more nearly exact at low values of p than at high. Moreover, the results would show that the relation becomes exact as p approaches zero (that is, as each of the constituents approaches perfect gas behavior). Hence, *for a mixture of perfect gases*, we may write

$$p = p_x + p_y + p_z, \tag{89}$$

where p_x, p_y, and p_z denote the pressures of the constituents when each occupies alone the volume of the mixture and is at the temperature of the mixture. This relation is shown schematically in Fig. 136. The pressures p_x, p_y, and p_z are called the **partial pressures** of constituents x, y, and z. Additional terms may be added to Eq. 89 if the mixture consists of more than three constituents.

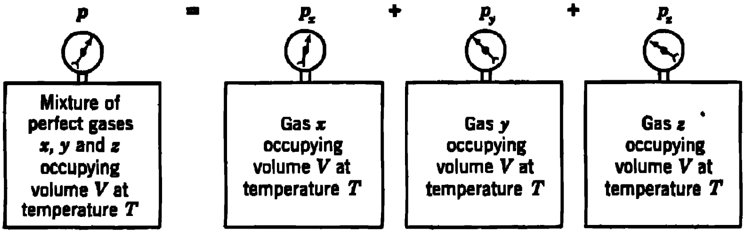
p
=
p_x
+
p_y
+
p_z
Mixture of perfect gases x, y and z occupying volume V at temperature T
Gas x occupying volume V at temperature T
Gas y occupying volume V at temperature T
Gas z occupying volume V at temperature T

FIG. 136. The relation between the pressure of a mixture of perfect gases and the partial pressures of its constituents.

According to Eq. 89, *each constituent of a mixture of perfect gases behaves as though the other constituents were not present* (at least as far as pressure is concerned). This statement is known as **Dalton's Law.** Although Dalton's Law is exact only for mixtures of perfect gases, it is sufficiently accurate for most engineering purposes if the mixture consists of actual gases at low pressures (atmospheric pressure, for example).

It will be recalled that the characteristic equation for a perfect gas is

$$pv = RT. \tag{3}$$

Let us write this equation for constituent x when x alone occupies the volume V of the mixture and is at the temperature T of the mixture. Since the mass of x is m_x, and since its pressure under the forenamed conditions is p_x, Eq. 3 takes the form

$$p_x\left(\frac{V}{m_x}\right) = R_xT,$$

or

$$p_xV = m_xR_xT, \tag{90}$$

where p_x = the partial pressure of constituent x, in pounds per square foot absolute,
V = the volume of the mixture, in cubic feet,
m_x = the mass of constituent x, in pounds,
R_x = the gas constant for constituent x, in feet per degree Fahrenheit, and
T = the temperature of the mixture, in degrees Fahrenheit absolute.

Equations similar to Eq. 90 may, of course, be written for the other constituents of the mixture.

It is evident from Eq. 90 that the density ρ_x of constituent x (the mass of x per cubic foot of the mixture) is

$$\rho_x = \frac{m_x}{V} = \frac{p_x}{R_xT}. \tag{90a}$$

Also, since

$$m = m_x + m_y + m_z,$$

we may write

$$\frac{m}{V} = \frac{m_x}{V} + \frac{m_y}{V} + \frac{m_z}{V},$$

or

$$\rho = \rho_x + \rho_y + \rho_z, \tag{91}$$

where ρ is the density of the mixture ($= m/V = 1/v$), and ρ_x, ρ_y, and ρ_z are the densities of each of its constituents.

ILLUSTRATIVE PROBLEMS

435. A mixture of N_2 and CO_2 occupies a volume of 20 cu ft when its pressure is 14.70 psia and its temperature is 70°F. If the mass of the N_2 in the mixture is 0.5 lb, what is the mass of the CO_2?

Solution. The mass of the CO_2 can be calculated by Eq. 90 after the partial pressure of the CO_2 has been determined. The partial pressure of the CO_2 can be found by Eq. 89 after the partial pressure of the N_2 has been calculated by Eq. 90. Thus, since the gas constant for N_2 is 55.1 ft per deg F (from Table I, Art. 10),

$$p_{N_2}V = m_{N_2}R_{N_2}T, \tag{90}$$

or

$$p_{N_2} \times 20 = 0.5 \times 55.1 \times 530,$$

whence

$$p_{N_2} = 730 \text{ psfa } (= 5.07 \text{ psia}).$$

Next, by Eq. 89,

$$p = p_{N_2} + p_{CO_2}, \tag{89}$$

or

$$14.70 \times 144 = 730 + p_{CO_2},$$

whence

$$p_{CO_2} = 1390 \text{ psfa } (= 9.63 \text{ psia}).$$

Finally, since the gas constant for CO_2 is 34.9 ft per deg F (from Table I),

$$p_{CO_2}V = m_{CO_2}R_{CO_2}T, \tag{90}$$

or

$$1390 \times 20 = m_{CO_2} \times 34.9 \times 530,$$

whence

$$m_{CO_2} = 1.50 \text{ lb}.$$

436. A mixture of gases consists of 0.3 lb of H_2, 0.5 lb of O_2, and 2.0 lb of N_2. What is the volume of the mixture if its pressure is 15 psia and its temperature is 80°F?

Solution. Since the gas constants for H_2, O_2, and N_2 are 767, 48.3, and 55.1 ft per deg F respectively (from Table I), writing Eq. 90 for each of the constituents gives us the three equations

$$p_{H_2}V = 0.3 \times 767 \times 540,$$

$$p_{O_2}V = 0.5 \times 48.3 \times 540,$$

and

$$p_{N_2}V = 2.0 \times 55.1 \times 540.$$

Equation 89 gives us a fourth equation:

$$15 \times 144 = p_{H_2} + p_{O_2} + p_{N_2}.$$

Eliminating the three partial pressures from these four equations and solving for the volume V of the mixture, we obtain

$$V = 91.4 \text{ cu ft}.$$

PROBLEMS

437. A 5-cu-ft tank contains dry air at 15.0 psia and 70°F. If the introduction of a certain quantity of steam produces a mixture of air and water vapor at 16.7 psia and 100°F, how much steam is introduced? Assume that the water vapor behaves as a perfect gas. *Ans.* 0.0127 lb.

438. A mixture consisting of 1.6 lb of CO_2 and 0.2 lb of H_2 is heated at a constant pressure of 25 psia. To what temperature must the mixture be heated in order for it to occupy a volume of 40 cu ft?

439. A 10-cu-ft tank contains nitrogen at 20 psia and 70°F. If 0.500 lb of O_2 are introduced into the tank, and if the resulting mixture is heated to 120°F, what will be its pressure? *Ans.* 31.6 psia.

440. Suppose that on a certain day the atmosphere is at 14.5 psia and 70°F and consists of 0.015 lb of water vapor per pound of dry air. (*a*) What is its density? (*b*) What would its density be if it were at 14.5 psia and 70°F but consisted of dry air only?

441. A mixture of CO_2 and N_2 has a density of 0.0830 lb per cu ft when the mixture is at 14.7 psia and 70°F. How many pounds of carbon dioxide are present in 1 lb of the mixture? *Ans.* 0.346 lb.

137. The Internal Energy and the Enthalpy of a Mixture of Perfect Gases. It was explained in Art. 29 that the difference between the internal energy of a system at any one state and its internal energy at any other state can be determined by (*a*) measuring the heat added to and the work done by the system when it changes by any convenient process from the one state to the other, and then (*b*) calculating the change in its internal energy by subtracting the work done from the heat added. It may be noted that internal energy differences can be determined in this way for any system, regardless of whether the system is a pure substance or a mixture of different substances.

Now let us again consider a mixture consisting of m_x, m_y, and m_z pounds of gases x, y, and z; and let us denote the total internal energy of the mixture at any state by the symbol U ($= mu$). The difference ΔU between its internal energies at any two states may be determined by the method explained in the preceding paragraph. Also, if the mixture is separated into its constituents, and if each of these is caused to change from an initial to a final temperature the same as those of the mixture, the differences ΔU_x, ΔU_y, and ΔU_z between the initial and final internal energies of each may be determined by the same method.

If the foregoing measurements are made, it is found that the values of ΔU_x, ΔU_y, and ΔU_z are each smaller than ΔU but that their sum is approximately equal to ΔU; that is, it is found that

$$\Delta U \approx \Delta U_x + \Delta U_y + \Delta U_z.$$

If these measurements are made at several pressures, it is found that the above relation is more nearly exact at low pressures than at high and that it becomes exact as the pressure of the mixture approaches zero; that is, the relation becomes exact as each gas approaches perfect gas behavior. Hence, *for a mixture of perfect gases,* we may write

$$\Delta U = \Delta U_x + \Delta U_y + \Delta U_z.$$

From the foregoing result, we may conclude that at any one state the internal energy U of a mixture of perfect gases is equal to the sum of the internal energies U_x, U_y, and U_z of its constituents when the constituents are separate from the mixture but are each at the temperature of the mixture; that is, for a mixture of perfect gases, we may conclude that

$$\left.\begin{aligned} U &= U_x + U_y + U_z, \\ \text{or} \quad U &= m_x u_x + m_y u_y + m_z u_z. \end{aligned}\right\} \tag{92}$$

Equations similar to these may be written for perfect gas mixtures consisting of any number of constituents.

Equation 92 further substantiates Dalton's Law since it shows that, as far as internal energy is concerned, each constituent of a mixture of perfect gases behaves as though the others were not present. Although Eq. 92, like Eq. 89, is exact only for mixtures of perfect gases, it is sufficiently accurate for most engineering purposes if the mixture consists of actual gases at low pressures.

We can now show that the total enthalpy H $(= mh)$ of a mixture of perfect gases is equal to the sum of the enthalpies H_x, H_y, and H_z of its constituents when the constituents are separate from the mixture but are each at the temperature of the mixture. Thus, since the total enthalpy H of the mixture is defined as

$$H = U + \frac{pV}{J},$$

by making use of Eqs. 89 and 92 we may write

$$\begin{aligned} H &= (U_x + U_y + U_z) + \frac{(p_x + p_y + p_z)V}{J} \\ &= \left(U_x + \frac{p_x V}{J}\right) + \left(U_y + \frac{p_y V}{J}\right) + \left(U_z + \frac{p_z V}{J}\right). \end{aligned}$$

But the pressure and internal energy of constituent x are p_x and U_x respectively when x occupies alone the volume V of the mixture and is at the temperature of the mixture. Hence, the term $U_x + p_x V/J$ is equal to H_x. Similarly, the other two groups of terms on the right side of the last equation are H_y and H_z. Therefore, finally, for a mixture of perfect gases,

$$\left.\begin{aligned} H &= H_x + H_y + H_z \\ &= m_x h_x + m_y h_y + m_z h_z. \end{aligned}\right\} \tag{93}$$

Additional terms may, of course, be added to these equations if the mixture consists of more than three constituents. Note also that, since the enthalpy of a perfect gas depends only upon its temperature (see Art. 38), the enthalpy H of a mixture of perfect gases is equal to the sum of the enthalpies of its constituents when the constituents are at the temperature of the mixture, regardless of what their pressures may be.

ILLUSTRATIVE PROBLEM

442. A mixture consists of 2 lb of N_2 and 3 lb of CO_2. Assuming that these constituents behave as perfect gases, calculate the change in the internal energy and in the enthalpy of the mixture when its temperature rises from 70° to 150°F.

Solution. The change ΔU in the internal energy of the mixture is the sum of the changes in the internal energies of its constituents; that is,

$$\begin{aligned}\Delta U &= \Delta U_{N_2} + \Delta U_{CO_2}\\ &= m_{N_2}(u_2 - u_1)_{N_2} + m_{CO_2}(u_2 - u_1)_{CO_2}.\end{aligned}$$

If we assume that c_v for each constituent is a constant, we may calculate the change in internal energy per pound of either by Eq. 17*a*:

$$u_2 \quad u_1 = c_v(t_2 - t_1). \tag{17a}$$

Therefore, using the values 0.178 and 0.165 Btu per lb deg F as the values of c_v for the N_2 and for the CO_2 respectively (from Table IV), we obtain

$$\begin{aligned}\Delta U &= 2 \times 0.178(150 - 70) + 3 \times 0.165(150 - 70)\\ &= 68.1 \text{ Btu}.\end{aligned}$$

Similarly, the change ΔH in the enthalpy of the mixture is

$$\begin{aligned}\Delta H &= \Delta H_{N_2} + \Delta H_{CO_2}\\ &= m_{N_2}(h_2 - h_1)_{N_2} + m_{CO_2}(h_2 - h_1)_{CO_2}.\end{aligned}$$

Assuming that the values of c_p for the N_2 and for the CO_2 are each constant and equal to 0.249 and 0.210 Btu per lb deg F respectively (Table IV), we obtain, by making use of Eq. 18*a*,

$$\begin{aligned}\Delta H &= 2 \times 0.249(150 - 70) + 3 \times 0.210(150 - 70)\\ &= 90.2 \text{ Btu}.\end{aligned}$$

PROBLEMS

443. If a mixture consisting of 0.3 lb of H_2 and 2.2 lb of N_2 is heated at constant volume, how much heat is required to raise its temperature from 60° to 240°F?

444. Suppose that the mixture in the preceding problem undergoes a non-flow process during which its pressure remains constant at 15 psia. If the temperature of the mixture rises from 60° to 240°F during the process, (*a*) how much heat is added to the mixture, and (*b*) how much work does it do?

Ans. (*a*) 286.5 Btu; (*b*) 81.5 Btu.

445. A 12-cu-ft tank contains a mixture of O_2 and CO_2 at 20 psia and 80°F. If adding 30 Btu of heat to the mixture raises its temperature to 200°F, how many pounds of oxygen and how many pounds of carbon dioxide does the tank contain?

446. A mixture consisting of 90% oxygen and 10% hydrogen by weight enters a long pipe at the rate of 200 cfm. The mixture enters at 18 psia and 150°F and leaves at 15 psia and 80°F. Determine (*a*) the rate at which the mixture loses heat as it flows through the pipe, and (*b*) the volume of the mixture leaving the pipe per minute. Assume that the velocities of the mixture entering and leaving the pipe are low. *Ans.* (*a*) 269 Btu per min; (*b*) 212 cfm.

447. A rigid container is divided into two compartments by a partition. One compartment contains 6 cu ft of CO_2 at 30 psia and 200°F, and the other compartment contains 4 cu ft of N_2 at 15 psia and 100°F. If the partition is removed, and if the two gases are allowed to mix adiabatically, what will be the pressure and temperature of the mixture after equilibrium is reached?

138. Mixtures of Air and Water Vapor.

As explained at the beginning of the chapter, the atmosphere consists of nitrogen, oxygen, argon, traces of certain other gases, and water vapor. Whereas the amount of water vapor present varies over a wide range, the other constituents are present always in approximately constant proportions. For this reason they may be regarded as a single gas. We shall call them **air** and shall assume that both they and the water vapor behave as perfect gases.

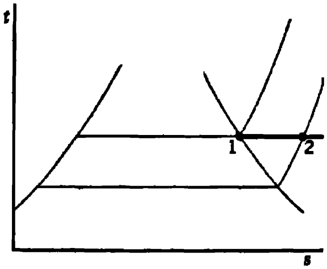

FIG. 137.

Let us suppose that a mixture of air and water vapor is confined inside a rigid container. Now, if by some means we remove all the air from the container, and if we keep the remaining water vapor at the temperature of the original mixture, we shall find that the water vapor is either saturated or superheated. Thus, its state might be that represented by point 1 in Fig. 137, or it might be that represented by point 2. If the vapor is saturated, the original air-water vapor mixture is said to be a **saturated mixture.** We shall also call it **saturated air** (although it is the water vapor and not the air that is saturated). If the vapor is superheated, we shall call the original mixture **unsaturated air.**

Let us consider next how the density of the water vapor in saturated air compares with the density of the water vapor in unsaturated air if both mixtures are at the same temperature. Let us use the symbols ρ_s and p_s to denote the density and partial pressure of the water vapor in saturated air and the symbols ρ_H and p_H to denote the density and partial pressure of the water vapor in unsaturated air. Also, let us use the symbol R_H (= 85.8 ft per deg F) to denote the gas constant for water vapor. Introducing these symbols into Eq. 90*a* we obtain

$$\rho_s = \frac{p_s}{R_H T}$$

and

$$\rho_H = \frac{p_H}{R_H T}.$$

It is evident from these equations that *at any given temperature* the density of the water vapor is directly proportional to its partial pressure. Therefore, since the vapor pressure for saturated vapor (point 1 in Fig. 137) is higher than the pressure for superheated vapor at the same temperature (point 2), the density of the water vapor in a mixture at any given temperature is a maximum if the mixture is saturated.

Suppose, for example, that a mixture of air and water vapor is at a temperature of 70°F. If the mixture is saturated, the partial pressure p_s of the water vapor is equal to the saturation pressure of water at 70°F (= 0.3631 psia, from Table 1 of the *Steam Tables*). Therefore, the density ρ_s of the water vapor in the mixture is

$$\rho_s \quad \frac{0.3631 \times 144}{85.8 \times 530}$$

$$= 0.00115 \text{ lb per cu ft of mixture.}$$

If, on the other hand, the water vapor is superheated, its partial pressure p_H will be some value less than 0.3631 psia. It might, for instance, be 0.250 psia. The density ρ_H of the water vapor in the mixture will then be

$$\rho_H = \frac{0.250 \times 144}{85.8 \times 530}$$

$$= 0.00079 \text{ lb per cu ft of mixture.}$$

Now let us consider how the internal energy and the enthalpy of the water vapor in an air-water vapor mixture can be determined. Since the water vapor in such a mixture is always at a low pressure, and since at low pressures it behaves nearly like a perfect gas, its internal energy and its enthalpy must depend, at least approximately, upon its temperature only. That this is true may be verified by examining the constant temperature lines for superheated steam on the Mollier diagram in the back of the *Steam Tables*. The 60°, 80°, and 100°F lines, for example, coincide very nearly with the constant enthalpy lines. Hence, *the enthalpy h of superheated water vapor at low pressures is approximately* *equal to the enthalpy h_g of saturated water vapor at the same temperature.* To illustrate, the enthalpy of superheated water vapor at 70°F and 0.250 psia (or at any other pressure less than the saturation pressure of 0.3631 psia at 70°F) is approximately equal to 1092.3 Btu per lb ($= h_g$ at 70°F). Similarly, *the internal energy u of superheated water vapor at low pressures is approximately equal to the internal energy u_g of saturated water vapor at the same temperature.*

Whereas approximate values of u and h for superheated water vapor at low pressures can usually be determined most easily by finding the values of u_g and h_g listed in the *Steam Tables*, it is sometimes convenient to have them expressed as a function of the temperature. The following two equations reproduce the values of u_g and h_g listed in the *Steam Tables* to within an accuracy of ±0.3 Btu per lb in the temperature range 0° to 150°F:

$$u = 1011.4 + 0.32t, \tag{94a}$$

and

$$h = 1061.9 + 0.43t, \tag{94b}$$

where the units of u and h are British thermal units per pound and the temperature t is in degrees Fahrenheit. It is sometimes convenient also to know that the internal energy and the enthalpy of *water* at low temperatures are nearly equal and can be determined approximately by the following simple equation:

$$u = h = t - 32. \tag{94c}$$

ILLUSTRATIVE PROBLEM

448. Determine the density ρ_s of the water vapor and the density ρ_a of the air in saturated air at 14.20 psia and 80°F. Also determine the density of the mixture.

Solution. From Table 1 of the *Steam Tables*, the partial pressure p_s of saturated water vapor at 80°F is 0.5069 psia. Therefore,

$$\rho_s = \frac{0.5069 \times 144}{85.8 \times 540}$$

$$= 0.00158 \text{ lb vapor per cu ft of mixture.}$$

This value can also be obtained by finding the reciprocal of v_g listed in the *Steam Tables*. Thus,

$$\rho_s = \frac{1}{v_g} = \frac{1}{633.1} = 0.00158 \text{ lb vapor per cu ft of mixture.}$$

The partial pressure p_a of the air, by Eq. 89, is equal to the difference between the pressure of the mixture and the partial pressure of the water vapor. Hence,

$$\rho_a = \frac{(14.20 - 0.5069) \times 144}{53.3 \times 540}$$

$$= 0.0685 \text{ lb air per cu ft of mixture.}$$

The density ρ of the mixture, by Eq. 91, is equal to the sum of the densities of its constituents. Therefore,

$$\rho = \rho_a + \rho_s$$

$$= 0.0685 + 0.00158$$

$$= 0.0701 \text{ lb per cu ft.}$$

PROBLEMS

449. A saturated mixture of air and water vapor is at a temperature of 90°F. Determine the density of the mixture if the mixture is at a pressure of (*a*) 14.70 psia, and (*b*) 14.20 psia.

450. A tank contains 1.5 lb of dry air at 14.50 psia and 75°F. (*a*) How much water vapor must be introduced to produce a saturated mixture at 75°F? (*b*) What will be the pressure of the mixture? *Ans.* (*a*) 0.0276 lb; (*b*) 14.93 psia.

451. (*a*) How much water vapor must be added to 1.5 lb of dry air to produce a saturated mixture at 14.50 psia and 75°F? (*b*) What will be the volume of the mixture?

452. The density of a mixture of air and water vapor at 14.70 psia and 80°F is 0.0730 lb per cu ft. Determine the density of the water vapor in the mixture. *Ans.* 0.00090 lb per cu ft.

453. A mixture of air and water vapor initially at 14.70 psia and 100°F is heated at constant volume. If the mixture is initially saturated, and if it is heated to 150°F, how much heat is added to each pound?

454. If the initial mixture in the preceding problem were cooled at constant volume, it would remain saturated, but some of the water vapor present would condense. How much heat would have to be absorbed from 1 lb of the mixture to cool it to 50°F? Assume that the condensate would also be cooled to 50°F. (*Note:* The temperature of the mixture is changed by 50 degrees in both this and the preceding problem. Compare the quantities of heat transferred.) *Ans.* 41.7 Btu.

455. A mixture of air and water vapor initially at 14.50 psia and 85°F is heated at constant volume. If the mixture is initially saturated, and if 5.0 Btu of heat are added to each pound, what will be the final temperature?

456. An air-water vapor mixture initially at 15.00 psia and 80°F undergoes a non-flow process during which its pressure remains constant. If the mixture is initially saturated, and if in its final state its temperature is 120°F, how much heat is added to each pound? *Ans.* 9.78 Btu.

457. If the initial mixture in the preceding problem is cooled at constant pressure to 40°F, how much heat must be absorbed from each pound? Assume that the final temperature of the condensate is 40°F. Compare your answer with that for the preceding problem.

458. An air-water vapor mixture initially at 14.5 psia and 90°F is composed of 0.050 lb of water vapor and 2.00 lb of air. If the mixture undergoes a constant pressure, non-flow process during which it absorbs 15.0 Btu of heat, what is its final temperature? *Ans.* 119.8°F.

459. A small pressure cooker is equipped with a safety valve which opens when the pressure in the cooker reaches 15 psi gage. If the cooker initially contains water at 70°F plus saturated air at 14.50 psia and 70°F, what will be the temperature in the cooker when the safety valve first opens? Assume that the volume of the water remains constant during the process.

139. Specific Humidity (Humidity Ratio).

The composition of an air-water vapor mixture can be described by specifying the weight of air and of water vapor present in a cubic foot of the mixture (that is, by specifying the densities ρ_a and ρ_H of the air and water vapor). A more convenient description, however, is the ratio of the pounds of vapor per pound of air. This ratio is called the **specific humidity,** or **humidity ratio,** of the mixture. It will be denoted by the symbol w; that is,

$$w = \frac{m_H}{m_a}, \tag{95}$$

where m_H denotes the pounds of water vapor, and m_a the pounds of air in the mixture.

A relation among the specific humidity w, the partial pressure p_H of the water vapor, and the partial pressure p_a of the air in an air-water vapor mixture can be easily derived. Writing Eq. 90 first for the water vapor and then for the air, we get

$$p_H V = m_H R_H T,$$

and

$$p_a V = m_a R_a T,$$

where R_a denotes the gas constant for air (= 53.3 ft per deg F) and the other symbols have the meanings previously stated. Dividing the first equation by the second, we obtain

$$\frac{p_H}{p_a} = w\frac{R_H}{R_a}. \tag{96}$$

We may write an alternative form of this equation by making use of the fact that, by Eq. 89, $p = p_a + p_H$. Thus,

$$\frac{p_H}{p - p_H} = w\frac{R_H}{R_a}. \tag{96a}$$

Or, if the mixture is saturated, we may write the last equation in the form

$$\frac{p_s}{p - p_s} = w_s\frac{R_H}{R_a}, \tag{96b}$$

where w_s denotes the specific humidity of the saturated mixture.

To determine the specific humidity w_s of *saturated* air, we need only to know the pressure and the temperature of the mixture. For saturated air at 14.70 psia and 70°F, for example, the partial pressure p_s of the saturated water vapor is 0.3631 psia (from Table 1 of the *Steam Tables*); and therefore, by Eq. 96*b*,

$$\frac{0.3631}{14.70 - 0.3631} = w_s \times \frac{85.8}{53.3},$$

whence

$$w_s = 0.0157 \text{ lb vapor per lb air.}$$

The specific humidity w_s of saturated air at 14.70 psia and any other temperature can be determined in a similar manner. The results of such calculations for the temperature range 0° to 100°F are shown in Fig. 138. Note that the value of w_s (at any given pressure p) decreases as the temperature of the mixture decreases.

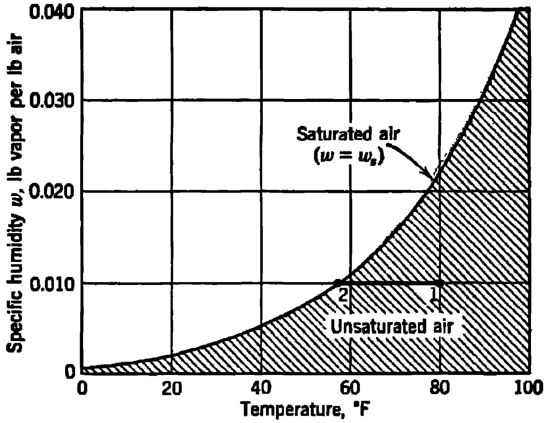

FIG. 138.

It was explained in Art. 138 that, for an air-water vapor mixture *at any given temperature*, the partial pressure p_H of the water vapor has its maximum possible value when the air is saturated (see Fig. 137). Therefore, according to Eq. 96*a*, the specific humidity w of a mixture *at any given pressure and temperature* is a maximum if the mixture is saturated (that is, the maximum possible value of w is w_s); and the points representing possible values of w for mixtures at a pressure of 14.70 psia and temperatures between 0° and 100°F all fall in the shaded portion of Fig. 138.

Whereas we can determine the specific humidity w_s of saturated air if we know only the pressure and temperature of the mixture, these two properties do not permit us to determine the specific humidity w of unsaturated air. We shall consider in the next two articles methods by which the specific humidity of such mixtures can be determined.

PROBLEMS

460. Determine the specific humidity of saturated air at a pressure of 14.20 psia and a temperature of (*a*) 100°F, (*b*) 50°F, and (*c*) 0°F.

461. Determine the specific humidity of saturated air at a temperature of 70°F and a pressure of (*a*) 15.00 psia, (*b*) 12.50 psia, and (*c*) 10.00 psia.

462. A mixture of air and water vapor at 14.60 psia and 80°F has a specific humidity of 0.018 lb of vapor per pound of air. Determine the density of the mixture.

Ans. 0.0723 lb per cu ft.

463. A saturated mixture of air and water vapor is at a pressure of 14.60 psia and has a specific humidity of 0.016 lb of vapor per pound of air. Determine the density of the mixture.

464. An air-water vapor mixture at 14.2 psia and 85°F has a specific humidity of 0.019 lb of vapor per pound of air. If the mixture is compressed isothermally, at what pressure will it become saturated? *Ans.* 20.1 psia.

465. A mixture of air and water vapor at 14.70 psia and 90°F has a density of 0.0715 lb per cu ft. Determine the specific humidity of the mixture.

Ans. 0.0165 lb vapor per lb air.

140. The Dew Point. As explained in the preceding article, the shaded portion of Fig. 138 includes all points representing possible values of the specific humidity w for air-water vapor mixtures at a pressure of 14.7 psia and temperatures between 0° and 100°F. Similar figures can be drawn for mixtures at any other pressure. Since the values of w_s, which form the upper boundaries of the shaded portions of these figures, decrease as the temperature decreases, it follows that cooling any unsaturated mixture at constant pressure and constant specific humidity will cause the mixture to reach a temperature at which it becomes saturated. This temperature is called the dew point of the mixture; that is, *the dew point of an air-water vapor mixture is the temperature at which the mixture becomes saturated if it is cooled at constant pressure and constant specific humidity.*

Consider, for example, a mixture at 14.70 psia and 80°F having a specific humidity of 0.0100 lb of vapor per pound of air. Its state is represented by point 1 in Fig. 138. If the mixture is cooled at constant pressure and constant specific humidity, it will reach the state represented by point 2. At state 2 the mixture will be saturated, and its temperature (= 57.3°F) will be the dew point of the original mixture.

It is evident that the specific humidity w of any mixture of air and water vapor can be found simply by determining the dew point of the mixture. Thus, if its dew point is known, its specific humidity can be calculated by Eq. 96*b* because at its dew point the mixture is saturated. Suppose, for example, that a mixture at 14.20 psia and 70°F has a dew point of 50°F. The value of p_s at 50°F is 0.17811 psia, and therefore, by Eq. 96*b*,

$$\frac{0.17811}{14.20 - 0.17811} = w_s \frac{85.8}{53.3},$$

whence

$$w_s = 0.0079 \text{ lb vapor per lb air.}$$

This is also the specific humidity of the mixture at 14.20 psia and 70°F.

Experimentally, one way to determine the dew point of an air-water vapor mixture is to bring the mixture into contact with a polished metal surface whose temperature can be both controlled and measured. If the metal surface is slowly cooled, the portion of the mixture in contact with it will also be cooled; and, when this portion reaches its dew point, the metal surface will become fogged. Therefore, by measuring the temperature of the surface when the fog first appears, we can determine the dew point of the mixture. The reason for having the surface polished is to make the appearance of the fog readily visible.

It may be noted that the dew point of an air-water vapor mixture can be calculated if the pressure and specific humidity of the mixture are known. The partial pressure p_s of the water vapor at the dew point is first determined by Eq. 96*b*; and the dew point, which is equal to the saturation temperature corresponding to this pressure, is then found by means of Table 1 in the *Steam Tables*. Suppose, for example, that a mixture at 14.20 psia has a specific humidity w of 0.0100 lb of vapor per pound of air. The partial pressure p_s of its water vapor at the dew point may be calculated by Eq. 96*b*:

$$\frac{p_s}{14.20 - p_s} = 0.0100 \times \frac{85.8}{53.3},$$

whence

$$p_s = 0.225 \text{ psia.}$$

The dew point of the mixture, therefore, is 56.4°F.

PROBLEMS

466. A mixture of air and water vapor at 14.50 psia and 80°F has a dew point of 65°F. Determine (*a*) the specific humidity, and (*b*) the density of the mixture.

467. An air-water vapor mixture at 14.70 psia and 75°F has a specific humidity of 0.0115 lb of vapor per pound of air. Determine the dew point of the mixture.
Ans. 61.3°F.

468. A mixture of air and water vapor at 13.50 psia and 70°F has a dew point of 60°F. If the pressure of the mixture is raised to 14.50 psia, what does its dew point become?

469. An air-water vapor mixture at 14.70 psia and 85°F has a specific volume of 13.85 cu ft per lb. Determine the dew point of the mixture. *Ans.* 69.4°F.

470. A stream of air and water vapor at 14.70 psia and 70°F is flowing at the rate of 200 cfm. A second stream at 14.70 psia and 120°F is flowing at the rate of 100 cfm. If the dew point of the first stream is 60°F and that of the second is 90°F, what will be the dew point of a stream formed by mixing the two streams? Assume that the final mixture is also at 14.70 psia.

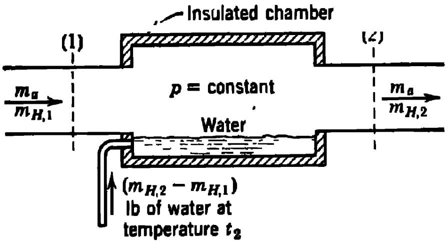

FIG. 139.

141. The Adiabatic Saturation Temperature and the Wet-Bulb Temperature. The specific humidity of an air-water vapor mixture can be determined experimentally by finding either the dew point of the mixture, as explained in the preceding article, or its so-called *adiabatic saturation temperature*. To define the latter term, let us consider the apparatus shown in Fig. 139. It consists of an insulated chamber containing some water. The mixture whose adiabatic saturation temperature is to be determined flows through the chamber at constant pressure and comes into contact with the water. The area of contact is large enough so that the mixture leaves the apparatus *saturated*. Since this process takes place *adiabatically*, the energy needed to evaporate the water must come from the mixture. Consequently, the temperature of the saturated mixture leaving the apparatus is lower than that of the unsaturated mixture entering. The temperature at which the saturated mixture leaves after the flow has continued long enough for the temperature to reach a constant value is called the **adiabatic saturation temperature** of the entering mixture.

Since the *adiabatic saturation process* described in the preceding paragraph takes place in a steady-flow manner, we can derive an equation for determining the specific humidity w_1 of the entering mixture by applying the steady-flow energy equation (Eq. 14) to the process. First, we note that, to have steady flow, water must be supplied to the apparatus at a rate equal to its rate of evaporation within the apparatus.

We shall assume that this water is supplied at a temperature equal to the adiabatic saturation temperature of the entering mixture. Next, let us denote the rate at which air enters and leaves the apparatus by m_a, the rate at which water vapor enters by $m_{H,1}$, the rate at which water vapor leaves by $m_{H,2}$, the temperature of the entering mixture by t_1, and the temperature of the saturated mixture leaving (that is, the adiabatic saturation temperature of the entering mixture) by t_2. The rate at which water must be supplied to the apparatus is evidently equal to $m_{H,2} - m_{H,1}$, and the temperature of this water is t_2.

Now, assuming that the velocity of each stream entering and leaving the apparatus is negligible, and noting that $\bar{Q}$ and $\bar{W}'$ are zero, we see from Eq. 14 that the sum of the enthalpies of the fluids entering the apparatus per unit time is equal to the sum of the enthalpies of the fluids leaving per unit time. Hence, we may write

$$m_a h_{a,1} + m_{H,1} h_{H,1} + (m_{H,2} - m_{H,1}) h_{f,2} = m_a h_{a,2} + m_{H,2} h_{H,2},$$

where $h_{a,1}$ and $h_{a,2}$ denote the enthalpies of each pound of air entering and leaving, $h_{H,1}$ and $h_{H,2}$ denote the enthalpies of each pound of water vapor entering and leaving, and $h_{f,2}$ denotes the enthalpy of a pound of water at temperature t_2. Assuming that the specific heat c_p of air is constant and equal to 0.241 Btu per lb deg F (Table IV), by Eq. 18*a* we may write

$$h_{a,1} - h_{a,2} = 0.241(t_1 - t_2).$$

Also, as explained in Art. 138, we may assume that $h_{H,1} = h_{g,1}$ and $h_{H,2} = h_{g,2}$, where $h_{g,1}$ and $h_{g,2}$ denote the enthalpies per pound of saturated water vapor at temperatures t_1 and t_2. Making use of these relations, we may write the foregoing steady-flow equation in the form

$$m_{H,1} h_{g,1} + (m_{H,2} - m_{H,1}) h_{f,2} = m_{H,2} h_{g,2} - m_a \times 0.241(t_1 - t_2).$$

Or, noting that the specific humidity w_1 of the entering mixture is equal to $m_{H,1}/m_a$, and that the specific humidity $w_{s,2}$ of the saturated mixture leaving is equal to $m_{H,2}/m_a$, we may write the last equation in the form

$$w_1 h_{g,1} + (w_{s,2} - w_1) h_{f,2} = w_{s,2} h_{g,2} - 0.241(t_1 - t_2).$$

Finally, since $h_{g,2} - h_{f,2} = h_{fg,2}$, we obtain

$$w_1 = \frac{w_{s,2} h_{fg,2} - 0.241(t_1 - t_2)}{h_{g,1} - h_{f,2}}, \tag{97}$$

where w_1 = the specific humidity of the mixture entering the apparatus, and

$w_{s,2}$ = the specific humidity of the *saturated* mixture leaving.

Note that $h_{g,1}$ must be evaluated at the temperature t_1, and $h_{f,2}$ and $h_{fg,2}$ at the adiabatic saturation temperature t_2.

To illustrate how we can determine the specific humidity of an air-water vapor mixture by Eq. 97 if we know the pressure, temperature, and adiabatic saturation temperature of the mixture, let us suppose that a mixture at 14.70 psia and 70°F has an adiabatic saturation temperature of 60°F. We can determine the specific humidity $w_{s,2}$ of saturated air at 60°F by Eq. 96*b*. Since $p_s = 0.2563$ psia for saturated vapor at 60°F (from Table 1 of the *Steam Tables*), Eq. 96*b* becomes

$$\frac{0.2563}{14.70 - 0.2563} = w_{s,2} \times \frac{85.8}{53.3},$$

whence

$$w_{s,2} = 0.0110 \text{ lb vapor per lb air.}$$

Now, taking the value of h_g at 70°F and the values of h_f and h_{fg} at 60°F from Table 1 of the *Steam Tables*, we may write Eq. 97 as follows:

$$w_1 = \frac{0.0110 \times 1059.9 - 0.241(70 - 60)}{1092.3 - 28.06}$$

$$= 0.0087 \text{ lb vapor per lb air.}$$

This is the specific humidity of the mixture.

Note that for a *saturated* mixture of air and water vapor $t_1 = t_2$, and that for such a mixture Eq. 97 reduces to $w_1 = w_2$. Note also that the adiabatic saturation temperature of an *unsaturated* mixture is higher than the dew point of the mixture. That this is true can be seen as follows: An unsaturated mixture can be caused to become saturated either by cooling it to its dew point or by making it undergo an adiabatic saturation process. Its specific humidity remains constant in the first process but increases in the second. Therefore, since the temperature of a saturated mixture having a high specific humidity is higher than the temperature of one having a lower specific humidity (see Fig. 138), the adiabatic saturation temperature of an unsaturated mixture is higher than the dew point of the mixture.

Finally, let us consider the so-called **wet-bulb temperature** of an air-water vapor mixture. This is the temperature a thermometer will indicate if its bulb is covered by a wick which has been dipped in water. It is easily determined since it is necessary only that the wick be wet and that the thermometer be placed in a stream of the mixture.

The wet-bulb temperature of unsaturated air is always lower than the true temperature of the air (frequently called its **dry-bulb temperature**) because the wet-bulb thermometer is cooled by the evaporation of water from the wick. Note that the wet-bulb thermometer reaches a state of equilibrium when the rate at which it receives heat from the warmer air surrounding it is equal to the rate at which it supplies heat to the water evaporating from the wick. Note also that the wet-bulb and dry-bulb temperatures of saturated air are the same because no evaporation takes place.

The wet-bulb temperature is of interest to us because, *for mixtures of air and water vapor, the wet-bulb temperature and the adiabatic saturation temperature are almost identical.* Therefore, we can determine the specific humidity of a mixture by Eq. 97 if we know the pressure, dry-bulb temperature, and wet-bulb temperature of the mixture. In effect, we determine the adiabatic saturation temperature of the mixture, not by causing it to undergo an adiabatic saturation process, but simply by measuring its wet-bulb temperature.

PROBLEMS

471. An air-water vapor mixture at 14.30 psia has a dry-bulb temperature of 81°F and a wet-bulb temperature of 75°F. Determine (*a*) the specific humidity, and (*b*) the density of the mixture.

472. If the dry-bulb temperature of an air-water vapor mixture at 14.50 psia is 72°F, and if the wet-bulb temperature is 65°F, what is the dew point of the mixture? *Ans.* 61.3°F.

473. Air at 14.40 psia and 250°F is supplied to a laundry drier at the rate of 100 cfm. If the air has a wet-bulb temperature of 100°F, if it leaves the drier saturated, and if no heat is lost from the drier, how many pounds of water are evaporated per minute? *Ans.* 0.192 lb per min.

474. Water at a temperature of 125°F enters a cooling tower at the rate of 5000 lb per min. Part of it evaporates within the tower, and the remainder leaves at 90°F. If the air entering the tower has a dry-bulb temperature of 80°F and a wet-bulb temperature of 68°F, and if the air leaving is at 100°F and is saturated, how many cubic feet of air enter the tower per minute? Assume that the atmospheric pressure is 14.70 psia and that the tower operates adiabatically. (*Suggestion:* First determine by the steady-flow energy equation the pounds of dry air flowing through the tower per minute.)

142. Relative Humidity and Percentage Saturation.

An air-water vapor mixture can be described by specifying its pressure, temperature, and specific humidity. It can also be described by specifying its pressure, temperature, and either its *relative humidity* or its *percentage saturation.*

Relative humidity is defined as the ratio of the density ρ_H of the water vapor in the mixture to the density ρ_s of the water vapor *in a saturated mixture at the same temperature;* that is,

$$\phi = \frac{\rho_H}{\rho_s}, \tag{98}$$

where ϕ denotes the relative humidity of the mixture. By making use of the expressions for ρ_H and ρ_s stated in Art. 138, we can derive an alternative expression for ϕ in terms of the partial pressures p_H and p_s of the water vapor. Thus, since

$$\rho_H = \frac{p_H}{R_H T} \quad \text{and} \quad \rho_s = \frac{p_s}{R_H T},$$

we may write

$$\phi = \frac{p_H}{p_s}, \tag{98a}$$

where p_H is the partial pressure of the water vapor in the mixture and p_s is the saturation pressure of water vapor *at the temperature of the mixture.*

Percentage saturation is defined as the ratio of the specific humidity w of the mixture to the specific humidity w_s of a saturated mixture at the same temperature; * that is,

$$\mu = \frac{w}{w_s}, \tag{99}$$

where μ denotes the percentage saturation of the mixture.

It is of interest to see how the relative humidity and percentage saturation are related. To develop this relation, we note first that, according to Eq. 96*a*,

$$w = \frac{p_H}{(p - p_H)} \times \frac{R_a}{R_H},$$

and, according to Eq. 96*b*,

$$w_s = \frac{p_s}{(p - p_s)} \times \frac{R_a}{R_H}.$$

Dividing the first equation by the second, we obtain

$$\frac{w}{w_s} = \frac{p_H}{p_s} \times \frac{(p - p_s)}{(p - p_H)},$$

whence

$$\mu = \phi\left(\frac{p - p_s}{p - p_H}\right). \tag{99a}$$

Since p_H is always less p_s, the percentage saturation μ is always smaller than the relative humidity ϕ. However, since under ordinary conditions both p_H and p_s are small compared to the pressure p, the values of μ and ϕ are usually nearly equal. This is true especially if the mixture is at a low temperature (since both p_s and p_H are then quite small) or if the mixture is nearly saturated (since p_s and p_H are then nearly equal).

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* This ratio is also called the **degree of saturation,** the **percentage humidity,** and the **saturation ratio.**

It should be noted that, if the pressure, temperature, and specific humidity w of a mixture are known, its relative humidity ϕ and percentage saturation μ can be calculated. Suppose, for example, that a mixture at 14.70 psia and 70°F has a specific humidity of 0.0100 lb of vapor per pound of air. The partial pressure p_H of the water vapor can be calculated by Eq. 96*a*:

$$\frac{p_H}{14.70 - p_H} = 0.0100 \times \frac{85.8}{53.3},$$

whence

$$p_H = 0.233 \text{ psia}.$$

Then, since $p_s = 0.3631$ psia for saturated water vapor at 70°F (from Table 1 of the *Steam Tables*),

$$\phi = \frac{0.233}{0.3631}$$

$$= 0.642 \text{ (or } 64.2\%\text{)}.$$

To determine the percentage saturation μ of the mixture, we must first calculate the specific humidity w_s of saturated air at 70°F by Eq. 96*b*. Since $p_s = 0.3631$ psia at 70°F,

$$\frac{0.3631}{14.70 - 0.3631} = w_s \times \frac{85.8}{53.3},$$

whence

$$w_s = 0.0157 \text{ lb vapor per lb air},$$

Therefore,

$$\mu = \frac{0.0100}{0.0157}$$

$$= 0.636 \text{ (or } 63.6\%\text{)}.$$

It should also be noted that, if the pressure, temperature, and either the relative humidity ϕ or the percentage saturation μ of a mixture are known, the specific humidity w of the mixture can be calculated. Suppose, for example, that a mixture at 14.70 psia and 70°F has a relative humidity of 50%. The partial pressure p_H of the water vapor can be calculated by Eq. 98*a*. Thus, since $p_s = 0.3631$ psia at 70°F,

$$0.50 = \frac{p_H}{0.3631},$$

or

$$p_H = 0.182 \text{ psia.}$$

Then, by Eq. 96*a*,

$$\frac{0.182}{14.70 - 0.182} = w \times \frac{85.8}{53.3},$$

whence

$$w = 0.0078 \text{ lb vapor per lb air.}$$

Or, if the mixture at 14.70 psia and 70°F has a percentage saturation μ of 50%, the specific humidity w_s of a saturated mixture at 70°F must be calculated by Eq. 96*b*:

$$\frac{0.3631}{14.70 - 0.3631} = w_s \times \frac{85.8}{53.3},$$

whence

$$w_s = 0.0157 \text{ lb vapor per lb air.}$$

Therefore, by Eq. 99,

$$w = 0.50 \times 0.0157$$
$$= 0.0078 \text{ lb vapor per lb air.}$$

A problem that frequently arises in air-conditioning work is the determination of the weight of air m_a and the weight of water vapor m_H in a given volume of mixture when the pressure, the temperature, and either the relative humidity ϕ or the percentage saturation μ of the mixture are known. Suppose, for example, that we wish to determine the weight of air and of water vapor in 1000 cu ft of a mixture which is at 14.70 psia and 70°F and has a relative humidity of 50%. We can determine the partial pressure p_H of the water vapor by Eq. 98*a*. Since $p_s = 0.3631$ psia at 70°F,

$$p_H = 0.50 \times 0.3631$$
$$= 0.182 \text{ psia.}$$

Then to determine m_H, we may write Eq. 90 in the form

$$p_H V = m_H R_H T,$$

or

$$0.182 \times 144 \times 1000 = m_H \times 85.8 \times 530,$$

whence

$$m_H = 0.576 \text{ lb water vapor.}$$

To determine m_a, we note that, by Eq. 89, $p_a = p - p_H$. Hence, we may also write Eq. 90 in the form

$$(p - p_H)V = m_a R_a T,$$

or

$$(14.70 - 0.182) \times 144 \times 1000 = m_a \times 53.3 \times 530,$$

whence

$$m_a = 74.0 \text{ lb air.}$$

If, instead of the relative humidity ϕ, we have given the percentage saturation μ, we must first calculate the specific humidity w of the mixture by the method illustrated in the preceding paragraph. Then, knowing the value of w, we can calculate the partial pressure p_H of the water vapor by Eq. 96*a*. Finally, having determined the value of p_H, we can calculate m_H and m_a by Eq. 90, as illustrated above.

PROBLEMS

475. An air-water vapor mixture at 14.20 psia and 75°F has a specific humidity of 0.0085 lb of vapor per pound of air. Determine (*a*) the relative humidity, and (*b*) the percentage saturation of the mixture. *Ans.* (*a*) 44.5%; (*b*) 43.8%.

476. A thousand cubic feet of an air-water vapor mixture at 14.40 psia and 80°F has a relative humidity of 60%. Determine (*a*) the specific humidity, (*b*) the weight of air, and (*c*) the weight of water vapor in the mixture.

477. If the mixture in the preceding problem, instead of having a relative humidity of 60%, had a percentage saturation of 60%, what would be (*a*) the specific humidity, (*b*) the weight of air, and (*c*) the weight of water vapor in the mixture? *Ans.* (*b*) 70.5 lb; (*c*) 0.959 lb.

478. A mixture of air and water vapor at 14.70 psia and 70°F has a dew point of 40°F. Determine (*a*) the relative humidity, and (*b*) the percentage saturation of the mixture.

479. A thousand cubic feet of an air-water vapor mixture at 14.50 psia has a dry-bulb temperature of 85°F and a wet-bulb temperature of 70°F. Determine (*a*) the specific humidity, (*b*) the dew point, (*c*) the relative humidity, (*d*) the percentage saturation, (*e*) the weight of air, and (*f*) the weight of water vapor in the mixture.

480. The specific humidity of an air-water vapor mixture at 14.30 psia is 0.0150 lb of vapor per pound of air. At what temperature will the mixture have a relative humidity of 20%? *Ans.* 119.8°F.

481. The dew point of a mixture of air and water vapor is 55°F. If its pressure is kept constant, to what temperature must the mixture be heated in order for its relative humidity to drop to 40%?

482. Suppose that an air-water vapor mixture having a specific humidity of 0.0100 lb of vapor per pound of air is compressed at a constant temperature of 90°F. At what pressure will the mixture have a relative humidity of 50%? *Ans.* 22.0 psia.

483. Five hundred cubic feet of air at 14.70 psia and 80°F enter a compressor per minute. The air leaves the compressor at 120 psia and cools at constant pressure back to 80°F. If the air has a relative humidity of 56% when it enters the compressor, how many pounds of water are condensed out of it per minute?

143. Psychrometric Charts. The relation among the dry-bulb temperature, specific humidity, relative humidity (or percentage saturation), and wet-bulb temperature of air-water vapor mixtures *at any given pressure* can be shown graphically on a diagram such as Fig. 140. Such diagrams are called **psychrometric charts.** More detailed charts than the one shown have been prepared by various manufacturers of air-conditioning equipment and may be obtained from these companies. Many of these charts include, in addition to the quantities shown in Fig. 140, values of the enthalpy of the mixture.

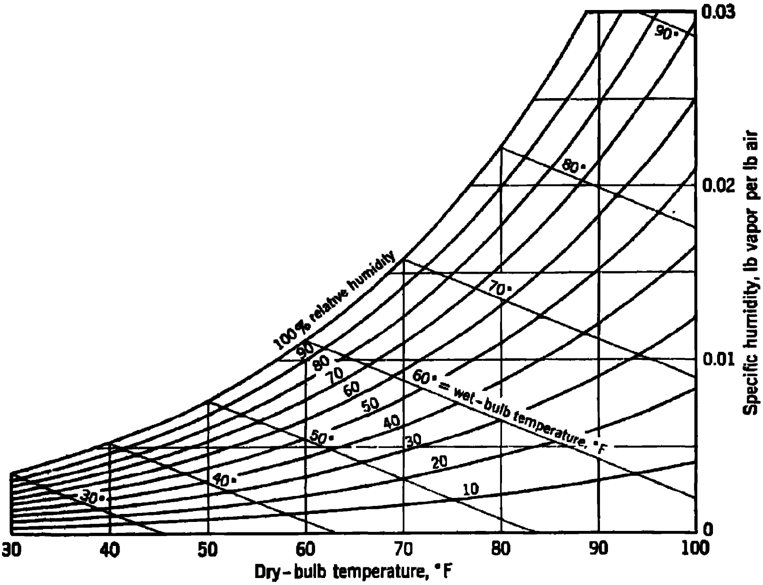


FIG. 140. Psychrometric chart.

To construct an accurate psychrometric chart, it is necessary to determine experimentally the relation among the various quantities. Less accurate charts can be constructed if it is assumed (*a*) that air and water vapor behave as perfect gases, and (*b*) that the wet-bulb temperature and the adiabatic saturation temperature of any mixture are identical. These assumptions permit charts to be constructed by means of the equations developed in the foregoing articles.

Note that Fig. 140 is the same as Fig. 138 except that lines of constant relative humidity and constant wet-bulb temperature have been added. Note also that, as explained in connection with Fig. 138, the dew point of any mixture is the temperature at the intersection of a horizontal line through the point representing the state of the mixture and the 100% relative humidity curve. For example, the dew point of a mixture which is at 14.70 psia and 90°F and has a relative humidity of 50% is approximately 69°F.

PROBLEM

484. Construct a psychrometric chart for a barometric pressure of 28.9 in. Hg abs and for the temperature range 40° to 100°F. Assume (*a*) that air and water vapor behave as perfect gases, and (*b*) that the wet-bulb and adiabatic saturation temperatures are identical. Proceed as follows:

(*a*) Calculate the values of w_s corresponding to dry-bulb temperatures of 40°, 60°, 80°, and 100°F. Plot these values against the dry-bulb temperature.

(*b*) Calculate the values of w corresponding to the following pairs of wet- and dry-bulb temperatures:

Wet-Bulb Temperature, °F	Dry-Bulb Temperature, °F
40	60
60	80
80	100

Plot these values of w against the dry-bulb temperature. Then, *assuming that the constant wet-bulb temperature lines are straight*, draw the 40°, 60°, and 80°F wet-bulb temperature lines. (*Note:* More accurate curves can be drawn if values of w for each of these wet-bulb temperatures and several dry-bulb temperatures are calculated.)

(*c*) Calculate the values of w corresponding to relative humidities of 25%, 50%, and 75% at dry-bulb temperatures of 40°, 60°, 80°, and 100°F. Draw the 25%, 50%, and 75% relative humidity curves.

144. Some Air-Conditioning Processes. Most problems that arise in air-conditioning work are of two kinds: In one kind, a stream of air is changed from one temperature and humidity to some other temperature and humidity. The problem is to find at what rate heat and water vapor must be added to or removed from the stream in order to effect this change. In the second kind, the air within an enclosure is maintained at some specified temperature and humidity while heat and water vapor enter or leave the enclosure. To maintain the specified conditions, a stream of air is removed from the enclosure and is replaced by another stream having a different temperature and humidity. The problem is to determine what the temperature and humidity of the second stream must be. In this article we shall consider how to solve these two kinds of problems. We shall also consider how to find the temperature and humidity of a stream formed by the adiabatic mixing of two streams which are at different temperatures and have different humidities.

Since air-conditioning processes are always carried out in a steady-flow manner, to solve problems involving such processes we shall need to make use of the steady-flow energy equation; and, since more than one kind of fluid is involved, we shall need to use this equation in the form stated in Eq. 14. Because the entering or leaving velocity of each stream is invariably low, we may simplify Eq. 14 somewhat by dropping the kinetic energy terms from it. Then, if the process is one in which heat is supplied to the fluids flowing through the apparatus, we may write the equation in the form

$$\bar{Q}_{\text{in}} + \bar{W}_{\text{in}} = \Sigma(mh)_{\text{out}} - \Sigma(mh)_{\text{in}}, \tag{100a}$$

where $\bar{Q}_{\text{in}}$ and $\bar{W}_{\text{in}}$ denote the *rates* at which heat and work are supplied to the fluids, m denotes the *rate* at which each fluid is entering or leaving the apparatus, and h denotes the enthalpy per pound of the fluid. Or, if the process is one in which heat is absorbed from the fluids, we may write the equation in the form

$$\bar{Q}_{\text{out}} = \bar{W}_{\text{in}} + \Sigma(mh)_{\text{in}} - \Sigma(mh)_{\text{out}}, \tag{100b}$$

where $\bar{Q}_{\text{out}}$ denotes the *rate* at which heat is absorbed. It may be noted that the term $\bar{W}_{\text{in}}$ is frequently zero or at least is negligibly small. Let us now consider some specific air-conditioning processes:

(*a*) *Heating or Cooling at Constant Specific Humidity.* Perhaps the simplest kind of air-conditioning process is one in which a stream of air and water vapor flows without change in composition over a hot or cold surface. Such a process occurs, for example, when an air-water vapor mixture flows over a steam radiator or over tubes through which cold water circulates (provided that the temperature of the water is above the dew point of the mixture). If no fan or blower forces the stream over the hot or cold surface, the rate at which heat is transferred to or from the stream is equal to the difference between the sums of the enthalpies of the entering and leaving streams, as is evident from Eq. 100*a* or 100*b*. The solution of this type of problem is illustrated in Illustrative Problem 485 following this article.

(*b*) *Heating and Humidifying.* A common kind of air-conditioning process is one in which a stream of air and water vapor is both heated and humidified. Such a process occurs, for example, when air flows through an ordinary warm-air furnace equipped with a humidifier (usually simply an open pan of water). The rate at which heat must be supplied to the mixture can be calculated by Eq. 100*a*. Although the term $\bar{W}_{\text{in}}$ is not zero if the furnace is equipped with a fan or blower, it is usually small and may be disregarded.

Another example of the heating and humidifying of a stream of air and water vapor is the process which occurs in summer when the temperature of the air inside a building is kept below the outdoor temperature. The air inside the building is heated by the conduction of heat through the walls of the building, by people within the building, by any electrical equipment operating within the building, etc.; and its humidity is raised by the evaporation of moisture from people, by various drying processes, by cooking, etc. To maintain any desired temperature and humidity, it is necessary to remove continuously some of the air from the building and to supply in its place a stream of air at a lower temperature and lower specific humidity. The temperature and humidity at which this stream must be supplied can be determined by Eq. 100*a* if the rates at which heat and humidity are supplied to the air within the building are known. A trial-and-error solution can be avoided by making use of Eqs. 94*b* and 94*c*, given in Art. 138. Note that the building itself is considered the apparatus through which the steady-flow process takes place. The solution of this type of problem is illustrated in Illustrative Problem 486 following this article.

(*c*) *Cooling and Dehumidifying.* Another air-conditioning process of interest is one in which a stream of air and water vapor is cooled and dehumidified. This process takes place, for example, in air-conditioning plants designed for summer-time air conditioning of buildings. The mixture to be cooled and dehumidified is caused to flow over tubes in which a refrigerant circulates, or to pass through a spray of cold water. The mixture is cooled below its dew point, and therefore some of its water vapor is condensed and can be removed. If the resulting dehumidified mixture is then heated (at constant specific humidity), its relative humidity (or percentage saturation) decreases. Since the amount of water vapor condensed out of the original mixture depends upon the temperature to which the mixture is cooled, a mixture having any desired temperature and humidity can be produced simply by controlling (*a*) the temperature to which the original mixture is cooled, and (*b*) the temperature to which the dehumidified mixture is reheated. The rate at which heat must be removed from the mixture during the cooling process can be calculated by Eq. 100*b*. The rate at which heat must be supplied during the reheating process can be calculated by Eq. 100*a*, as already explained. This type of problem is illustrated in Illustrative Problem 487 following this article.

Cooling and dehumidifying of air also take place in winter when the air within a building is maintained at a temperature above the outdoor temperature. The air is cooled by the conduction of heat through the walls of the building and is dehumidified by the infiltration of outdoor air having a low specific humidity. To maintain any desired temperature and humidity within the building, it is necessary to remove continuously some of the air from the building and to replace it with a stream of air at a higher temperature and higher specific humidity. The temperature and humidity at which this stream must be supplied can be determined by Eq. 100*b* if the rates at which heat and humidity are lost from the building are known. Note that the building itself must again be considered the apparatus through which the steady-flow process is taking place. Note also that a trial-and-error solution can be avoided by means of Eqs. 94*b* and 94*c*.

(*d*) *Adiabatic Mixing of Streams.* A process frequently occurring in air-conditioning installations is the adiabatic mixing of two streams which differ in temperature and in composition. To determine the temperature and composition of the resulting stream, we note that the weights of air and water vapor in this stream are equal to the sums of the weights of air and water vapor in the two initial streams. Therefore, we can find the composition of the final mixture. We can then determine its temperature by applying the steady-flow energy equation to the process. Note that, for an adiabatic mixing process, both Eqs. 100*a* and 100*b* reduce to

$$\Sigma(mh)_{\text{in}} = \Sigma(mh)_{\text{out}}.$$

A trial-and-error solution can again be avoided by means of Eq. 94*b*. The solution of this type of problem is illustrated in Illustrative Problem 488, which follows.

ILLUSTRATIVE PROBLEMS

485. One thousand cubic feet per minute of an air-water vapor mixture at 65°F and 40% relative humidity are heated at a constant pressure of 14.70 psia to a temperature of 120°F. If the process takes place in a steady-flow manner, at what rate must heat be supplied to the mixture? What is the relative humidity of the mixture leaving the heater?

Solution. The weight of air m_a and the weight of water vapor m_H flowing through the heater per minute must first be determined. These quantities can be calculated by Eq. 90 after the partial pressure p_H of the water vapor has been found by Eq. 98*a*. Thus, for the mixture entering the heater,

$$\begin{aligned} p_H &= \phi p_s \qquad (98a) \\ &= 0.40 \times 0.3056 \\ &= 0.122 \text{ psia.} \end{aligned}$$

Therefore, by Eq. 90,

$$0.122 \times 144 \times 1000 = m_H \times 85.8 \times 525,$$

whence

$$m_H = 0.390 \text{ lb per min.}$$

The partial pressure p_a of the air, by Eq. 89, is 14.70 − 0.122 = 14.58 psia. Therefore, by Eq. 90,

$$14.58 \times 144 \times 1000 = m_a \times 53.3 \times 525,$$

whence

$$m_a = 75.0 \text{ lb per min.}$$

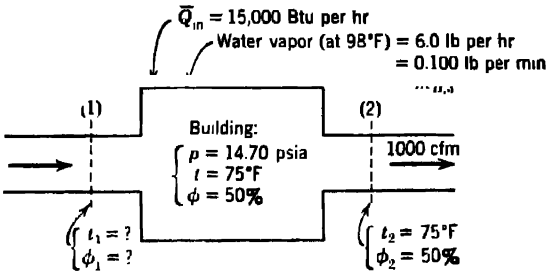
The rate at which heat must be supplied to the mixture can now be calculated by Eq. 100*a*:

$$\begin{aligned}\bar{Q}_{\text{in}} &= (m_a h_{a,2} + m_H h_{H,2}) - (m_a h_{a,1} + m_H h_{H,1})\\ &= m_a(h_{a,2} - h_{a,1}) + m_H(h_{H,2} - h_{H,1})\\ &= 75.0 \times 0.241(120 - 65) + 0.390(1113.7 - 1090.2)\\ &= 1004 \text{ Btu per min.}\end{aligned}$$

Since both the specific humidity and the pressure of the mixture remain constant during the process, it is evident from Eq. 96*a* that the partial pressure p_H of the water vapor also remains constant. Therefore, the relative humidity ϕ_2 of the mixture leaving the heater is, by Eq. 98*a*,

$$\begin{aligned}\phi_2 &= \frac{0.122}{1.6924}\\ &= 0.072 \text{ (or } 7.2\%).\end{aligned}$$

486. The air in a building is to be maintained during the summer at a temperature of 75°F and at a relative humidity of 50%. It is estimated that heat will be supplied to the air at the rate of 15,000 Btu per hr (by conduction through the walls, by people within the building, by electric lights, etc.) and that water vapor will be supplied at the rate of 6.0 lb per hr (by evaporation from people within the building). It is proposed that the desired conditions be maintained by withdrawing air at the rate of 1000 cfm and replacing it with air at a lower temperature and lower specific humidity. What must be the temperature and relative humidity of the air supplied to the building? Assume that the barometric pressure is 14.70 psia and that the water vapor supplied to the building is at 98°F.



Solution. The weight of air m_a and of water vapor $m_{H,2}$ carried out of the building per minute (by the 1000 cfm of mixture leaving at 75°F and 50% relative humidity) must first be determined. By Eq. 98*a*, the partial pressure $p_{H,2}$ of the water vapor in this mixture is

$$\begin{aligned}p_{H,2} &= 0.50 \times 0.4298\\ &= 0.215 \text{ psia.}\end{aligned}$$

Therefore, by Eq. 90,

$$0.215 \times 144 \times 1000 = m_{H,2} \times 85.8 \times 535,$$

whence

$$m_{H,2} = 0.675 \text{ lb per min.}$$

The partial pressure of the air in this mixture is 14.70 − 0.215 = 14.48 psia, and therefore

$$14.48 \times 144 \times 1000 = m_a \times 53.3 \times 535,$$

whence

$$m_a = 73.0 \text{ lb per min.}$$

The mixture supplied to the building must evidently carry air in at the rate of 73.0 lb per min and must carry water vapor in at the rate of 0.675 − 6.0/60 = 0.575 lb per min ($= m_{H,1}$). The temperature t_1 at which this mixture must be supplied can be calculated by Eq. 100*a*:

$$\begin{aligned}\bar{Q}_{\text{in}} &= (m_a h_{a,2} + m_{H,2} h_{H,2}) - (m_a h_{a,1} + m_{H,1} h_{H,1} + m_{H,3} h_{H,3})\\ &= m_a(h_{a,2} - h_{a,1}) + m_{H,2} h_{H,2} - m_{H,1} h_{H,1} - m_{H,3} h_{H,3}.\end{aligned}$$

Since, by Eq. 94*b*,

$$h_{H,1} = 1061.9 + 0.43 t_1,$$

the last equation becomes

$$\begin{aligned}\frac{15{,}000}{60} = 73.0 \times 0.241(75 - t_1) &+ 0.675 \times 1094.5\\ &- 0.575(1061.9 + 0.43 t_1) - 0.100 \times 1104.4,\end{aligned}$$

whence

$$t_1 = 61.0°\text{F}.$$

The relative humidity ϕ_1 of this mixture can be calculated by Eq. 98*a* after the partial pressure $p_{H,1}$ of its water vapor has been found by Eq. 96*a*. Thus,

$$\frac{p_{H,1}}{14.70 - p_{H,1}} = \frac{0.575}{73.0} \times \frac{85.8}{53.3},$$

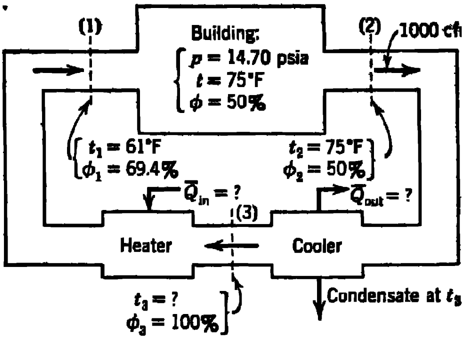
whence

$$p_{H,1} = 0.184 \text{ psia.}$$

Finally,

$$\begin{aligned}\phi_1 &= \frac{0.184}{0.2655}\\ &= 0.694 \text{ (or } 69.4\%).\end{aligned}$$

487. To maintain the conditions desired within the building described in the preceding problem, 1000 cfm of air at a temperature of 75°F and a relative humidity of 50% must be changed into air at a temperature of 61°F and a relative humidity of 69.4%. This can be accomplished by cooling the mixture to a certain temperature t_3 (below the dew point of the mixture) and then reheating it to 61°F, as shown on the accompanying sketch. Assuming that the pressure remains constant at 14.70 psia, and that the mixture leaving the cooler is saturated, find what the temperature t_3 must be. Also, find the rate at which heat must be removed from the mixture flowing through the cooler and the rate at which heat must be supplied to the mixture flowing through the heater.



Solution. Since both the pressure and specific humidity of the stream flowing through the *heater* remain constant, it is evident from Eq. 96*a* that the partial pressure of its water vapor must also remain constant; that is,

$$p_{H,3} = p_{H,1} = 0.184 \text{ psia (from the preceding problem).}$$

Since the mixture is assumed to be saturated on entering the heater, its temperature t_3 is the saturation temperature corresponding to a vapor pressure of 0.184 psia. Therefore, from Table 1 of the *Steam Tables*,

$$t_3 \approx 51\text{°F}.$$

Using the values of m_a, $m_{H,1}$, and $m_{H,2}$ calculated in the preceding problem, and noting that $m_{H,1} = m_{H,3}$, we can calculate the rate at which heat must be removed from the mixture flowing through the cooler by Eq. 100*b*:

$$\begin{aligned}
\bar{Q}_{out} &= [m_a h_{a,2} + m_{H,2} h_{H,2}] - [m_a h_{a,3} + m_{H,3} h_{H,3} + (m_{H,2} - m_{H,3}) h_{f,3}] \\
&= [m_a(h_{a,2} - h_{a,3})] + [m_{H,2} h_{H,2} - m_{H,3} h_{H,3} - (m_{H,2} - m_{H,3}) h_{f,3}] \\
&= [73.0 \times 0.241(75 - 51)] + [0.675 \times 1094.5 - 0.575 \times 1084.1 \\
&\qquad - (0.675 - 0.575) \times 19.07] \\
&= 424 + 114 \\
&= 538 \text{ Btu per min.}
\end{aligned}$$

We can calculate the rate at which heat must be supplied to the mixture flowing through the heater by Eq. 100*a*:

$$\begin{aligned}
\bar{Q}_{in} &= (m_a h_{a,1} + m_{H,1} h_{H,1}) - (m_a h_{H,3} + m_{H,1} h_{H,3}) \\
&= m_a(h_{a,1} - h_{a,3}) + m_{H,1}(h_{H,1} - h_{H,3}) \\
&= 73.0 \times 0.241(61 - 51) + 0.575(1088.4 - 1084.1) \\
&= 176 + 2.5 \\
&= 179 \text{ Btu per min.}
\end{aligned}$$

It is of interest to note that the heat (2.5 Btu per min) required to raise the temperature of the water vapor flowing through the heater is a negligible fraction of the total heat supplied; whereas the heat (114 Btu per min) which must be absorbed to cool and partially condense the water vapor flowing through the cooler is an appreciable fraction of the total heat absorbed. Note also that a fan or blower would be required in an actual installation to cause the air from the building to flow through the cooler and the heater.

488. It was found in Illustrative Problem 486 that, to maintain a temperature of 75°F and a relative humidity of 50% in the building described, air at 61°F and 69.4% relative humidity must be supplied to the building. Suppose that this air, instead of being supplied directly, is first mixed with 500 cfm of air taken from the building. What will be the temperature and relative humidity of the resulting mixture?

Solution. Let us indicate the air at 61°F by subscript 1, the air at 75°F by subscript 2, and the resulting mixture by subscript 3. From Illustrative Problem 486,

$$m_{a,1} = 73.0 \text{ lb per min,}$$

and

$$m_{H,1} = 0.575 \text{ lb per min.}$$

Also, for 500 cfm of air taken from the building,

$$m_{a,2} = 73.0 \times \tfrac{500}{1000} = 36.5 \text{ lb per min,}$$

and

$$m_{H,2} = 0.675 \times \tfrac{500}{1000} = 0.338 \text{ lb per min.}$$

Hence,

$$m_{a,3} = 73.0 + 36.5 = 109.5 \text{ lb per min,}$$

and

$$m_{H,3} = 0.575 + 0.338 = 0.913 \text{ lb per min.}$$

The temperature of the final mixture can now be determined by the equation

$$\Sigma(mh)_{\text{in}} = \Sigma(mh)_{\text{out}}.$$

Thus,

$$m_{a,1}h_{a,1} + m_{H,1}h_{H,1} + m_{a,2}h_{a,2} + m_{H,2}h_{H,2} = m_{a,3}h_{a,3} + m_{H,3}h_{H,3}.$$

Making use of Eq. 94*b*, we obtain

$$73.0 \times 0.241 \times 61 + 0.575 \times 1088.4 + 36.5 \times 0.241 \times 75 + 0.338 \times 1094.5$$
$$= 109.5 \times 0.241t_3 + 0.913(1061.9 + 0.43t_3),$$

whence

$$t_3 = 65.5°\text{F}.$$

To determine the relative humidity ϕ_3 of the final mixture, we must first calculate the partial pressure $p_{H,3}$ of its water vapor by Eq. 96*a*:

$$\frac{p_{H,3}}{14.70 - p_{H,3}} = \frac{0.913}{109.5} \times \frac{85.8}{53.3},$$

whence

$$p_{H,3} = 0.195 \text{ psia.}$$

Therefore, finally, by Eq. 98*a*,

$$\phi_3 = \frac{0.195}{0.311}$$

$$= 0.627 \text{ (or } 62.7\%).$$

PROBLEMS

489. Air at 14.20 psia and 75°F enters a heater at the rate of 1000 cfm. It enters with a relative humidity of 45%, flows through the heater at constant pressure, and leaves at a temperature of 150°F. Determine (*a*) the rate at which heat is supplied to the air, and (*b*) the relative humidity of the air leaving the heater.

490. If the stream of air in the preceding problem, instead of being heated at constant pressure to 150°F, were cooled to its dew point, at what rate would heat have to be removed from the stream? *Ans.* 395 Btu per min.

491. A thousand cubic feet of air per minute at a temperature of 60°F and a relative humidity of 60% enter a warm-air furnace. The air leaves the furnace at a temperature of 120°F and a relative humidity of 12%. If the air flows through the furnace at a pressure of 14.50 psia, and if water is supplied to the humidifier at a temperature of 65°F, (*a*) how many pounds of water must be supplied per hour, and (*b*) at what rate must heat be supplied?

492. To maintain the air within a building at a temperature of 72°F and a relative humidity of 40% during the summer, 1500 cu ft of air are to be withdrawn per minute and are to be replaced with air at a lower temperature and lower specific humidity. It is estimated that 20,000 Btu of heat and 10.0 lb of water vapor will be added to the air within the building each hour. If the water vapor is at 98°F, and if the barometric pressure is 29.1 in. Hg abs, what must be the temperature and relative humidity of the air supplied to the building? *Ans.* 59.2°F; 48.3%.

493. The relative humidity of a stream of air at 85°F is to be changed from 60% to 30% by first cooling the stream to condense out part of the water vapor and then heating the stream back to 85°F. If the stream enters the dehumidifying apparatus at the rate of 1000 cfm, and if it leaves the cooler saturated, (*a*) at what rate must heat be removed from the stream in the cooler, and (*b*) at what rate must heat be supplied to the stream in the heater? Also determine (*c*) the volume of the stream leaving the heater per minute. Assume that the processes take place at a pressure of 14.70 psia.

494. During the winter the air within a building is to be maintained at a temperature of 70°F and a relative humidity of 40%. Because the building is not absolutely airtight, air will be lost from the building continuously and will be replaced by air from the out-of-doors. If air is lost at the rate of 300 cfm, and if the outdoor air is at a temperature of 0°F and has a relative humidity of 50%, at what rates must heat and water vapor be supplied to the building to compensate for the heat and humidity lost because of the infiltration of the outdoor air? Assume that the supply of water vapor is obtained by evaporating water initially at 60°F, and assume that the barometric pressure is 14.70 psia. *Ans.* 30,800 Btu per hr; 7.75 lb per hr.

495. A stream of air at 14.50 psia and 65°F is flowing at the rate of 500 cfm. A second stream at 14.50 psia and 125°F is flowing at the rate of 300 cfm. The two streams are mixed adiabatically to form a third stream at 14.50 psia. If the relative humidity of the first stream is 40% and that of the second is 15%, what is the temperature and what is the relative humidity of the third stream?

145. Some Additional Relations Pertaining to Mixtures of Perfect Gases. Although the material discussed in this article is not needed to solve air-conditioning problems, it is appended to the present chapter because, like the material on air conditioning, it has to do with mixtures of gases.

By way of introduction, it may be noted that a mixture of different substances undergoing any process behaves as though it were a pure substance if during the process its composition remains unchanged. Air, for example, has been treated in the preceding chapters as though it were a pure substance even though it is actually a mixture of different gases. The relations among the various properties of any mixture can be determined experimentally, just as the relations among the properties of a pure substance are determined. This is unnecessary if the mixture is made up of different perfect gases, however, because *the properties of a mixture of perfect gases can be deduced from the properties of the components of the mixture.* Let us consider first how the *p-v-T* relation for a mixture of perfect gases can be deduced from the *p-v-T* relations for its components.

(*a*) *The p-v-T Relation for a Mixture of Perfect Gases.* Suppose that a mixture of perfect gases consists of m_x pounds of gas x, m_y pounds of gas y, and m_z pounds of gas z. The partial pressure p_x of gas x, by Eq. 90, is equal to m_xR_xT/V. If we substitute this expression for p_x and the corresponding expressions for p_y and p_z into Eq. 89, we obtain

$$p = p_x + p_y + p_z \tag{89}$$

$$= m_xR_x\frac{T}{V} + m_yR_y\frac{T}{V} + m_zR_z\frac{T}{V},$$

or

$$pV = (m_xR_x + m_yR_y + m_zR_z)T. \tag{a}$$

Now, if we divide both sides of the last equation by the weight m of the mixture, we obtain

$$pv = \left(\frac{m_x}{m}R_x + \frac{m_y}{m}R_y + \frac{m_z}{m}R_z\right)T.$$

Therefore, on comparing this equation with Eq. 3 ($pv = RT$), we see that the *p-v-T* relation for a mixture of perfect gases is the same as that for a single perfect gas provided that we take the gas constant R for the mixture as

$$R = \frac{m_x}{m}R_x + \frac{m_y}{m}R_y + \frac{m_z}{m}R_z. \tag{101}$$

An alternative method of finding the relation among the pressure p, total volume V, and temperature T of a mixture of perfect gases can be developed as follows: In accordance with Eq. 4 in Art. 11, the gas constant R_x for gas x is equal to R_0/M_x, where R_0 denotes the universal gas constant and M_x the molecular weight of gas x. Introducing this expression for R_x and the corresponding expressions for R_y and R_z into Eq. *a* above, we obtain

$$pV = \left(\frac{m_x}{M_x} R_0 + \frac{m_y}{M_y} R_0 + \frac{m_z}{M_z} R_0\right) T.$$

But m_x/M_x is the number of moles n_x of gas x in the mixture (see Art. 11). Similarly, m_y/M_y and m_z/M_z are the numbers of moles n_y and n_z of gases y and z in the mixture. Therefore, we may write the preceding equation in the form

$$pV = (n_x + n_y + n_z)R_0T$$

or

$$\left.\begin{aligned} pV &= nR_0T, \end{aligned}\right\} \tag{101a}$$

where n denotes the total number of moles of gas in the mixture.

(*b*) *The Volumetric Analysis of a Mixture of Perfect Gases.* Suppose that a mixture of perfect gases consisting of m_x, m_y, and m_z pounds of gases x, y, and z respectively is at some pressure p and temperature T. Suppose, next, that by some means the mixture is separated into its constituents and that *each constituent is brought to the pressure p and temperature T of the original mixture.* Let us denote the volumes (total, not specific) of gases x, y, and z under these conditions by the symbols V_x, V_y, and V_z respectively. Since the specific volume of gas x is V_x/m_x, we may write its characteristic equation (Eq. 3) in the form

$$p\left(\frac{V_x}{m_x}\right) = R_xT,$$

or

$$pV_x = m_xR_xT. \tag{102}$$

We may, of course, write similar equations for gases y and z. Now, comparing Eqs. 90 and 102, we see that

$$p_xV = pV_x,$$

or

$$p_x = p\left(\frac{V_x}{V}\right).$$

Hence, introducing this expression for p_x together with the corresponding expressions for p_y and p_z into Eq. 89, we obtain

$$p = p_x + p_y + p_z' \tag{89}$$

$$= p\left(\frac{V_x}{V}\right) + p\left(\frac{V_y}{V}\right) + p\left(\frac{V_z}{V}\right),$$

or

$$1 = \frac{V_x}{V} + \frac{V_y}{V} + \frac{V_z}{V} \tag{103}$$

The ratio V_x/V is called the **volume fraction of constituent x in the mixture,** and the three ratios V_x/V, V_y/V, and V_z/V are called the **volumetric analysis** of the mixture.

It is of interest to determine the relation between the volume fraction V_x/V and the *mole fraction* n_x/n of constituent x in a mixture of perfect gases. Since $R_x = R_0/M_x$ (Eq. 4), we may write Eq. 102 in the form

$$pV_x = \left(\frac{m_x}{M_x}\right) R_0 T$$

$$= n_x R_0 T, \tag{102a}$$

where n_x denotes the number of moles of gas x in the mixture. Comparing the last equation with Eq. 101*a* ($pV = nR_0T$), we see that

$$\frac{V_x}{V} \quad \frac{n_x}{n} \tag{104}$$

Thus, the volume fraction and the mole fraction of any constituent of a mixture of perfect gases are equal.

(*c*) *Conversion of a Volumetric to a Gravimetric Analysis, and Vice Versa.* If a mixture of perfect gases consists of m_x, m_y, and m_z pounds of gases x, y, and z respectively, and if the total weight of the mixture is denoted by the symbol m $(= m_x + m_y + m_z)$, the ratios m_x/m, m_y/m, and m_z/m are called the **gravimetric analysis** of the mixture.

The gravimetric analysis of a mixture of perfect gases can be easily determined if the volumetric analysis of the mixture is known. Thus, we note first that, by Eq. 102,

$$m_x = \frac{pV_x}{R_xT}.$$

Introducing this expression for m_x and the corresponding expressions for m_y and m_z into the equation

$$\frac{m_x}{m} = \frac{m_x}{m_x + m_y + m_z},$$

we obtain

$$\frac{m_x}{m} = \frac{V_x/R_x}{V_x/R_x + V_y/R_y + V_z/R_z},$$

or

$$\frac{m_x}{m} = \frac{(V_x/V)/R_x}{(V_x/V)/R_x + (V_y/V)/R_y + (V_z/V)/R_z}. \tag{105}$$

Note that, since R for any constituent is equal to R_0/M, an alternative form of this equation is

$$\frac{m_x}{m} = \frac{(V_x/V)M_x}{(V_x/V)M_x + (V_y/V)M_y + (V_z/V)M_z}. \tag{105a}$$

It is also easy to determine the volumetric analysis of a mixture of perfect gases if the gravimetric analysis of the mixture is known. We can determine it, for example, by first calculating the number of moles n_x, n_y, and n_z of each constituent ($n_x = m_x/M_x$, etc.) and then making use of Eq. 104. Or we can determine it by noting that, by Eq. 102,

$$V_x = \frac{m_x R_x T}{p}.$$

Introducing this expression for V_x and the corresponding expressions for V_y and V_z into the equation

$$\frac{V_x}{V} = \frac{V_x}{V_x + V_y + V_z},$$

we obtain

$$\frac{V_x}{V} = \frac{(m_x/m)R_x}{(m_x/m)R_x + (m_y/m)R_y + (m_z/m)R_z}. \tag{104a}$$

(*d*) *The Specific Heats c_v and c_p of a Mixture of Perfect Gases.* To develop a relation between the specific heat c_v for a mixture of perfect gases x, y, and z and the specific heats $c_{v,x}$, $c_{v,y}$, and $c_{v,z}$ of its constituents, let us first recall that a perfect gas has been defined, in effect, as a substance (*a*) whose p-v-T relation is $pv = RT$, and (*b*) whose internal energy is a function only of its temperature. We have shown that the p-v-T relation for a mixture of perfect gases is $pv = RT$ (where R for the mixture is defined by Eq. 101). We have also shown that the internal energy U of the mixture is equal to the sum of the internal energies of its constituents (Eq. 92). Since the internal energy of each constituent depends only upon its temperature, the internal energy U of the mixture is also a function only of its temperature. Therefore, a mixture of perfect gases is itself a perfect gas.

It was shown in Art. 37 that for a perfect gas

$$u_2 - u_1 = \int c_v \, dt. \tag{17}$$

Since this equation may be written in the differential form,

$$du = c_v \, dt,$$

for m pounds of a mixture of perfect gases we may write

$$dU = mc_v \, dt,$$

where c_v denotes the specific heat at constant volume *for the mixture.* But it was shown in Art. 137 that for a mixture of perfect gases

$$U = m_x u_x + m_y u_y + m_z u_z. \tag{92}$$

If we differentiate this equation and replace du_x by $c_{v,x} \, dt$, du_y by $c_{v,y} \, dt$, and du_z by $c_{v,z} \, dt$, we obtain

$$\begin{aligned} dU &= m_x \, du_x + m_y \, du_y + m_z \, du_z \\ &= m_x c_{v,x} \, dt + m_y c_{v,y} \, dt + m_z c_{v,z} \, dt. \end{aligned}$$

Finally, therefore, equating the two foregoing expressions for dU and solving for c_v, we obtain

$$c_v = \left(\frac{m_x}{m}\right) c_{v,x} + \left(\frac{m_y}{m}\right) c_{v,y} + \left(\frac{m_z}{m}\right) c_{v,z}. \tag{106}$$

By means of this equation we can determine the specific heat c_v for a mixture of perfect gases from the values of c_v for its constituents.

To determine a relation between the specific heat c_p for a mixture of perfect gases and the specific heats $c_{p,x}$, $c_{p,y}$, and $c_{p,z}$ of its constituents, we recall that in Art. 138 it was shown that for a perfect gas

$$h_2 - h_1 = \int c_p \, dt. \tag{18}$$

Since this equation may be written in the differential form,

$$dh = c_p \, dt,$$

for m pounds of a mixture of perfect gases we may write

$$dH = mc_p \, dt,$$

where c_p denotes the specific heat at constant pressure *for the mixture.* But in Art. 137 it was shown that for a mixture of perfect gases

$$H = m_x h_x + m_y h_y + m_z h_z.$$

Therefore,

$$dH = m_x\,dh_x + m_y\,dh_y + m_z\,dh_z$$

$$= m_x c_{p,x}\,dt + m_y c_{p,y}\,dt + m_z c_{p,z}\,dt.$$

Finally, equating the two foregoing expressions for dH and solving for the specific heat c_p of the mixture, we obtain

$$c_p = \left(\frac{m_x}{m}\right) c_{p,x} + \left(\frac{m_y}{m}\right) c_{p,y} + \left(\frac{m_z}{m}\right) c_{p,z}. \qquad (107)$$

To determine the value of k ($= c_p/c_v$) for a mixture of perfect gases, we must first determine the values of c_v and c_p for the mixture by Eqs. 106 and 107. We can then find k by dividing c_p by c_v.

In summary, note that problems involving a mixture of perfect gases may be treated as though the mixture were a single perfect gas (that is, as though it were a pure substance) after the values of R, c_v, c_p, and k for the mixture have been found.

ILLUSTRATIVE PROBLEMS

496. Re-solve Illustrative Problem 436 first by means of Eq. 101 and then by means of Eq. 101*a*. Also find the volumetric analysis of the mixture.

Solution. Since the gas constants for H_2, O_2, and N_2 are 767, 48.3, and 55.1 respectively, and since the weight of the mixture is (0.3 + 0.5 + 2.0 =) 2.8 lb, the gas constant R for the mixture is, by Eq. 101,

$$R = \frac{0.3}{2.8} \times 767 + \frac{0.5}{2.8} \times 48.3 + \frac{2.0}{2.8} \times 55.1$$

$$= 130.2 \text{ ft per deg F.}$$

Hence, by Eq. 3,

$$v = \frac{130.2 \times 540}{15 \times 144}$$

$$= 32.6 \text{ cu ft per lb;}$$

and the total volume V of the mixture is 2.8 × 32.6 = 91.4 cu ft.

To find the total volume of the mixture by means of Eq. 101*a*, we note that the molecular weights of H_2, O_2, and N_2 are 2, 32, and 28 respectively. Hence, the mixture consists of 0.3/2 = 0.1500 mole of H_2, 0.5/32 = 0.0156 mole of O_2, and 2.0/28 = 0.0714 mole of N_2; and, by Eq. 101*a*,

$$15 \times 144 \times V = (0.1500 + 0.0156 + 0.0714) \times 1545 \times 540,$$

whence

$$V = 91.4 \text{ cu ft.}$$

The volumetric analysis can be calculated by either Eq. 104 or 104*a*. Since the moles of each constituent have already been determined in the preceding paragraph, let us use Eq. 104. The total number of moles n in the mixture is (0.1500 + 0.0156 + 0.0714 =) 0.2370 mole. Hence, $V_{H_2O}/V = 0.1500/0.2370 = 0.633$, $V_{O_2}/V = 0.0156/0.2370 = 0.066$, and $V_{N_2}/V = 0.0714/0.2370 = 0.301$.

497. A mixture of hydrogen and oxygen initially at 15 psia and 70°F is compressed isentropically to 50 psia. If the mixture consists of equal parts of hydrogen and oxygen *by volume*, what is its final temperature? If the process is non-flow, how much work is done on each pound of the mixture?

Solution. The values of c_v and c_p for the mixture must be calculated in order to find the value of k. First, however, the gravimetric analysis of the mixture must be determined. This can be done either by Eq. 105 or 105a. Let us use Eq. 105a. Since the molecular weights of hydrogen and oxygen are 2 and 32 respectively,

$$\frac{m_{H_2}}{m} = \frac{0.5 \times 2}{0.5 \times 2 + 0.5 \times 32} = 0.0588,$$

and therefore

$$\frac{m_{O_2}}{m} = 0.9412.$$

The values of c_v and c_p for the mixture can now be calculated by Eqs. 106 and 107. Using the values of c_v and c_p for hydrogen and oxygen listed in Table IV (Art. 36), we obtain

$$c_v = 0.0588 \times 2.49 + 0.9412 \times 0.156 = 0.293 \text{ Btu per lb deg F},$$

and

$$c_p = 0.0588 \times 3.48 + 0.9412 \times 0.218 = 0.410 \text{ Btu per lb deg F}.$$

The value of k for the mixture is therefore $0.410/0.293 = 1.40$.

The final temperature of the mixture can now be calculated by Eq. 28b:

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(k-1)/k} \qquad (28b)$$

$$= 530\left(\tfrac{50}{15}\right)^{0.4/1.4}$$

$$= 747\text{°F abs} \quad (t_2 = 287\text{°F}).$$

Finally, the work done *by* the mixture can be calculated by Eq. 27b:

$$\frac{W}{J} = c_v(t_1 - t_2) \qquad (27b)$$

$$= 0.293(70 - 287)$$

$$= -63.6 \text{ Btu per lb},$$

or 63.6 Btu of work must be done on each pound of the mixture during the process.

498. One pound of methane (CH_4) is burned at a pressure of 14.70 psia with twice the amount of air required for complete combustion. Assuming that the composition of air is 21% oxygen and 79% nitrogen by volume, find the dew point of the products of combustion. If the products are cooled to 70°F, how much water will condense?

Solution. Since the equation for the reaction of CH_4 with exactly the amount of O_2 required for complete combustion is

$$CH_4 + 2O_2 = CO_2 + 2H_2O,$$

and since there are (0.79/0.21 =) 3.76 moles of N_2 per mole of O_2 in the air (see Eq. 104), it follows that the equation for the reaction of CH_4 with twice the amount of air needed for complete combustion is

$$CH_4 + 4(O_2 + 3.76N_2) = CO_2 + 2H_2O + 2O_2 + 15.04N_2.$$

It was shown in part *b* of the preceding article that, for any constituent x of a mixture of perfect gases,

$$p_xV = pV_x.$$

Therefore, making use of Eq. 104, we may write

$$\frac{p_x}{p} = \frac{V_x}{V} = \frac{n_x}{n};$$

and the partial pressure p_H of the water vapor in the products of combustion is

$$\begin{aligned} p_H &= \left(\frac{2}{1 + 2 + 2 + 15.04}\right) \times 14.70 \\ &= 1.467 \text{ psia}. \end{aligned}$$

Since at the dew point the water vapor in the products of combustion is saturated, the dew point of the products is the saturation temperature corresponding to this pressure. Hence, from Table 1 of the *Steam Tables*,

$$t_{d.p.} = 114.9°\text{F}.$$

Since the molecular weight of CH_4 is 16 and that of H_2O is 18, the water vapor formed per pound of methane burned is (2 × 18)/16 = 2.250 lb. To determine how much of this vapor condenses when the products of combustion are cooled to 70°F, we must first find how much of it is still in the vapor phase. To do this, we note that at 70°F the gaseous portion of the products will consist, *per mole of CH_4 burned,* of 1 mole of CO_2, n_{H_2O} moles of saturated water vapor, 2 moles of O_2, and 15.04 moles of N_2. Since the partial pressure of the saturated water vapor at 70°F is 0.3631 psia (from Table 1 of the *Steam Tables*), and since, as shown in the preceding paragraph, for any constituent x of a mixture of perfect gases,

$$\frac{p_x}{p} = \frac{n_x}{n},$$

we may write

$$\frac{0.3631}{14.70} = \frac{n_{H_2O}}{1 + n_{H_2O} + 2 + 15.04},$$

whence $n_{H_2O} = 0.457$ mole H_2O vapor per mole CH_4 burned. Therefore, the weight of H_2O still in the gaseous phase at 70°F is (0.457 × 18)/16 = 0.514 lb per lb CH_4 burned; and, finally, the weight of water vapor condensed is 2.250 − 0.514 = 1.736 lb.

PROBLEMS

499. A mixture of gases at 20 psia and 70°F consists of 0.10 lb of O_2, 1.35 lb of N_2, and 0.25 lb of CO_2. Determine (*a*) the volume of the mixture, and (*b*) its volumetric analysis.

500. A 20-cu-ft tank contains 2 lb of N_2 and 1 lb of CO_2 at a pressure of 25 psia. How much heat must be added to the mixture to raise its pressure to 40 psia?
Ans. 156 Btu.

501. A 10-cu-ft tank contains a mixture of H_2 and N_2 at 16 psia and 90°F. If the mixture weighs 0.400 lb, what is its (*a*) gravimetric analysis, and (*b*) volumetric analysis?

502. A mixture of air and water vapor at 14.20 psia and 100°F has a relative humidity of 60%. Determine (*a*) the volumetric analysis and (*b*) the gravimetric analysis of the mixture.

503. A mixture consisting of equal parts of oxygen and hydrogen by volume undergoes a non-flow process during which its pressure remains constant at 20 psia and its temperature rises from 70° to 160°F. If the mixture has an initial volume of 15 cu ft, how much heat does it absorb during the process? *Ans.* 33.2 Btu.

504. A mixture consisting of 0.35 lb of O_2 and 0.55 lb of CO_2 undergoes a non-flow process during which its temperature remains constant at 80°F. If the mixture has an initial volume of 5 cu ft, how much work is required to compress it to 100 psia?
Ans. 25,400 ft-lb.

505. Two hundred cubic feet per minute of a mixture consisting of 85% H_2 and 15% CO_2 by volume enter a compressor at 14.7 psia and 60°F and leave at 75 psia. If the mixture is compressed isentropically, and if it enters and leaves the compressor at low velocities, how many horsepower are required to drive the machine?

506. A pound of acetylene (C_2H_2) is burned at atmospheric pressure with exactly the amount of oxygen required for complete combustion. Determine (*a*) the dew point of the products of combustion. If the products are cooled to 80°F, (*b*) how many pounds of water vapor will condense, and (*c*) what will be the volume of the remaining gases? Assume that atmospheric pressure is 14.70 psia.
Ans. (*a*) 161.4°F; (*b*) 0.644 lb; (*c*) 31.4 cu ft.

507. A pound of octane (C_8H_{18}) is burned at a pressure of 14.70 psia with one and a half times the amount of air required for complete combustion. Assuming that air consists of 21% O_2 and 79% N_2 by volume, determine (*a*) the dew point of the products of combustion. If the products are cooled to 80°F, (*b*) how many pounds of water vapor will condense, and (*c*) what will be the volume of the remaining gases?

SYMBOLS

c_p	specific heat at constant pressure
c_v	specific heat at constant volume
h	enthalpy per unit mass
H	total enthalpy
J	mechanical equivalent of heat
k	ratio of c_p to c_v
m	mass
m	mass rate of flow

n	moles
p	pressure
$\dot{Q}$	rate at which heat is added to (or removed from) a system in steady flow
R	gas constant
R_0	universal gas constant
t	temperature
T	absolute temperature
u	internal energy per unit mass
U	total internal energy
v	specific volume
V	total volume
w	specific humidity (humidity ratio)
$\dot{W}$	rate at which shaft-work is done on a system in steady flow
μ	percentage saturation (degree of saturation)
ρ	density
ϕ	relative humidity

SUBSCRIPTS

a	air
H	water vapor
s	saturated water vapor
x, y, z	gases x, y, *and* z

APPENDIX

THERMODYNAMIC PROPERTIES OF AMMONIA

(Abridged from "Tables of Thermodynamic
Properties of Ammonia," Circular 142 of
the Bureau of Standards, 1923)

TABLE 1

SATURATED AMMONIA: TEMPERATURE TABLE

(v = cu ft per lb; h = Btu per lb; s = Btu per lb deg F)

		Specific Volume		Enthalpy			Entropy			
Temp., °F	Press., psia	Sat. liquid	Sat. vapor	Sat. liquid	Evap.	Sat. vapor	Sat. liquid	Evap.	Sat. vapor	Temp., °F
t	p	v_f	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	t
−60	5.55	0.02278	44.73	−21.2	610.8	589.6	−0.0517	1.5286	1.4769	−60
−50	7.67	0.02299	33.08	−10.6	604.3	593.7	−0.0256	1.4753	1.4497	−50
−40	10.41	0.02322	24.86	0.0	597.6	597.6	0.0000	1.4242	1.4242	−40
−30	13.90	0.02345	18.97	10.7	590.7	601.4	0.0250	1.3751	1.4001	−30
−20	18.30	0.02369	14.68	21.4	583.6	605.0	0.0497	1.3277	1.3774	−20
−10	23.74	0.02393	11.50	32.1	576.4	608.5	0.0738	1.2820	1.3558	−10
0	30.42	0.02419	9.116	42.9	568.9	611.8	0.0975	1.2377	1.3352	0
5	34.27	0.02432	8.150	48.3	565.0	613.3	0.1092	1.2161	1.3253	5
10	38.51	0.02446	7.304	53.8	561.1	614.9	0.1208	1.1949	1.3157	10
20	48.21	0.02474	5.910	64.7	553.1	617.8	0.1437	1.1532	1.2969	20
30	59.74	0.02503	4.825	75.7	544.8	620.5	0.1663	1.1127	1.2790	30
40	73.32	0.02533	3.971	86.8	536.2	623.0	0.1885	1.0733	1.2618	40
50	89.19	0.02564	3.294	97.9	527.3	625.2	0.2105	1.0348	1.2453	50
60	107.6	0.02597	2.751	109.2	518.1	627.3	0.2322	0.9972	1.2294	60
70	128.8	0.02632	2.312	120.5	508.6	629.1	0.2537	0.9603	1.2140	70
80	153.0	0.02668	1.955	132.0	498.7	630.7	0.2749	0.9242	1.1991	80
86	169.2	0.02691	1.772	138.9	492.6	631.5	0.2875	0.9029	1.1904	86
90	180.6	0.02707	1.661	143.5	488.5	632.0	0.2958	0.8888	1.1846	90
100	211.9	0.02747	1.419	155.2	477.8	633.0	0.3166	0.8539	1.1705	100
110	247.0	0.02790	1.217	167.0	466.7	633.7	0.3372	0.8194	1.1566	110
120	286.4	0.02836	1.047	179.0	455.0	634.0	0.3576	0.7851	1.1427	120

TABLE 2

SATURATED AMMONIA: PRESSURE TABLE

(v = cu ft per lb; h = Btu per lb; s = Btu per lb deg F)

Press., psia	Temp., °F	Specific Volume		Enthalpy			Entropy			Press., psia
		Sat. liquid	Sat. vapor	Sat. liquid	Evap.	Sat. vapor	Sat. liquid	Evap.	Sat. vapor	
p	t	v_f	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g	p
5	−63.11	0.02271	49.31	−24.5	612.8	588.3	−0.0599	1.5456	1.4857	5
10	−41.34	0.02319	25.81	−1.4	598.5	597.1	−0.0034	1.4310	1.4276	10
15	−27.29	0.02351	17.67	13.6	588.8	602.4	0.0318	1.3620	1.3938	15
20	−16.64	0.02377	13.50	25.0	581.2	606.2	0.0578	1.3122	1.3700	20
30	− 0.57	0.02417	9.236	42.3	569.3	611.6	0.0962	1.2402	1.3364	30
40	11.66	0.02451	7.047	55.6	559.8	615.4	0.1246	1.1879	1.3125	40
50	21.67	0.02479	5.710	66.5	551.7	618.2	0.1475	1.1464	1.2939	50
60	30.21	0.02504	4.805	75.9	544.6	620.5	0.1668	1.1119	1.2787	60
80	44.40	0.02546	3.655	91.7	532.3	624.0	0.1982	1.0563	1.2545	80
100	56.05	0.02584	2.952	104.7	521.8	626.5	0.2237	1.0119	1.2356	100
120	66.02	0.02618	2.476	116.0	512.4	628.4	0.2452	0.9749	1.2201	120
140	74.79	0.02649	2.132	126.0	503.9	629.9	0.2638	0.9430	1.2068	140
170	86.29	0.02692	1.764	139.3	492.3	631.6	0.2881	0.9019	1.1900	170
200	96.34	0.02732	1.502	150.9	481.8	632.7	0.3090	0.8666	1.1756	200
230	105.30	0.02770	1.307	161.4	472.0	633.4	0.3275	0.8356	1.1631	230
260	113.42	0.02806	1.155	171.1	462.8	633.9	0.3441	0.8077	1.1518	260

TABLE 3
SUPERHEATED AMMONIA VAPOR

(v = volume, cu ft per lb; h = enthalpy, Btu per lb; s = entropy, Btu per lb deg F)

Temp., °F	Pressure, psia (saturation temperature in italics)												Temp., °F
	5 *−63.11°*			10 *−41.34°*			15 *−27.29°*			20 *−16.64°*			
	v	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	
Sat.	*49.31*	*588.3*	*1.4857*	*25.81*	*597.1*	*1.4276*	*17.67*	*602.4*	*1.3938*	*13.50*	*606.2*	*1.3700*	*Sat.*
−50	51.05	595.2	1.5025	......	.....	......	.....	.....	......	.....	.....	......	−50
−40	52.36	600.3	.5149	25.90	597.8	1.4293	.....	.....	......	.....	.....	......	−40
−30	53.67	605.4	.5269	26.58	603.2	.4420	.....	.....	......	.....	.....	......	−30
−20	54.97	610.4	.5385	27.26	608.5	.4542	18.01	606.4	1.4031	.....	.....	......	−20
−10	56.26	615.4	.5498	27.92	613.7	.4659	18.47	611.9	.4154	13.74	610.0	1.3784	−10
0	57.55	620.4	1.5608	28.58	618.9	1.4773	18.92	617.2	1.4272	14.09	615.5	1.3907	0
10	58.84	625.4	.5716	29.24	624.0	.4884	19.37	622.5	.4386	14.44	621.0	.4025	10
20	60.12	630.4	.5821	29.90	629.1	.4992	19.82	627.8	.4497	14.78	626.4	.4138	20
30	61.41	635.4	.5925	30.55	634.2	.5097	20.26	633.0	.4604	15.11	631.7	.4248	30
40	62.69	640.4	.6026	31.20	639.3	.5200	20.70	638.2	.4709	15.45	637.0	.4356	40
50	63.96	645.5	1.6125	31.85	644.4	1.5301	21.14	643.4	1.4812	15.78	642.3	1.4460	50
60	65.24	650.5	.6223	32.49	649.5	.5400	21.58	648.5	.4912	16.12	647.5	.4562	60
70	66.51	655.5	.6319	33.14	654.6	.5497	22.01	653.7	.5011	16.45	652.8	.4662	70
80	67.79	660.6	.6413	33.78	659.7	.5593	22.44	658.9	.5108	16.78	658.0	.4760	80
90	69.06	665.6	.6506	34.42	664.8	.5687	22.88	664.0	.5203	17.10	663.2	.4856	90
100	70.33	670.7	1.6598	35.07	670.0	1.5779	23.31	669.2	1.5296	17.43	668.5	1.4950	100
110	71.60	675.8	.6689	35.71	675.1	.5870	23.74	674.4	.5388	17.76	673.7	.5042	110
120	72.87	680.9	.6778	36.35	680.3	.5960	24.17	679.6	.5478	18.08	678.9	.5133	120
130	74.14	686.1	.6865	36.99	685.4	.6049	24.60	684.8	.5567	18.41	684.2	.5223	130
140	75.41	691.2	.6952	37.62	690.6	.6136	25.03	690.0	.5655	18.73	689.4	.5312	140
150	76.68	696.4	1.7038	38.26	695.8	1.6222	25.46	695.3	1.5742	19.05	694.7	1.5399	150
160	77.95	701.6	.7122	38.90	701.1	.6307	25.88	700.5	.5827	19.37	700.0	.5485	160
170	79.21	706.8	.7206	39.54	706.3	.6391	26.31	705.8	.5911	19.70	705.3	.5569	170
180	80.48	712.1	.7289	40.17	711.6	.6474	26.74	711.1	.5995	20.02	710.6	.5653	180
190	.....	.....	......	40.81	716.9	.6556	27.16	716.4	.6077	20.34	715.9	.5736	190
200	.....	.....	......	41.45	722.2	1.6637	27.59	721.7	1.6158	20.66	721.2	1.5817	200
220	.....	.....	......	......	.....	.....	28.44	732.4	.6318	21.30	732.0	.5978	220
240	.....	.....	......	......	.....	......	.....	.....	......	21.94	742.8	.6135	240

Temp., °F	Pressure, psia (saturation temperature in italics)												Temp., °F
	30 *−0.57°*			40 *11.66°*			50 *21.67°*			60 *30.21°*			
	v	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	
Sat.	*9.236*	*611.6*	*1.3364*	*7.047*	*615.4*	*1.3125*	*5.710*	*618.2*	*1.2939*	*4.805*	*620.5*	*1.2787*	*Sat.*
0	9.250	611.9	1.3371	......	.....	......	.....	.....	......	.....	.....	......	0
10	9.492	617.8	.3497	......	.....	......	.....	.....	......	.....	.....	......	10
20	9.731	623.5	.3618	7.203	620.4	1.3231	.....	.....	......	.....	.....	......	20
30	9.966	629.1	.3733	7.387	626.3	.3353	5.838	623.4	1.3046	.....	.....	......	30
40	10.20	634.6	.3845	7.568	632.1	.3470	5.988	629.5	.3169	4.933	626.8	1.2913	40
50	10.43	640.1	1.3953	7.746	637.8	1.3583	6.135	635.4	1.3286	5.060	632.9	1.3035	50
60	10.65	645.5	.4059	7.922	643.4	.3692	6.280	641.2	.3399	5.184	639.0	.3152	60
70	10.88	650.9	.4161	8.096	648.9	.3797	6.423	646.9	.3508	5.307	644.9	.3265	70
80	11.10	656.2	.4261	8.268	654.4	.3900	6.564	652.6	.3613	5.428	650.7	.3373	80
90	11.33	661.6	.4359	8.439	659.9	.4000	6.704	658.2	.3716	5.547	656.4	.3479	90
100	11.55	666.9	1.4456	8.609	665.3	1.4098	6.843	663.7	1.3816	5.665	662.1	1.3581	100
110	11.77	672.2	.4550	8.777	670.7	.4194	6.980	669.2	.3914	5.781	667.7	.3681	110
120	11.99	677.5	.4642	8.945	676.1	.4288	7.117	674.7	.4009	5.897	673.3	.3778	120
130	12.21	682.9	.4733	9.112	681.5	.4381	7.252	680.2	.4103	6.012	678.9	.3873	130
140	12.43	688.2	.4823	9.278	686.9	.4471	7.387	685.7	.4195	6.126	684.4	.3966	140
150	12.65	693.5	1.4911	9.444	692.3	1.4561	7.521	691.1	1.4286	6.239	689.9	1.4058	150
160	12.87	698.8	.4998	9.609	697.7	.4648	7.655	696.6	.4374	6.352	695.5	.4148	160
170	13.08	704.2	.5083	9.774	703.1	.4735	7.788	702.1	.4462	6.464	701.0	.4236	170
180	13.30	709.6	.5168	9.938	708.5	.4820	7.921	707.5	.4548	6.576	706.5	.4323	180
190	13.52	714.9	.5251	10.10	714.0	.4904	8.053	713.0	.4633	6.687	712.0	.4409	190
200	13.73	720.3	1.5334	10.27	719.4	1.4987	8.185	718.5	1.4716	6.798	717.5	1.4493	200
220	14.16	731.1	.5495	10.59	730.3	.5150	8.448	729.4	.4880	7.019	728.6	.4658	220
240	14.59	742.0	.5653	10.92	741.3	.5309	8.710	740.5	.5040	7.238	739.7	.4819	240
260	15.02	753.0	.5808	11.24	752.3	.5465	8.970	751.6	.5197	7.457	750.9	.4976	260
280	.....	.....	......	11.56	763.4	.5617	9.230	762.7	.5350	7.675	762.1	.5130	280
300	.....	.....	......	11.88	774.6	1.5766	9.489	774.0	1.5500	7.892	773.3	1.5281	300

TABLE 3 (*Continued*)

SUPERHEATED AMMONIA VAPOR

(v = volume, cu ft per lb; h = enthalpy, Btu per lb; s = entropy, Btu per lb deg F)

Temp., °F	Pressure, psia (saturation temperature in italics)												Temp., °F
	80 *44.40°*			100 *56.05°*			120 *66.02°*			140 *74.79°*			
	v	h	s	v	h	s	v	h	s	v	h	s	
Sat.	*3.655*	*624.0*	*1.2545*	*2.952*	*626.5*	*1.2356*	*2.476*	*628.4*	*1.2201*	*2.132*	*629.9*	*1.2068*	*Sat.*
50	3.712	627.7	1.2619	.....	.....	......	.....	.....	......	.....	.....	......	50
60	3.812	634.3	.2745	2.985	629.3	1.2409	.....	.....	......	.....	.....	......	60
70	3.909	640.6	.2866	3.068	636.0	.2539	2.505	631.3	1.2255	.....	.....	......	70
80	4.005	646.7	.2981	3.140	642.6	.2661	2.576	638.3	.2386	2.166	633.8	1.2140	80
90	4.098	652.8	.3092	3.227	649.0	.2778	2.645	645.0	.2510	2.228	640.9	.2272	90
100	4.190	658.7	1.3199	3.304	655.2	1.2891	2.712	651.6	1.2628	2.288	647.8	1.2396	100
110	4.281	664.6	.3303	3.380	661.3	.2999	2.778	658.0	.2741	2.347	654.5	.2515	110
120	4.371	670.4	.3404	3.454	667.3	.3104	2.842	664.2	.2850	2.404	661.1	.2628	120
130	4.460	676.1	.3502	3.527	673.3	.3206	2.905	670.4	.2956	2.460	667.4	.2738	130
140	4.548	681.8	.3598	3.600	679.2	.3305	2.967	676.5	.3058	2.515	673.7	.2843	140
150	4.635	687.5	1.3692	3.672	685.0	1.3401	3.029	682.5	1.3157	2.569	679.9	1.2945	150
160	4.722	693.2	.3784	3.743	690.8	.3495	3.089	688.4	.3254	2.622	686.0	.3045	160
170	4.808	698.8	.3874	3.813	696.6	.3588	3.149	694.3	.3348	2.675	692.0	.3141	170
180	4.893	704.4	.3963	3.883	702.3	.3678	3.209	700.2	.3441	2.727	698.0	.3236	180
190	4.978	710.0	.4050	3.952	708.0	.3767	3.268	706.0	.3531	2.779	704.0	.3328	190
200	5.063	715.6	1.4136	4.021	713.7	1.3854	3.326	711.8	1.3620	2.830	709.9	1.3418	200
210	5.147	721.3	.4220	4.090	719.4	.3940	3.385	717.6	.3707	2.880	715.8	.3507	210
220	5.231	726.9	.4304	4.158	725.1	.4024	3.442	723.4	.3793	2.931	721.6	.3594	220
230	5.315	732.5	.4386	4.226	730.8	.4108	3.500	729.2	.3877	2.981	727.5	.3679	230
240	5.398	738.1	.4467	4.294	736.5	.4190	3.557	734.9	.3960	3.030	733.3	.3763	240
250	5.482	743.8	1.4547	4.361	742.2	1.4271	3.614	740.7	1.4042	3.080	739.2	1.3846	250
260	5.565	749.4	.4626	4.428	747.9	.4350	3.671	746.5	.4123	3.129	745.0	.3928	260
280	5.730	760.7	.4781	4.562	759.4	.4507	3.783	758.0	.4281	3.227	756.7	.4088	280
300	5.894	772.1	.4933	4.695	770.8	.4660	3.895	769.6	.4435	3.323	768.3	.4243	300

Temp., °F	Pressure, psia (saturation temperature in italics)												Temp., °F
	170 *86.29°*			200 *96.34°*			230 *105.30°*			260 *113.42°*			
	v	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*	
Sat.	*1.764*	*631.6*	*1.1900*	*1.502*	*632.7*	*1.1756*	*1.307*	*633.4*	*1.1631*	*1.155*	*633.9*	*1.1518*	*Sat.*
90	1.784	634.4	1.1952	....	.....	......	.....	.....	......	.....	.....	......	90
100	1.837	641.9	1.2087	1.520	635.6	1.1809	.....	.....	......	.....	.....	......	100
110	1.889	649.1	.2215	1.567	643.4	.1947	1.328	637.4	1.1700	.....	.....	......	110
120	1.939	656.1	.2336	1.612	650.9	.2077	1.370	645.4	.1840	1.182	639.5	1.1617	120
130	1.988	662.8	.2452	1.656	658.1	.2200	1.410	653.1	.1971	1.220	647.8	.1757	130
140	2.035	669.4	.2563	1.698	665.0	.2317	1.449	660.4	.2095	1.257	655.6	.1889	140
150	2.081	675.9	1.2669	1.740	671.8	1.2429	1.487	667.6	1.2213	1.292	663.1	1.2014	150
160	2.127	682.3	.2773	1.780	678.4	.2537	1.524	674.5	.2325	1.326	670.4	.2132	160
170	2.172	688.5	.2873	1.820	684.9	.2641	1.559	681.3	.2434	1.359	677.5	.2245	170
180	2.216	694.7	.2971	1.859	691.3	.2742	1.594	687.9	.2538	1.391	684.4	.2354	180
190	2.260	700.8	.3066	1.897	697.7	.2840	1.629	694.4	.2640	1.422	691.1	.2458	190
200	2.303	706.9	1.3159	1.935	703.9	1.2935	1.663	700.9	1.2738	1.453	697.7	1.2560	200
210	2.346	713.0	.3249	1.972	710.1	.3029	1.696	707.2	.2834	1.484	704.3	.2658	210
220	2.389	719.0	.3338	2.009	716.3	.3120	1.729	713.5	.2927	1.514	710.7	.2754	220
230	2.431	724.9	.3426	2.046	722.4	.3209	1.762	719.8	.3018	1.543	717.1	.2847	230
240	2.473	730.9	.3512	2.082	728.4	.3296	1.794	726.0	.3107	1.572	723.4	.2938	240
250	2.514	736.8	1.3596	2.118	734.5	1.3382	1.826	732.1	1.3195	1.601	729.7	1.3027	250
260	2.555	742.8	.3679	2.154	740.5	.3467	1.857	738.3	.3281	1.630	736.0	.3115	260
270	2.596	748.7	.3761	2.189	746.5	.3550	1.889	744.4	.3365	1.658	742.2	.3200	270
280	2.637	754.6	.3841	2.225	752.5	.3631	1.920	750.5	.3448	1.686	748.4	.3285	280
290	2.678	760.5	.3921	2.260	758.5	.3712	1.951	756.5	.3530	1.714	754.5	.3367	290
300	2.718	766.4	1.3999	2.295	764.5	1.3791	1.982	762.6	1.3610	1.741	760.7	1.3449	300
320	2.798	778.3	.4153	2.364	776.5	.3947	2.043	774.7	.3767	1.796	772.9	.3608	320
340	2.878	790.1	.4303	2.432	788.5	.4099	2.103	786.8	.3921	1.850	785.2	.3763	340
360	.....	.....	......	2.500	800.5	.4247	2.163	798.9	.4070	1.904	797.4	.3914	360
380	.....	.....	......	2.568	812.5	.4392	2.222	811.1	.4217	1.957	809.6	.4062	380

Index

Absolute Centigrade scale, 19
Absolute Fahrenheit scale, 19
Absolute perfect gas scale of temperature, 14, 188
Absolute pressure, 9
Absolute zero of temperature, 194
Absorber, 312
Absorption refrigeration cycle, 311
  determination of ideal coefficient of performance of, 313
Acoustic velocity, 346, 354, 355
Actual flow through nozzles, 358
Actual thermal efficiency, of a steam power plant, 267
  of an internal-combustion engine, 285
Adiabatic mixing of streams, 424
Adiabatic non-flow processes, 124, 217
Adiabatic process, definition of, 38
  reversible non-flow, 124, 217
  reversible steady-flow, 139, 217
Adiabatic saturation process, 412
Adiabatic saturation temperature, 412
  and wet-bulb temperature, 415
Adiabatic steady-flow processes, 139, 217
Air, constituents of, 397, 404
  liquid, 318
  saturated, 404
  specific heats of, 94, 95
Air compressors, *see* Compressors
Air conditioning, definition of, 397
Air-conditioning processes, 421–424
Air leakage into refrigerating machines, 302
Air-preheater, 266
Air refrigerating machines, 164, 292
  effect of friction on, 297
Air refrigeration cycle, 164, 292
  determination of ideal coefficient of performance of, 294
  volume of air compressed, 296
Air-standard cycles, 271
  Diesel, 277
Air-standard cycles, dual-combustion, 277
  gas turbine, 281
  Otto, 271
Air-water vapor mixtures, 404
Ammonia, 23, 303
Ammonia absorption cycle, 311
Ammonia-water solutions, 311
Angle of divergence, 355
Avogadro's Law, 23
Axial thrust, 374

Barnard, W. N., 361
Binary-vapor cycle, 262
  determination of ideal efficiency of, 263
  flow-diagram for, 262
Blades, 361
Bled steam, 258
Blower, 321
Boiler efficiency, 267
Bomb calorimeter, 265
Booster compressor, 321
Boundaries of a system, 7
Bourdon gage, 9
Brake horsepower, definition of, 286
British thermal unit (Btu), definition of, 38, 39
Buckets, 361

Calorie, definition of, 38
Calorimeter, throttling, 143
Carbon dioxide, specific heats of, 95
Carbon monoxide, specific heats of, 95
Carnot power cycle, 158
  efficiency of, 162
  impracticability of, 245
Carnot Principle, 178, 243
  a corollary of, 180, 245
Carnot refrigeration cycle, 169
  coefficient of performance of, 173
  impracticability of, 291
Cascade refrigeration system, 308

Centigrade scale, 19
Centrifugal compressors, 321
  power ideally required by, 341
Change of entropy of perfect gases, 208
Change of state, 8
Characteristic equation, 20
Clausius, 177
Clearance volume, definition of, 322
Closed-type feedwater heater, 258
Coefficient of discharge, definition of, 384
  for flow nozzles, 387
  for thin-plate orifices, 390
  for venturi meters, 385
Coefficient of performance, definition of, 165, 290
  of absorption refrigeration cycle, 313
  of air refrigeration cycle, 294
  of Carnot refrigeration cycle, 173
  of reversible refrigerating machines, 181
  of vapor compression refrigeration cycle, 299
  relation to horsepower per ton, 291
Compressed liquid, 31
Compression efficiency, 339
  equivalent, 339
Compression ratio, 272
Compression refrigeration cycle, 298
Compressor efficiency, 340
Compressor stages, limitations on number of, 331
Compressors, 321
  centrifugal, 321
    power ideally required by, 341
  for refrigerating machines, 303
  ideal multistage, 329
    power required by, 332
  ideal single-stage, 322
    power required by, 325
  lubrication of, 301, 305
Condensation, 8
Condition curve, 375
Conduction of heat, 38
Conservation of energy, 61
Constant pressure gas thermometer, 12
Constant pressure process, non-flow, 117
Constant temperature process, non-flow, 120
Constant volume gas thermometer, 12
Constant volume process, non-flow, 115
Continuity equation, 81
Converging-diverging nozzles, 354
  actual flow through, 358
  isentropic flow through, 356
Converging nozzles, 345
  actual flow through, 358
  isentropic flow through, 347
Converging passages, isentropic flow through, 381
Cooling and dehumidifying, 423
Cooling at constant specific humidity, 422
Cooling tower, 415
Corner taps, 389
Critical point, 26, 226, 228
Critical pressure, 27
  in nozzles, 347
    determination of, 352, 359
Critical pressure ratio, 347
  determination of, 352, 359
  effect of, on rate of discharge, 350
Critical temperature, 27
  of common refrigerants, 303
Critical temperature ratio, 353
Curved passages, flow through, 370
Cycle, definition of, 2, 65, 151
Cycles, air refrigeration, 292
  binary-vapor, 262
  Carnot power, 158, 169
    efficiency of, 162
  Carnot refrigeration, 291
    coefficient of performance of, 173
  ideal regenerative, 256
  non-flow, definition of, 151
  power, definition of, 153
  Rankine, 152, 248
  refrigeration, definition of, 164
  reheating, 254
  reversible, definition of, 157
  steady-flow, definition of, 151
  vapor compression refrigeration, 298

Dalton's Law, 397, 402
Degree of saturation, 416
Degrees of superheat, definition of, 229
Dehumidifying, 423
De Laval nozzle, 84
Density, definition of, 10
  of a constituent of a mixture, 399
  of a mixture of gases, 399
  of air in unsaturated air, 404
  of water vapor in unsaturated air, 404

Detonation, 274
Dew point, definition of, 410
  determination of, 411
Diagrams, Mollier, 227
  pressure-volume, 55
  temperature-entropy, 219
    for a perfect gas, 220
    for steam, 223
Diaphragms, 369
Diesel engine, 275
  determination of air-standard efficiency for, 277
Diffuser, 321
Discharge coefficient, definition of, 384
  for flow nozzles, 387
  for thin-plate orifices, 390
  for venturi meters, 385
Discharge region, 345
Dissociation, 268, 271
Divergence, angle of, 355
Double-acting compressor, 322
Dry air, 397
Dry-bulb temperature, 414
Dry compression, 300
Dry ice, manufacture of, 316
Dual-combustion engine, 275
  determination of air-standard efficiency for, 279

Economizer, 266
Efficiency, definition of, actual over-all, 267
    combined boiler and furnace, 267
    compression, 339
    compressor, 340
    engine, 253
    equivalent compression, 339
    furnace, 267
    mechanical, 339
    nozzle, 358
    stage, 375
    thermal, 153
    turbine, 252, 267
    volumetric, 335
  thermal, determination of, for air-standard Diesel cycle, 277
    for air-standard dual-combustion cycle, 279
    for air-standard gas turbine cycle, 283
    for air-standard Otto cycle, 272
Efficiency, thermal, determination of, for binary-vapor cycle, 263
    for Carnot power cycle, 162
    for internal-combustion engine, 285
    for Rankine cycle, 249
    for regenerative cycle, 259
    for reheating cycle, 255
    for reversible engine, 180
Ejector, steam jet, 304
Electrical work, 37
Electrolux refrigerating machine, 315
Ellenwood, F. O., 361
Energy, conservation of, 61
  definition of, 39
  internal, definition of, 63
  of an elevated fluid in motion, 75
  units of, 39
Energy equation, non-flow, 67
  steady-flow, 76–81
Engine efficiency, 253
Engineering thermodynamics, objectives of, 1-3
Entering section of a nozzle, 345, 355
Enthalpy, definition of, 80
  determination of, by means of tables, 104
  of a mixture of perfect gases, 401
  of a perfect gas, 99
  of water at low temperatures, 406
  of water vapor at low pressures, 405
Entropy, 197
  definition of, 207
  evaluation of, by means of tables, 212
  of a perfect gas, 208
  units of, 208
  uses of, 230
Entropy Principle, 230, 252, 297, 301, 376
Equality of temperature, 10
Equations of state, 19
  for liquids and vapors, 26
  for perfect gases, 22
Equivalent compression efficiency, 339
Erosion of turbine blades, 251
Evaporation, 8
Excess air, 266
Expansion ratio, 277

Fahrenheit perfect gas scale, 189
Fahrenheit scale, 19
Fan, 321
  power ideally required by, 341

Feedwater heaters, 258
Feedwater heating, regenerative, 258
First Law, statements of, 61, 67
Flange taps, 389
Flash chamber, 306
Flow coefficient, 392
Flow-diagram, for absorption refrigerating machine, 312
  for air refrigerating machine, 293
  for binary-vapor power plant, 262
  for dry-ice-manufacturing plant, 317
  for Electrolux refrigerating machine, 315
  for gas turbine power plant, 281
  for Linde liquid-air machine, 319
  for Rankine cycle, 153
  for regenerative cycle, 259, 260
  for reheating cycle, 255
  for two-stage compressor, 330
  for vapor compression refrigeration cycle, 298, 307–310
Flow nozzle, 386
Flow process, 36
Flow through curved passages, 370
Flow through nozzles, actual, 358
  characteristics of, 345, 355
  isentropic, 347, 356
Flow-work, 76
Four-cycle Diesel engine, 276
Four-cycle Otto engine, 269
Freezing, 8
Freon, 303
Friction, effect of, on air refrigerating machine, 297
    on vapor compression refrigerating machine, 301
  in a turbine, 252
Fuel consumption, specific, 286
Furnace efficiency, 267

Gage pressure, 9
Gas, perfect, 18, 88, 91
    enthalpy of, 99
    entropy of, 208
    equation of state for, 22
    internal energy of, 91, 94
    mixtures of, 397, 401, 430
    specific heats of, 92
    temperature-entropy diagram for, 220
Gas constant, 22
Gas constant, of a mixture, 430
  table of values, 23
  universal, 23, 24
Gas thermometers, 12
Gas turbine, determination of air-standard efficiency for, 283
Gas turbine power plant, 280
Generator, absorption refrigerating machine, 312
Gerry, H. T., 26
Governing, of Diesel engines, 277
  of Otto engines, 270
Gravimetric analysis, conversion of, to volumetric, 432
  of a mixture of perfect gases, 432

Heat, added during a non-flow process, 115
  added during a constant pressure non-flow process, 117
  added during a constant volume non-flow process, 115
  added during an isothermal non-flow process, 120
  added during a polytropic non-flow process, 133
  added during a steady-flow process, 137
  conversion of, into work, 49
  definition of, 37
  dependence of, on process, 197
  methods of transferring, 37
  of combustion, 265
  of vaporization, 104
  sign convention for, 38
  units of, 38
Heat engine, definition of, 153
  ideal fluid for, 247
  maximum possible efficiency of, 157, 178
  reversible, 178
  thermal efficiency of, 153
Heat pump, 302
Heat reservoir, definition of, 41
Heating and humidifying, 422
Heating at constant specific humidity, 422
Heating value of fuel, definition of, 265
Higher heating value, 265
Hirshfeld, C. F., 361
Hottel, H. C., 271

Humidifying, 422
Humidity, relative, 415
  specific, 408
Humidity ratio, definition of, 408
Hydraulic equation, 383, 395
Hydrogen, specific heats of, 95

Ideal centrifugal compressor, power required by, 341
Ideal coefficient of performance, for absorption refrigeration cycle, 313
  for air refrigeration cycle, 294
  for vapor compression refrigeration cycle, 299
  for various refrigerants, 304
Ideal fluid for a power plant, 247
Ideal gas, *see* Perfect gas
Ideal multistage compressor, power required by, 332
Ideal regenerative cycle, 256
Ideal single-stage compressor, 322
  power required by, 325
Ideal thermal efficiency, for air-standard Diesel cycle, 277
  for air-standard dual-combustion cycle, 279
  for air-standard gas turbine cycle, 283
  for air-standard Otto cycle, 272
  for binary-vapor cycle, 263
  for Rankine cycle, 249
  for regenerative cycle, 259
  for reheating cycle, 255
Ideal volumetric efficiency, definition of, 336
Impact pressure, definition of, 395
Impact tube, 393
Impulse turbines, multistage, 364
  single-stage, 361
Indicated horsepower, determination of, 338
Indicator diagrams, 268
  for four-cycle Diesel engine, 276
  for four-cycle Otto engine, 269
  for ideal single-stage compressor, 323
    compared with actual diagram, 324
  for two-stage compressor, 331
Integral of dQ/T, 197–207
Integral taps, 389
Intercooler pressure, optimum, 333
Intercooling, perfect, 332
Internal-combustion engine and steam power plant compared, 286
Internal energy, a property, 63
  definition of, 63
  determination of, by means of tables, 109
    experimental, 68
  existence of, 62
  of mixtures of perfect gases, 401
  of perfect gases, 91, 94
  of water at low temperatures, 406
  of water vapor at low pressures, 406
Internally reversible process, definition of, 41
International temperature scale, 195
Irreversibility in a turbine, 252
Isentropic flow, through converging passages, 381
  through nozzles, 347, 356
Isentropic processes, analysis of, 217
  definition of, 218
Isothermal reversible processes, analysis of, 120, 215

Joule's experiment, 90
Joule's Law, 88

Keenan, J. H., 26
Keller, Allen, 368
Kelvin-Planck statement of Second Law, 177
Kelvin scale, 19
Keyes, F. G., 26
Kinetic pressure, definition of, 392

Leaving loss, 363
Limiting temperatures for heat engines, 243
Linde liquid-air machine, 318
Liquid air, production of, 318
Liquid-in-glass thermometer, 13
Liquid line on a T-s diagram, 225
Losses in a steam power plant, 264
Lubrication of compressors, 301, 305
Lyons, J. M., 368

Maxwell relations, 212
Mean effective pressure, 338
Mechanical efficiency of a compressor, 339
Mechanical equivalent of heat, 40

Mechanical work, 35
  definition of, 36
Melting, 8
Mercury, temperature-entropy diagram for, 248
Mercury-in-glass thermometer, 13
Mercury-steam power plant, 262
Metallurgical limit, 244, 288
Mixing of streams, adiabatic, 424
Mixtures of air and water vapor, 404
Mixtures of perfect gases, c_v and c_p for, 433
  density of, 399
  determination of k for, 435
  enthalpy of, 401
  gravimetric analysis of, 432
  internal energy of, 401
  p-v-T relation for, 430
  relations pertaining to, 430
  volumetric analysis of, 431
Mole, 23
Mollier diagram for steam, 227
Mouth of a nozzle, 345, 355
Multiple refrigerant system, 308
Multistage compressor, 329
  power ideally required by, 332
Multistage impulse turbine, 364, 368

Negative temperatures, 194
Newman, L. E., 368
Nitrogen, specific heats of, 95
Non-flow cycle, definition of, 151
Non-flow energy equation, 67
Non-flow processes, 114–135
  adiabatic reversible, 124, 217
  constant pressure, 117
  constant volume, 115
  definition of, 35
  isentropic, 217
  isothermal reversible, 120, 215
  polytropic, 131
Nozzle mouth, 345, 355
Nozzle throat, 355
Nozzles, 345
  actual flow through, 358
  angle of divergence of, 355
  characteristics of flow through, 345, 355
  converging, 345
  converging-diverging, 354
  critical-pressure ratio for, 352
Nozzles, efficiency of, 358
  flow, 386
  isentropic flow through, 347, 356
  supersaturation in, 360
  velocity of sound in, 346, 355

Open-type feedwater heater, 258
Optimum intercooler pressure, 333
Orifice taps, 389
Orifices, 388
Otto cycle, determination of air-standard efficiency for, 272
Otto engine, 268
  actual thermal efficiency of, 274
  analysis of, 270
Oxygen, specific heats of, 95

Partial pressure, 315, 398
Percentage humidity, 416
Percentage saturation, calculation of, 417
  definition of, 416
  relation to relative humidity, 416
Perfect gas, alternative definition of, 91
  definition of, 18, 88
  enthalpy of, 99
  entropy of, 208
  equation of state for, 22
  internal energy of, 91, 94
  relation between c_p, c_v, and R, 102
  specific heat of, at constant pressure, 92, 95
    at constant volume, 92, 95
  temperature-entropy diagram for, 220
Perfect gas mixtures, c_v and c_p for, 433
  density of, 399
  determination of k for, 435
  enthalpy of, 401
  gravimetric analysis of, 432
  internal energy of, 401
  p-v-T relation for, 430
  relations pertaining to, 430
  volumetric analysis of, 431
Perfect gas scale, and thermodynamic scale, 192
  definition of, 18, 188
Perfect intercooling, definition of, 332
Phase, 8
Piezometer ring, 380
Pipe taps, 389
Piston displacement, definition of, 322
  of two-stage compressor, 334

Pitot tube, 392
Planck, Max, 177
Plate taps, 389
Platinum resistance thermometer, 14
Polytropic process, analysis of, 131
  definition of, 132
  determination of n for, 134
Positive displacement blower, 321
Positive displacement compressor, 321
Pound-mole, definition of, 23
Power cycles, 153
  binary-vapor, 262
  Carnot, 158, 245, 250
    efficiency of, 162
  Rankine, 248
  regenerative, 256, 258
  reheating, 254
Power ideally required, by centrifugal compressor, 341
  by fan or blower, 341
  by multistage compressor, 332
  by single-stage compressor, 325
Pressure, definition of, 9
  impact, 395
  kinetic, 392
  mean effective, 338
  partial, 398
  static, 379
Pressure staging, 365, 367
  advantages of, 369
Pressure-volume diagrams, 55
Principle of the increase of entropy, 230, 252, 297, 301, 376
Process, definition of, 35
  internally reversible, definition of, 41
  isentropic, definition of, 218
  non-flow, adiabatic reversible, 124, 217
    analysis of, 114–135
    constant pressure, 117
    constant volume, 115
    definition of, 35
    isothermal, 120, 215
    polytropic, 131
  reversible, definition of, 40
    alternative definition of, 184
  steady-flow, adiabatic reversible, 139, 217
    analysis of, 137
    definition of, 71
  throttling, 143
Properties, relation among, 342
Property, definition of, 8
Psychrometric charts, 419
p-v-t relation, for liquids and vapors, 26
  for mixtures of perfect gases, 430
  for perfect gases, 21

Quality, definition of, 28
  determination of, by throttling calorimeter, 143

Radiation, 38
Radius taps, 389
Rankine cycle, 152, 248
  determination of ideal efficiency of, 249
  flow-diagram for, 153
Rankine scale, 19
Reaction turbines, 370
Reciprocating compressors, 321
  ideal single-stage, 322
  power ideally required by, 325, 332
Rectifier, 313
Refrigerants, 302
  comparison of, 304
  some properties of, 303
Refrigerating Data Book, 314
Refrigerating machines, 164
  absorption, 311
  air, 164, 292
  Carnot, 169, 291
    coefficient of performance of, 173
    impracticability of, 291
  cascade, 308
  Electrolux, 315
  heat-driven, 314
  maximum possible coefficient of performance of, 169
  reversible, 182
    coefficient of performance of, 181
  vapor compression, 164, 298, 305
  water-vapor, 304
  with two evaporators, 308, 310
Refrigeration cycles, *see* Refrigerating machines
Regenerative feedwater heating, 258
Regenerative power cycle, 256
  determination of ideal efficiency of, 259
  flow-diagrams for, 259, 260
  practical approach to, 258
Reheat factor, 375
Reheater, 254
Reheating cycle, 254

Reheating cycle, determination of ideal efficiency of, 255
  flow-diagram for, 255
Reheating-regenerative cycle, 261
Relative humidity, definition of, 415
  determination of, 417
  relation to percentage saturation, 416
Reversible cycle, definition of, 157
  efficiency of engine operating on, 157
Reversible engine, definition of, 178
Reversible processes, adiabatic, 124, 217
  and heat, 47
  and work, 41
  definition of, 40
    alternative definition of, 184
  isothermal, 120, 215
  time required for, 43, 48
Reversible refrigerating machine, 181
Rotary compressor, 321
Rotation loss, 375

Satterfield, C. N., 271
Saturated air, definition of, 404
Saturated liquid, definition of, 28
Saturated mixture, definition of, 404
Saturated vapor, definition of, 28
Saturated vapor line on a T-s diagram, 226
Saturation pressure of common refrigerants, 303
Saturation ratio, 416
Saturation temperature, definition of, 28
Scale of temperature, definition of, 11, 187
  Fahrenheit, 189
  international, 195
  perfect gas, 188
  thermodynamic, 189
Sealing strip, 374
Second Law, 176
Shaft horsepower, definition of, 286
Shaft-work, definition of, 77
Sharp-edged orifice, 388
Shroud ring, 374
Single-acting compressor, 322
Single-stage compressor, 322
  comparison of ideal and actual indicator diagrams for, 324
  power ideally required by, 325
Single-stage impulse turbine, 361
Single-stage reaction turbine, 372
Smith, L. B., 26
Specific fuel consumption, definition of, 286
Specific heat, at constant pressure, 57
  at constant volume, 57
  average, 56
  definition of, 56
  instantaneous, 56
Specific heats, of certain gases, 95
  of dry air, 94
  of mixtures of perfect gases, 433
  of perfect gases, 92
Specific humidity, definition of, 408
  determination of, 409, 410, 412, 417
  of saturated air, 409
Specific volume, definition of, 10
Square-edged orifice, 388
Stage efficiency, 375
Staging, pressure, 365, 367
  velocity, 365
State, 8
Static pressure, 379
Static pressure tube, 393
Steady-flow cycle, definition of, 151
  example of, 152
Steady-flow energy equation, 76–81
Steady-flow processes, adiabatic reversible, 139
  analysis of, 137
  definition of, 71
  examples of, 72
Steam, enthalpy of, 104
    at low pressures, 406
  entropy of, 212
  internal energy of, 109
    at low pressures, 406
  Mollier diagram for, 227
  specific volume of, 29
  temperature-entropy diagram for, 223
Steam jet ejector, 304
Steam power plants, and internal-co bustion engines compared, 286
  losses in, 264
Steam rate, definition of, 253
Steam Tables, 26, 29, 30, 31
Subcooled liquid, definition of, 28
Sublimation, 8
Superheat, degrees of, 229
Superheated vapor, definition of, 28
Supersaturation, 360

Surroundings, definition of, 7
System, definition of, 7

Taps, location of, 389
Temperature, absolute perfect gas scale, 14, 188
  absolute zero of, 194
  adiabatic saturation, 412
  Centigrade scale, 19
  definition of a scale of, 11, 187
  dry-bulb, 414
  equality of, 10
  Fahrenheit scale, 19, 189
  international scale, 195
  Kelvin scale, 19
  limiting, 243
  of a moving fluid, 380
  of a retaining wall, 244
  perfect gas scale, 18, 188
  Rankine scale, 19
  thermodynamic scale, 189
    relation to perfect gas scale, 192
  wet-bulb, 414
Temperature-entropy diagrams, 219
  for air refrigeration cycle, 293
  for air-standard Diesel cycle, 278
  for air-standard dual-combustion cycle, 278
  for air-standard gas turbine cycle, 282
  for air-standard Otto cycle, 275
  for binary-vapor cycle, 263
  for dry-ice-manufacturing plant, 317
  for ideal regenerative cycle, 257
  for Linde liquid-air machine, 319
  for mercury, 248
  for perfect gases, 220
  for Rankine cycle, 248
  for regenerative cycle, 259
  for reheating cycle, 255
  for steam, 223
  for vapor compression refrigeration cycle, 298, 300, 301, 306
Temperature scales, *see* Temperature
Thermal efficiency, actual over-all, 267
  definition of, 153
  of an internal-combustion engine, 285
Thermocouple, 14
Thermodynamic and perfect gas scales, 192
Thermodynamic properties, relation among, 342
Thermodynamic Properties of Steam, 26
Thermodynamic temperature scale, 189
Thermodynamics, definition of, 2
Thermometers, 11, 12, 187
  constant pressure gas, 12
  constant volume gas, 12
  liquid-in-glass, 13
  platinum resistance, 14
Thermometric fluid, 12
Thermometric property, 12, 187
Thin-plate orifice, 388
Throat, of a flow nozzle, 386
  of a nozzle, 355
  of a venturi meter, 384
Throttle valve, Otto engine, 270
Throttling calorimeter, 143
Throttling process, 143
Ton-day, definition of, 291
Ton of refrigeration, definition of, 290
Trap, 260
Triple point, 26, 28
Turbine, multistage impulse, 364
  reaction, 370
  single-stage impulse, 361
Turbine blades, erosion of, 251
Turbine efficiency, 252, 267
Two-cycle Diesel engine, 277
Two-cycle Otto engine, 270
Two-stage compressor, 330
  piston displacement of, 334

Universal gas constant, 23, 24
Unsaturated air, definition of, 404

Vacuum, 10
Vacuum pump, 251, 321
Vapor compression refrigeration cycle, 164, 298, 305
  determination of ideal coefficient of performance of, 299, 307, 308
  effect of friction on, 301
  volume of vapor compressed, 304
  with two expansion valves, 306, 308, 310
Velocity, determination of, from pitot tube measurements, 394
Velocity compounding, 365
Velocity diagram, for single-stage impulse turbine, 362
  for single-stage reaction turbine, 371
Velocity-of-approach factor, 390

Velocity of sound, 346, 354, 355
Velocity staging, 365
Vena contracta taps, 389
Venturi meters, 384
Volume fraction, definition of, 432
Volumetric analysis, conversion of, to gravimetric, 432
  of a mixture of perfect gases, 431
Volumetric efficiency, definition of, 335
  factors affecting, 336

Wales, L. B., 368
Water, enthalpy of, at low temperatures, 406
  internal energy of, at low temperatures, 406
Water vapor, enthalpy of, at low pressures, 405
  internal energy of, at low pressures, 406
Water-vapor refrigerating machine, 304
Wet-bulb and adiabatic saturation temperatures, 415
Wet-bulb temperature, 414
Wet compression, 292
Williams, G. C., 271
Work, definition of, 37
  during a non-flow process, 50, 114, 217
    adiabatic reversible, 124, 217
    constant pressure, 117
    isothermal, 120, 215
    polytropic, 133
  during a steady-flow process, 138
    adiabatic reversible, 139, 217
  electrical, definition of, 37
  mechanical, definition of, 36
  on a piston, 51
  required in a reversible process, 46
  sign convention for, 36
  to force a fluid into an apparatus,
  units of, 36

Zero-angle turbine, 362, 366